ECSE 6520: Detection and Estimation Theory

Homework 1

Due February 6th, 2014

Read Chapter 1 pages 1-12, Chapter 5 pages 101 - 130 excluding MVU Estimator. Use the definition of the sufficient statistics or the Fisher-Neyman Factorization Theorem to solve the following problems.

1. Prove the Neyman-Fisher Factorization Theorem for a continuous random variable.

2. Let X be a random variable with density $f_\theta(x)$, and let

$$Y = \mathcal{T}(X)$$

   be an invertible transformation. Suppose $T = \tau(Y)$ is a sufficient statistic for θ in f_Y. Show that

$$\tilde{\tau}(X) = \tau(T(X))$$

   is a sufficient statistic for θ in f_X.

3. Let X_i, $i = 1, ..., N$ be Bernoulli random variables. Compare the number of bits required to represent the sufficient statistics

$$t = \sum_{i=1}^{N} x_i$$

   with the number of bits required to code the sequence x_i, $i = 1, ..., N$.

4. Let $X = [x_1, x_2, ..., x_M]^T$ denote a random vector with X_i i.i.d. random variables. For each of the following cases, find the sufficient statistics for θ using the Fisher-Neyman factorization theorem and find the distribution of the sufficient statistics.

a. $X_i \sim \mathcal{N}(0, \theta)\ \theta > 0.$

b. $P[X_i = x] = \left(\begin{array}{c} N + x - 1 \\ \\ x \end{array} \right) (1-p)^N p^x;\ x = 0, 1, ...$

c. $P[X_i = x] = e^{-\lambda} \frac{\lambda^x}{x!}.$

d. $P[X_i \leq x] = \int_0^x [\frac{1}{\Gamma(\theta_2)} \theta_1^{\theta_2}] e^{-\frac{y}{\theta_1}} y^{\theta_2 - 1} dy\ 0 \leq x < \infty.$