ECSE 6520: Detection and Estimation Theory

Homework 3

Due February 20, 2014

Read Textbook pages 15-22, pages 101-122. Solve the following problems using Rao-Blackwell theorem.

1. Let $\mathbf{X} = [X_o, X_1, \ldots, X_{M-1}]^T$ denote a random sample of $U[0, \theta]$ random variables:

$$f_\theta(x_n) = \frac{1}{\theta} I_{[0,\theta]}(x_n).$$

   Find the minimum variance unbiased estimator of θ.

2. Let $\mathbf{X} = [X_o, X_1, \ldots, X_{M-1}]^T$ denote a random sample of scalar i.i.d random variables. For each of the cases, find the minimum variance unbiased estimator of θ and compute the variance of the estimate.

   a. $X_n : \mathcal{N}[\theta_1, 1]$

   b. $X_n : \mathcal{N}[0, \theta_2], \quad \theta_2 > 0$

   c. $X_n : \mathcal{N}[\theta_1, \theta_2], \quad \theta_2 > 0$

   d. $P[X_n = x] = \begin{pmatrix} N + x - 1 \\ x \end{pmatrix} (1-p)^N p^x, \qquad x = 0, 1, ...$

   e. $P[X_n = x] = e^{-\lambda} \frac{\lambda^x}{x!}, \qquad x = 0, 1, ...$

   f. $P[X_n \le x] = \int_0^x \left[1/\Gamma(\theta_2)\theta_1^{\theta_2}\right] e^{-y/\theta_1} y^{(\theta_2 - 1)} dy; \qquad 0 \le x \le \infty$