ECSE 6520: Detection and Estimation Theory

Homework 4

Due February 27th, 2014

Read Textbook Chp. 3, pages 27-62 and Chp. 4 pages 83-97.

1. Textbook Problem 3.2.

2. Textbook Problem 3.15.

3. Let $\mathbf{X} = [X_1, X_2, \ldots, X_N]^T$ denote a vector of N i.i.d. exponential random variables with unknown parameter θ.

$$f_\theta(x_n) = \frac{1}{\theta} e^{-x_n/\theta}, \qquad 0 \leq x_n \leq \infty, \qquad 0 < \theta < \infty.$$

   Compute the Fisher information matrix.

4. Let $\mathbf{Y} \sim \mathcal{N}[\mathbf{x}, \sigma^2\mathbf{I}]$ denote a normal vector with mean $\mathbf{x}$ and covariance $\sigma^2\mathbf{I}$. Write $\mathbf{x}$ as $\beta\mathbf{u}_x$, where β is the norm of $\mathbf{x}$ and $\mathbf{u}_x$ is a unit vector in direction of $\mathbf{x}$. Compute the Fisher information matrix of

   a. β when $\mathbf{u}_x$ is known (σ^2 can be known or unknown);

   b. $\mathbf{u}_x$ (β and σ^2 can be known or unknown);

   c. β and $\mathbf{u}_x$ (σ^2 can be known or unknown);

   d. σ^2 when β and $\mathbf{u}_x$ are known;

   e. σ^2 and β when $\mathbf{u}_x$ is known;

   f. σ^2 and $\mathbf{u}_x$ when β is known;

   g. σ^2, β, and $\mathbf{u}_x$.