WAVEFORM PRECONDITIONING FOR CLUTTER REJECTION IN MULTIPATH FOR SPARSE DISTRIBUTED APERTURES

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ABSTRACT

The idea of *preconditioning* transmit waveforms for optimal clutter rejection in radar imaging is presented. Waveform preconditioning involves determining a map on the space of transmit waveforms, and then applying this map to the waveforms before transmission. The work applies to systems with an arbitrary number of transmit- and receive-antenna elements, and makes no assumptions about the elements being co-located. Waveform preconditioning for clutter rejection achieves efficient use of power and computational resources by distributing power over a frequency band in an effective way and by eliminating clutter filtering in receive processing.

Index Terms— Clutter rejection, waveform preconditioning, distributed aperture, sparse aperture, waveform design, imaging

1. INTRODUCTION

In radar applications the scene (everything in the radar beam) is composed of three classes: objects of interest, objects which are not of interest, and (known) background. Objects of interest are referred to as *targets*, while those objects which are not of interest are referred to as *clutter*. Clutter rejection is an important task, as scattering from clutter can overpower scattering from targets, thus rendering the targets difficult to detect or image. In this paper we show how, by appropriate filtering of the transmit signals, we can design waveforms that suppress clutter in the scattered signal. This filter will be referred to as a *preconditioning operator*; it is applied to the transmit waveform prior to transmission.

Although our primary application is radar imaging, it should be clear that our physics-based approach, which we formulate in terms of Green's functions and second-order random fields, is applicable to pulse-echo imaging in general, *e.g.* ultrasound imaging, sonar imaging and microwave imaging.

We formulate the processing of radar data from an arbitrary distribution of transmit- and receive-antennas in a stochastic framework.

Our approach is motivated by communication theory: the radar transmit signal which illuminates the target may be considered as a means for establishing a communications channel between the target and the observer. In this language, the effect of a complex environment is considered to be part of this communications channel. The goal is to design a filter which, when applied to the transmit signal, results in a received signal that is shaped in a desired manner. In our case, the goal is to emphasize scattering from the targets and suppress scattering from clutter.

2. PRELIMINARY THEORY AND MODELING

We will consider a radar antenna consisting of transmitting elements and receiving elements, all positioned arbitrarily.

For simplicity, we assume that each element is an isotropic point antenna. We assume also that we have a reference clock which is common to all of the elements; this makes possible *coherent* data processing.

We allow the different elements to transmit different waveforms. Let denote the waveform which emanates from the element. The transmit waveforms are arranged in a *transmit vector* . Similarly, if the measured signal at the receive element is denoted by , then the data which is collected by the distributed antenna may be arranged in a *measurement vector* .

2.1. Scattering model

The ability to distinguish a target depends on how much its electromagnetic wave speed deviates from a known background . We denote this deviation ("the scene") by the reflectivity function .

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We denote by the *channel* or *scattering operator* which maps the transmit vector to the measurement vector [*i.e.*,]. An explicit relationship between the scene and the scattering operator can be derived in terms of the *Green's function* for the background medium. The Green's function is the response measured at position from an impulse δ at position . Its form depends on the specific wave propagation model; here we assume a scalar, linearized (distorted-wave-Born-approximated (DWBA) [1]) wave propagation model. In particular, let the transmit element be located at position , and the receive element be located at position . If we define a $\times$ matrix with matrix elements

$$\int \quad \tau' \; \partial \quad - \tau' \quad \tau' \tag{1}$$

then we can model the relationship between the scene, the transmit vector, and the measurement vector as

$$\int \quad - \tau \quad \tau \quad \tau \tag{2}$$

Once the map is known, can be recovered by any one of a number of imaging techniques.

2.2. Target and clutter

We split the scene into two parts: , where represents target and represents clutter. The real interest lies in recovering , while suppressing .

In our development we assume that and are realizations of second-order random fields with known first- and second-order statistics. The background may be defined in such a way that the first-order statistics of are zero. For simplicity we will assume that both the target and clutter processes have zero mean and known auto-correlation functions

$$\left[\quad \overline{\quad\quad} \right] \tag{3}$$

$$\left[\quad \overline{\quad\quad} \right] \tag{4}$$

Finally, we will assume that the fields and are statistically independent.

Thus Eq. (2) directly relates the statistics of the electromagnetic scattering received by a radar antenna to the statistics of the scattering reflectivity function. We assume that Eq. (2) is defined in the mean-square sense.

3. THE PRECONDITIONING OPERATOR

The preconditioning idea is to suppress by modifying the transmit waveforms such that they produce mostly scattering from . In this work we choose to reject clutter in the minimum-mean-square-error (MMSE) sense. Our goal is therefore to determine a linear operator which minimizes

$$- \tag{5}$$

The solution is

$$* \quad \dagger \; * \tag{6}$$

Here $\dagger$ denotes pseudo-inverse, and the operators and are defined by

$$\int \quad \tau \quad \tau \quad \tau$$
$$\int \tag{7}$$

This operator may be applied to any transmit vector to yield a new transmit vector. Loosely speaking, emphasizes the parts of residing the subspaces in which the signal-to-clutter ratio is relatively high.

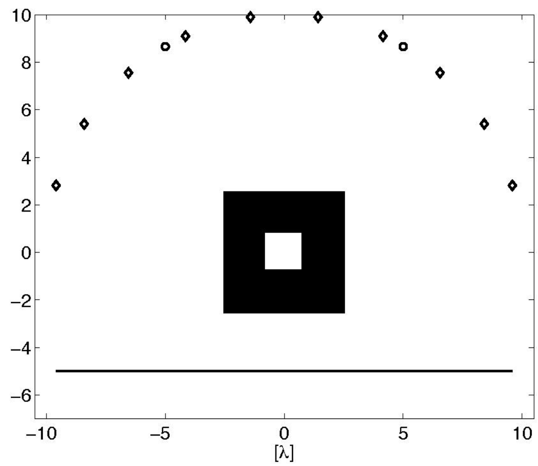

Fig. 1. Geometry for the numerical simulations.

4. NUMERICAL SIMULATION

We performed a set of numerical simulations for the case of two transmitting elements and ten receiving elements positioned on an arc around the target.

From the two transmitters we transmitted short chirp signals: transmitter 1 emitted a linear up-chirp, while transmitter 2 simultaneously emanated a linear down-chirp. All dimensions of the experiment were normalized according to the wavelength corresponding to the center frequency. We denote this wavelength by λ. In particular, the target was taken to be a square of side λ, and the transmitters and receivers were positioned on an arc of radius λ around the center of the target.

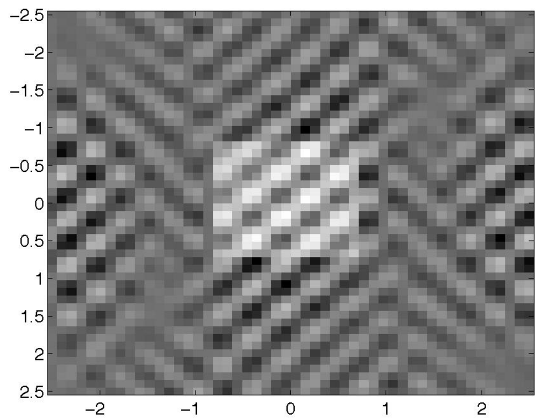

Fig. 2. Target with clutter.

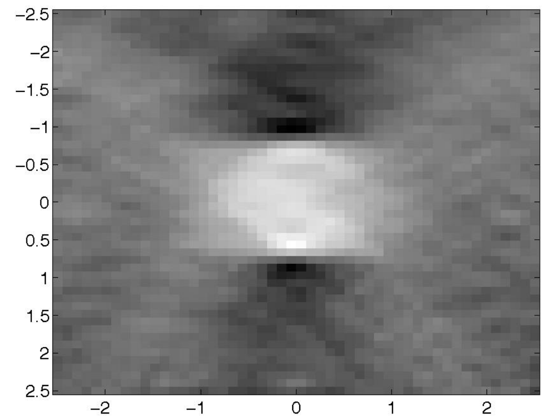

Fig. 3. Reconstruction from clutter-free scattering.

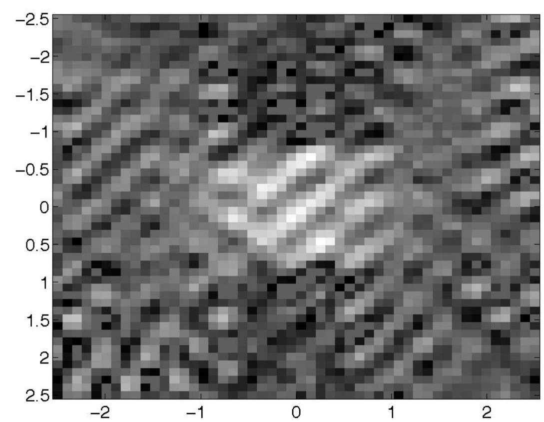

Fig. 4. Reconstruction of cluttered target from chirp.

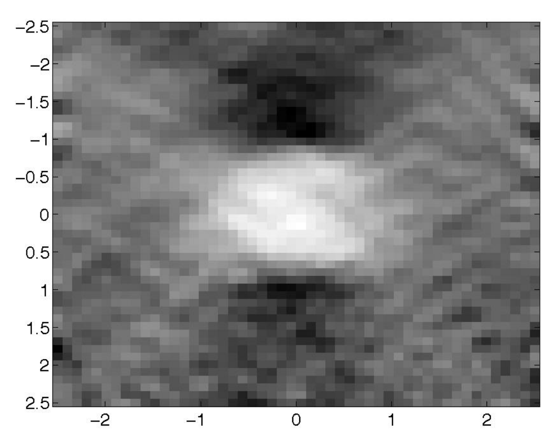

Fig. 5. Reconstruction from preconditioned chirp.

The target spectrum we constructed under the assumption that the target is a realization of a stationary random field. A high-frequency version of the stationary stochastic target model was then constructed and used to simulate different realizations of the surrounding clutter. This construction is explained further in Yazici *et al.*[2]. The compact support of the clutter was imposed by applying a spatial mask. For our purpose we used a clutter region of $\lambda \times \lambda$ around the target.

The preconditioning operator was constructed according to Eq. (6) by a Monte-Carlo approach where we used 500 realizations of the stochastic fields and . The spatial discretization for each scattering simulation was 15 samples per wavelength λ.

The signal-to-clutter ratio (SCR) in our simulations was set to -3.5 dB, when defined according to

$$\left(\frac{\int \; [\qquad]}{\int} \right) \tag{8}$$

The performance of the preconditioning was then evaluated by observing the square error in the reconstructed image when compared to the true scattering reflectivity function. The mean-square-error (MSE) was estimated by averaging over 10 clutter realizations.

To be consistent with the MMSE-criterion from which the preconditioning was constructed, we used an imaging reconstruction algorithm which is based on minimising the MSE of the final image[3].

5. DISCUSSION AND CONCLUDING REMARKS

The underlying propagation model which we have used for this work is derived from a scalar wave equation. This is a commonly-used model for many radar applications where polarization effects may be ignored. A propagation model based on a scalar Green's function is applicable in many other applications such as ultrasound, sonar and microwave

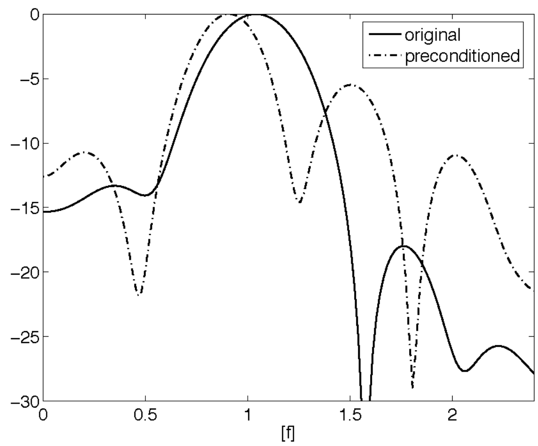

Fig. 6. Spectrum of transmit waveforms.

imaging. In situations where polarization is important, a similar framework can be developed using a dyadic Green's function.

In order to get explicit expressions in terms of Green's functions, we used a linearized scattering model, namely the distorted-wave Born approximation (DWBA). Note, however, that the minimization criterion (5) which we used to determine the preconditioning operator will make sense also without the DWBA. Thus this approach could be used to address similar issues with the full Maxwell equations as the propagation model, but in that case, the solution would not have the simple explicit form (**??**)

Multipath may be easily modeled by including multipath scattering in the background Green's function. When we perform clutter rejection we identify a transmit-vector subspace where the signal-to-clutter (SCR) ratio is high. Hence, for a fixed total transmit power, the SCR may be improved in the final image. Alternatively, for a given signal-to-noise ratio in the final image the total transmit power can be reduced.

Clutter filtering is an integral part of *space-time adaptive processing* (STAP). In STAP the statistics of the clutter is used to perform filtering of the measurement in order to reduce the clutter content[4]. Filtering of the measurement implies that parts of the scattered power will not contribute to the final result. In this sense it is wasted scattering power. We obtain a similar result by filtering the transmit signal, thereby avoiding transmitting power which will be predominantly used to produce scattering from clutter. In this sense, the preconditioned transmit vector will yield more efficient use of the transmitters. Furthermore, by avoiding clutter filtering of the measurements, we reduce the computational resources needed at each receiver element. Our scheme is therefore suited for applications where inexpensive receiver elements with limited computing power are employed, such as distributed sensing in urban environments.

The development in this paper was performed using the minimum-mean-square-error (MMSE) to define the optimal clutter rejection operator. This is a suitable in many applications. However, the idea can be extended to include other figures of merit, such as limited transmit power, limited bandwidth or absolute error. Although other figures of merit will change the optimal preconditioning operator, the same idea of preconditioning the transmit vectors can still be used.

6. REFERENCES

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