

**ECSE-2210 Microelectronics Technology**  
**Homework 4 – Solution**

1) a) The contact potential,  $V_{bi}$

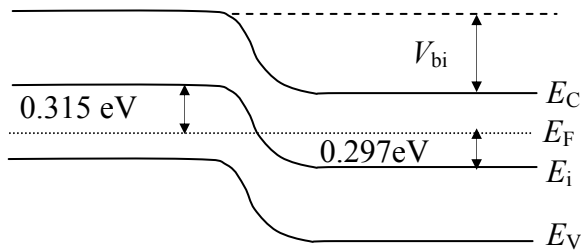


p-side:  $E_i - E_F = kT \ln\left(\frac{p}{n_i}\right) = 0.0258 \text{ eV} \ln\left(\frac{2 \times 10^{15}}{10^{10}}\right) = 0.315 \text{ eV}$

n-side:  $E_i - E_F = kT \ln\left(\frac{n}{n_i}\right) = 0.0258 \text{ eV} \ln\left(\frac{10^{15}}{10^{10}}\right) = 0.297 \text{ eV}$

$$V_{bi} = \frac{1}{q} [E_{i|p\text{-side}} - E_{i|n\text{-side}}] = \frac{1}{q} [E_F + 0.315 - (E_F - 0.297)] = 0.61 \text{ V}$$

Note:  $V_{bi} = \frac{kT}{q} \ln(n_n p_p / n_i^2) = 0.0258 \text{ V} \ln\left(\frac{10^{15} \times 2 \times 10^{15}}{10^{20}}\right) = 0.61 \text{ V}$



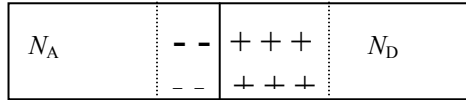
b) Total depletion layer width:  $W = \left[ \frac{2\epsilon V_{bi}}{q} \left( \frac{1}{N_A} - \frac{1}{N_D} \right) \right]^{1/2}$

$$W = [(2 \times 11.8 \times 8.85 \times 10^{-14} \text{ F/cm} \times 0.61 \text{ V}) / (1.6 \times 10^{-19} \text{ C}) \times \left( \frac{1}{2 \times 10^{15}} - \frac{1}{10^{15}} \right) \text{ cm}^3]^{1/2} = 1.09 \times 10^{-4} \text{ cm} = 1.09 \text{ } \mu\text{m}$$

see page 214 in the textbook:

$$x_p = W \times [N_D / (N_A + N_D)] = 0.09 \mu\text{m} \times 10^{15} / (2 \times 10^{15} + 10^{15}) = 0.36 \mu\text{m}$$

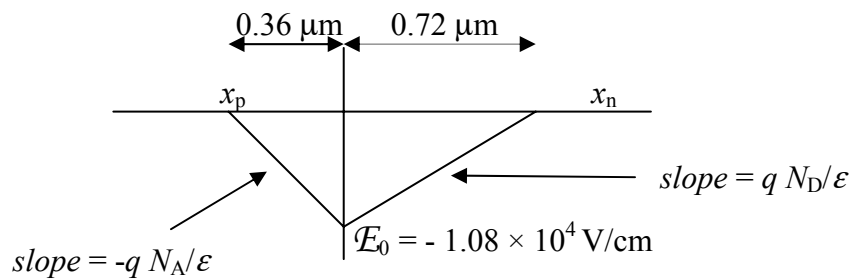
$$x_n = W \times [N_A / (N_A + N_D)] = 1.09 \mu\text{m} \times 2 \times 10^{15} / (2 \times 10^{15} + 10^{15}) = 0.72 \mu\text{m}$$



Note that,  $N_A x_p = N_D x_n$

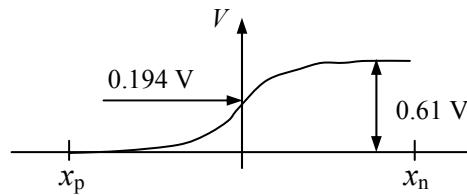
c) The maximum electric field occurs at the metallurgical junction.

$$\begin{aligned} \mathcal{E}_0 &= q/\epsilon N_D x_n = -q/\epsilon N_A x_p = \\ &= - [(1.6 \times 10^{-19} \text{ C}) / (11.8 \times 8.85 \times 10^{-14} \text{ F/cm})] \times 10^{15} \text{ cm}^{-3} \times 0.72 \times 10^{-4} \text{ cm} = \\ &= - 1.08 \times 10^4 \text{ V/cm} \end{aligned}$$

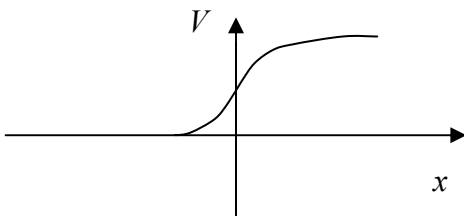
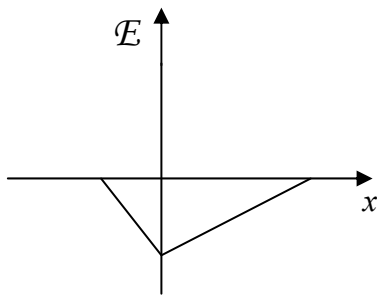
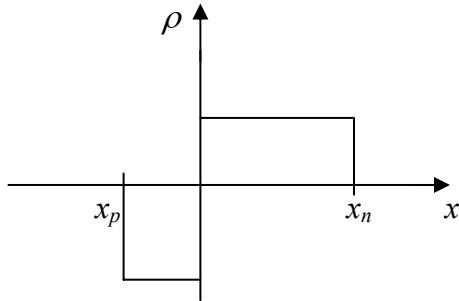


d) The potential at the metallurgical junction  $V(0)$ .

$$\begin{aligned} V(0) &= -\int \mathcal{E} dx = \mathcal{E}_0 \times 0.36 \mu\text{m} \times 0.5 = \\ &= 1.08 \times 10^4 \text{ V/cm} \times 0.36 \times 10^{-4} \text{ cm} = 0.194 \text{ V} \end{aligned}$$



e)



Note:

- Depletion layer extends further to lightly doped side.
- $\mathcal{E}$  field points in the negative  $x$  – direction everywhere.
- $\mathcal{E}$  – field varies linearly with  $x$  since doping constant everywhere:  $d\mathcal{E}/dx = |q N_A / \epsilon|$
- The potential varies as  $x^2$  since  $\mathcal{E}$ -field varies linearly with  $x$ ,  $V = -\int \mathcal{E} dx$

2. a)  $V_{bi} = (kT/q) \ln [(10^{17} \times 10^{15})/10^{20}] = 0.712 \text{ V}$   
Larger than before. Follows directly from band diagram.

b)  $W = [(2\epsilon V_{bi} / q) (1/N_A + 1/N_D)]^{1/2} = 0.95 \mu\text{m}$   
Narrower now, since the width on the p – side is very small.

$$x_p = 9.4 \times 10^{-3} \mu\text{m}$$

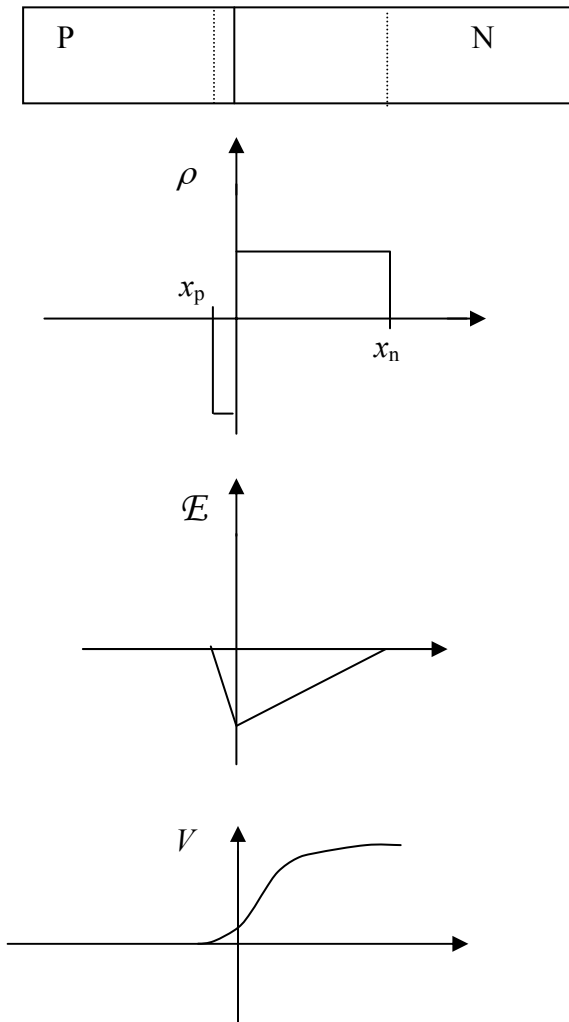
Much narrower depletion width on p-side since  $N_A \gg N_D$

$$x_n = 0.94 \mu\text{m}$$

$$\begin{aligned} \text{c) } \mathcal{E}_0 &= q N_D x_n = \\ &= -(1.6 \times 10^{-19} \text{ C}) / (11.8 \times 8.85 \times 10^{-14} \text{ F/cm}) \times 10^{15} \text{ cm}^{-3} \times 0.94 \times 10^{-4} \text{ cm} = \\ &= -1.44 \times 10^4 \text{ V/cm} \end{aligned}$$

$$\text{d) } V(x=0) = 1.44 \times 10^4 \text{ V/cm} \times 0.94 \times 10^{-4} \text{ cm} \times 0.5 = 6.7 \times 10^{-3} \text{ V}$$

e)

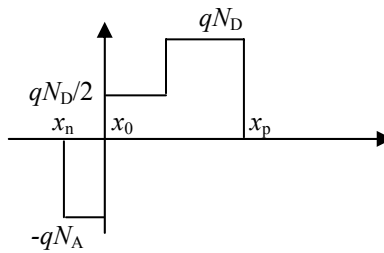


3. a) Refer to Eq. 5.8: The built-in voltage depends only on the majority carrier concentration at the edges of the depletion region.

Note (Eq. 5.9a,b):  $n(x_n) = N_D$  (since  $x_n > x_0$ ) and  $n(-x_p) = (n_i)^2/N_A$  (assuming the semiconductor is non-degenerately doped)

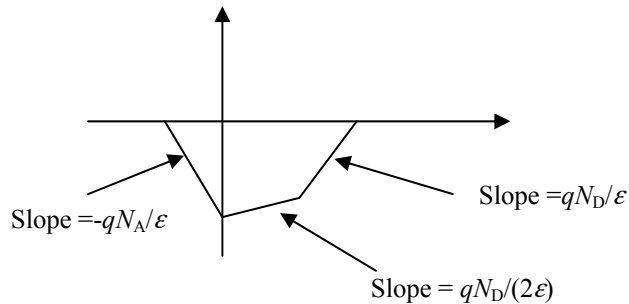
$$\Rightarrow V_{bi} = (kT/q) \ln (N_A N_D / n_i^2)$$

$$\begin{aligned} \text{b) } \rho \text{ (charge density)} &= 0 && \text{for } x < -x_p \text{ and } x_p > x_n \\ &= -q N_A && \text{for } -x_p < x < 0 \\ &= q N_D / 2 && \text{for } 0 < x < x_0 \\ &= q N_D && \text{for } x_0 < x < x_n \end{aligned}$$

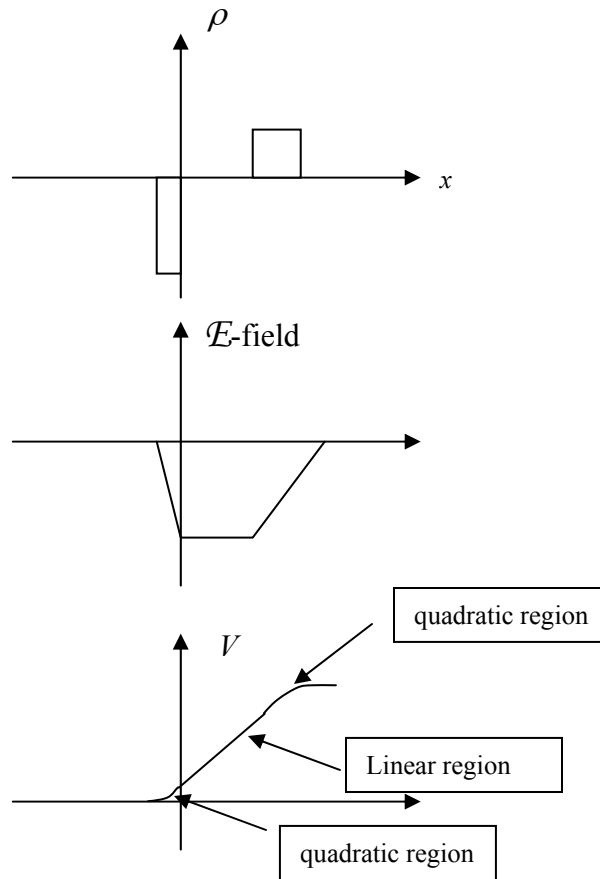


$$dE/dx = \rho / \epsilon \quad (\text{Poisson's equation})$$

$$\begin{aligned} dE/dx &= -qN_A/\epsilon && \text{for } -x_p < x < 0 \\ &= qN_D/(2\epsilon) && \text{for } 0 < x < x_0 \\ &= qN_D/\epsilon && \text{for } x_0 < x < x_n \\ &= 0 && \text{for } x < -x_p, x_p > x_n \end{aligned}$$



4. Charge density as a function of  $x$ :



Note: The banddiagram corresponds to the  $V$ -vs- $x$ . curve plotted an upside-down

b. The built in voltage will be equal to  $kT/q \ln [N_A N_D / n_i^2]$  where  $N_A$  and  $N_D$  are the dopant concentrations at the depletion region edges. This is the same reasoning as in problem 3 except that there is an undoped region.