ELECTRIC CIRCUITS
ECSE-2010-04
Fall 2001
Class 17

REVIEW

- **AC Steady State:**
  - **Input = x(t) =** $X\cos(w\,t + f_1)$
  - $\mathbf{y_{ss}(t)} =$ $Y\cos(w\,t + f_2)$
  - **Phasors; Express x(t) as $\underline{X}$:**
  - $\underline{X} = X_r + j\,X_i = X/\underline{f_1} = Xe^{jf_1}$
  - **Complex Math**
- **AC Steady State Impedances:**

$$Z_R = R;\quad Z_L = jwL;\quad Z_C = \frac{-j}{wC}$$

REVIEW

- **Time Domain:**
  - **v(t) =** $V\cos(w\,t + f)$
  - **KVL: Sum of v's around Mesh = 0**
  - **KCL: Sum of i's out of Node = 0**
  - $v_R = i_R\,R$; $i_C = C\,dv_C/dt$; $v_L = L\,di_L/dt$
- **Frequency Domain:**
  - $\underline{V} = V/\underline{f}$
  - **KVL: Sum of $\underline{V}$'s around Mesh = 0**
  - **KCL: Sum of $\underline{I}$'s out of Node = 0**
  - $\underline{V}_R = Z_R\,\underline{I}_R$; $\underline{V}_C = Z_C\,\underline{I}_C$; $\underline{V}_L = Z_L\,\underline{I}_L$

REVIEW

- **$\underline{V} = Z\,\underline{I}$; Ohm's Law for AC Steady State:**
  - **Z = R(w) + j X(w) = AC Steady State Impedance**
  - **R(w) = AC Steady State Resistance**
  - **X(w) = AC Steady State Reactance**
  - **Y = 1 / Z = G(w) + j B(w) = AC Admittance**
  - **G(w) = AC Steady State Conductance**
  - **B(w) = AC Steady State Susceptance**

FREQUENCY DEPENDENCE

- **Usually Interested in the Behavior of a Circuit as the <u>Frequency</u> of the Input is Varied:**
  - **Frequency Behavior of a Circuit**
- **Since Z = R(w) + j X(w) is Frequency Dependent:**
  - **Behavior of a Circuit in AC Steady State Varies Considerably with Frequency**

ACTIVITY 6-6

- **Treat w as a Variable:**
  - $Z_c = -j/w\,C = -j\,50/w$ **ohms**
  - $Z_L = jw\,L = jw\,1 = jw$ **ohms**
  - $Z_R = R = 10$ **ohms**
- **Find Z(w) = R(w) + j X(w):**

$$R(w) = \frac{10w^2}{100 + w^2}$$

$$X(w) = \frac{100w}{100 + w^2} - \frac{50}{w}$$

ACTIVITY 6-6

w	$R(w)$	$X(w)$	Comment
0 (DC)	0	$-\infty$	
1	$\approx .1$	≈ -49	Capacitive
10	5	0	Resistive
100	≈ 10	$\approx +.5$	Inductive
∞	10	0	

AC STEADY STATE CIRCUIT ANALYSIS

- **Express Currents and Voltages as Phasors:**
  - $v_1(t) \rightarrow \underline{V}_1$; $i_1(t) \rightarrow \underline{I}_1$; etc.
- **Express R, L, C as AC Impedances:**
  - $R \rightarrow Z_R = R$
  - $L \rightarrow Z_L = jw\,L$
  - $C \rightarrow Z_c = \frac{-j}{w\,C}$
- **Time Domain $\rightarrow$ Frequency Domain**

AC STEADY STATE CIRCUIT ANALYSIS

- **Use Ohm's Law for AC Steady State:**
  - $\underline{V} = Z\,\underline{I}$
  - Linear Relationship between $\underline{V}$ and $\underline{I}$
- **All Techniques from Unit I Can Now Be Used:**
  - No Differential Equations
  - All Math in Now Complex Math

AC STEADY STATE CIRCUIT ANALYSIS

- Node and Mesh Equations:

=> For Resistive Ckts: $[G]\ [V] = [I_s]$; Node Eq

$[R]\ [I] = [V_s]$; Mesh Eq

=> For AC Steady State: $[Y]\ [\underline{V}] = [\underline{I}_s]$; Node Eq

$[Z]\ [\underline{I}] = [\underline{V}_s]$; Mesh Eq

=> For Controlled Sources: Expand $[\underline{I}_s]$ and $[\underline{V}_s]$

e.g. $[\underline{I}_s] = \left[\tilde{\underline{I}}_s\right] + \left[\tilde{Y}\right][\underline{V}]$; $\left[Y - \tilde{Y}\right][V] = \left[\tilde{I}_s\right]$

PROBLEM 8

- **Frequency Dependence:**
  - w is a variable
- **Mesh Equations:**
  - 1 Unknown Mesh Current
- **Controlled Source:**
  - Will Need a Constraint Equation
  - Will Need to Expand [$\underline{V}_s$]

PROBLEM 8

Part a); Frequency Domain Diagram

Constraint Equation

See Diagram

Constraint Eq :

$\underline{V}_A = \frac{4}{jw}\underline{I}$; Relates $\underline{V}_A$ to Unknown Mesh Current, $\underline{I}$

PROBLEM 8

Part b); Mesh Equation

$[Z][\underline{I}] = [\underline{V}_s]$

$$\left(\frac{4}{jw} + jw2\right)\underline{I} = \underline{V} + jw2\left(\frac{\underline{V}_A}{8}\right)$$

Use Constraint Equation :

$$=> \left(\frac{4}{jw} + jw2 - 1\right)\underline{I} = \underline{V}$$

PROBLEM 8

$$Y = \frac{\underline{I}}{\underline{V}} = \frac{1}{\frac{4}{jw} + jw2\text{-}1} = \frac{jw}{4\text{-}2w^2\text{-}jw}$$

Rationalize: (multiply by $\frac{4\text{-}2w^2 + jw}{4\text{-}2w^2 + jw}$)

$$=> Y = \frac{-w^2}{(4\text{-}2w^2)^2 + w^2} + j\frac{4w - 2w^3}{(4\text{-}2w^2)^2 + w^2}$$

PROBLEM 8

$$G(w) = \text{Real}\,[Y] = \frac{-w^2}{(4\text{-}2w^2)^2 + w^2}$$

$$B(w) = \text{Imag}\,[Y] = \frac{4w - 2w^3}{(4\text{-}2w^2)^2 + w^2}$$

PROBLEM 8

Part c); Range of w for Capacitive Susceptance

Capacitive Susceptance: $B(w) > 0$

Need $w\text{-}2w^3 > 0 => w < \sqrt{2}$