

Course: Communication Systems  
ECSE 4520 - Section 01  
Fall Semester 2005

Electrical, Computer and Systems Engineering Department  
Rensselaer Polytechnic Institute

Homework 5

Matlab Problem <sup>1</sup>:

The message signal is defined as

$$m(t) = \begin{cases} 1 & , 0 \leq t < .05 \\ -2 & , .05 \leq t < .1 \\ 0 & , \text{otherwise} \end{cases}$$

This message DSB-SC AM modulates the carrier  $c(t) = \cos(2\pi \cdot 250 \cdot t)$ . We call the modulated signal  $u(t)$ .

1. Obtain the expression for  $u(t)$ .

$$u(t) = \left( \Pi\left(\frac{t - .025}{.05}\right) - 2\Pi\left(\frac{t - .075}{.05}\right) \right) \cos(500\pi t)$$

where  $\Pi$  is the square function.

2. Derive the spectra of  $m(t)$  and  $u(t)$ .

$$\begin{aligned} M(f) &= \frac{t_0}{3} e^{-j\pi f t_0/3} \text{sinc}\left(\frac{t_0 f}{3}\right) - \frac{2t_0}{3} e^{-j\pi f t_0} \text{sinc}\left(\frac{t_0 f}{3}\right) \\ &= \frac{t_0}{3} e^{-j\pi f t_0/3} \text{sinc}\left(\frac{t_0 f}{3}\right) \left(1 - 2e^{-j\pi f t_0/3}\right) \\ &= .05e^{-0.05j\pi f} \text{sinc}(0.05f) \left(1 - 2e^{-0.01j\pi f}\right) \end{aligned}$$

Therefore,

$$\begin{aligned} U(f) &= 0.025e^{-.05j\pi(f-250)} \text{sinc}(0.05(f-250))(1 - 2e^{-0.1j\pi(f-250)}) \\ &\quad + 0.025e^{-.05j\pi(f+250)} \text{sinc}(0.05(f+250))(1 - 2e^{-0.1j\pi(f+250)}) \end{aligned}$$

3. IN MATLAB (no points for doing it by hand) demodulate  $u(t)$  and recover  $m(t)$ . Plot the results in time AND frequency domain.

For expediency, we define the function `fftseq.m` with the following script:

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<sup>1</sup>From "Contemporary Communications Systems using Matlab" Second Edition, by Proakis, Salehi, and Bauch.

```

[M,m,df]=fftseq(m,ts,df)
% This function generates the FFT of m, denoted M.
% The frequency resolution is df, ts is the sampling interval.
fs=1/ts;
n1=fs/df;
n2=length(m);
n=2(max(nextpow2(n1),nextpow2(n2))); % This determines length of transform
M=fft(m,n);
m=[m,zeros(1,n-n2)]; % zero padding
df=fs/n; % output frequency resolution
return;

```

The main program may be written:

```

% Part 3 of Homework 5
t0=.15; % signal duration
ts = 1/1500; %sampling interval
fc=250; % carrier frequency
fs=1/ts; % sampling frequency
t=[0:ts:t0]; % time vector
df=.3; % the resolution for the frequency plot

%—
% message signal
%—
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];

%—
% modulated signal
%—
c=cos(2*pi*fc.*t); % carrier signal
u=m.*c; % modulated signal

%—
% at receiver
%—
y=u.*c; % mixing

```

```

%—
% spectral analysis
%—
[M,m,df1]=fftseq(m,ts,df); % Fourier TRansform of m
M=M/fs; % scaling
[U,u,df1]=fftseq(u,ts,df); % Fourier TRansform of u
U=U/fs; % scaling
[Y,y,df1]=fftseq(y,ts,df); % Fourier TRansform of y
Y=Y/fs; % scaling

%—
% design filter
%—
fcutoff=150; % cutoff frequency for the filter
ncutoff=floor(150/df1);
f=[0:df1:df1*(length(y)-1)]-fs/2;
H=zeros(size(f));
H(1:ncutoff)=2*ones(1,ncutoff);
H(length(f)-ncutoff+1:length(f))=2*ones(1,ncutoff);

%—
% filtering
%—
DEM=H.*Y; % output of filter in frequency domain
dem=real(ifft(DEM))*fs; % output in time domain

%—
% plots
%—
subplot(1,2,1),plot(t,m(1:length(t)))
xlabel('time')
ylabel('m(t)')
axis([0,.11,-2.3,2])
subplot(1,2,2),plot(t,u(1:length(t)))
xlabel('time')
ylabel('u(t)')
axis([0,.11,-2.3,2])
figure

```

```

subplot(1,3,1),plot(f,fftshift(abs(M)))
xlabel('frequency')
ylabel('M(f)')
subplot(1,3,2),plot(f,fftshift(abs(U)))
xlabel('frequency')
ylabel('U(f)')
subplot(1,3,3),plot(f,fftshift(abs(Y)))
xlabel('frequency')
ylabel('Y(f)')
figure
subplot(1,3,1),plot(f,fftshift(abs(H)))
xlabel('frequency')
ylabel('H(f)')
axis([0,1000,-.2,2.2])
subplot(1,3,2),plot(f,fftshift(abs(DEM)))
xlabel('frequency')
ylabel('Spectrum of demodulator Output')
subplot(1,3,3),plot(f,fftshift(abs(M)))
xlabel('frequency')
ylabel('M(f)')
figure
subplot(1,2,1),plot(t,m(1:length(t)))
xlabel('time')
ylabel('m(t)')
axis([0,.16,-2.5,1.5])
subplot(1,2,2),plot(t,dem(1:length(t)))
xlabel('time')
ylabel('demodulated signal in time')
axis([0,.16,-2.5,1.5])

```

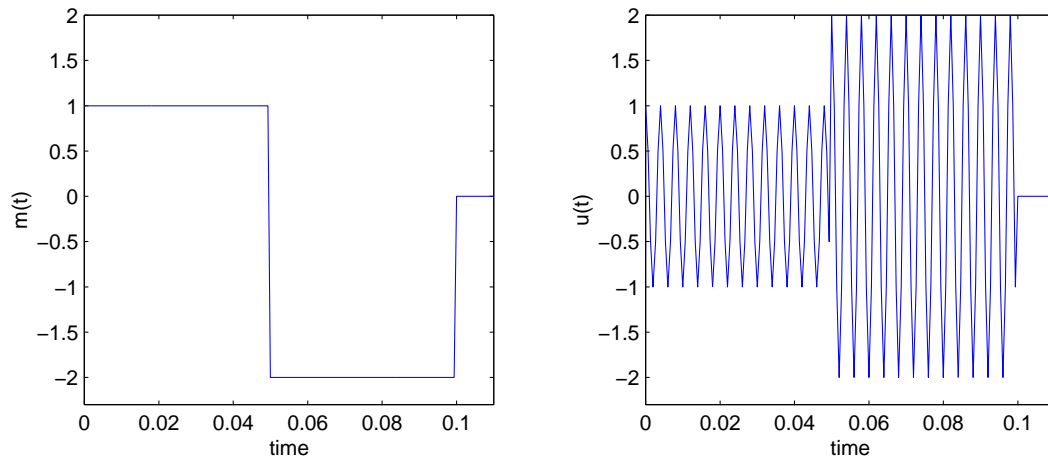


Figure 1: Message and modulated signal.

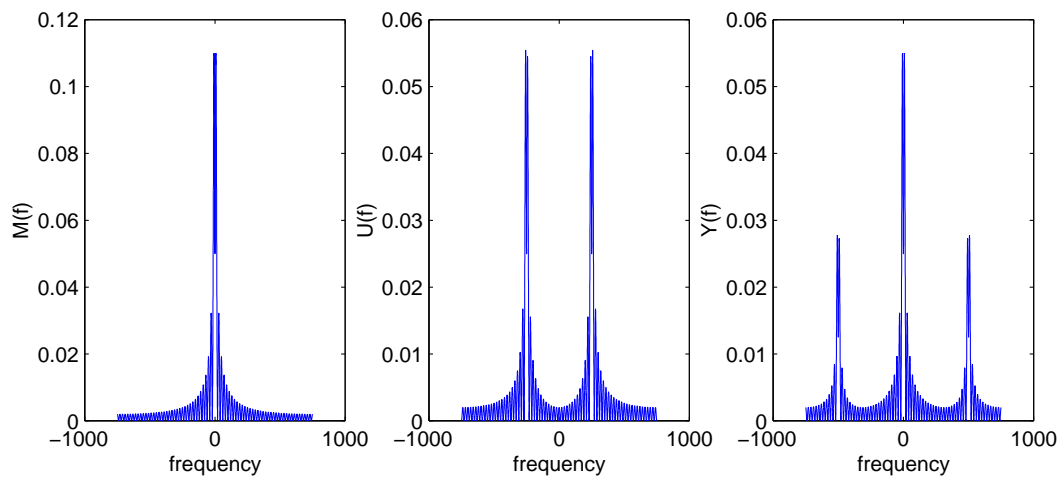


Figure 2: Message, modulated signal, and mixer output in frequency domain.

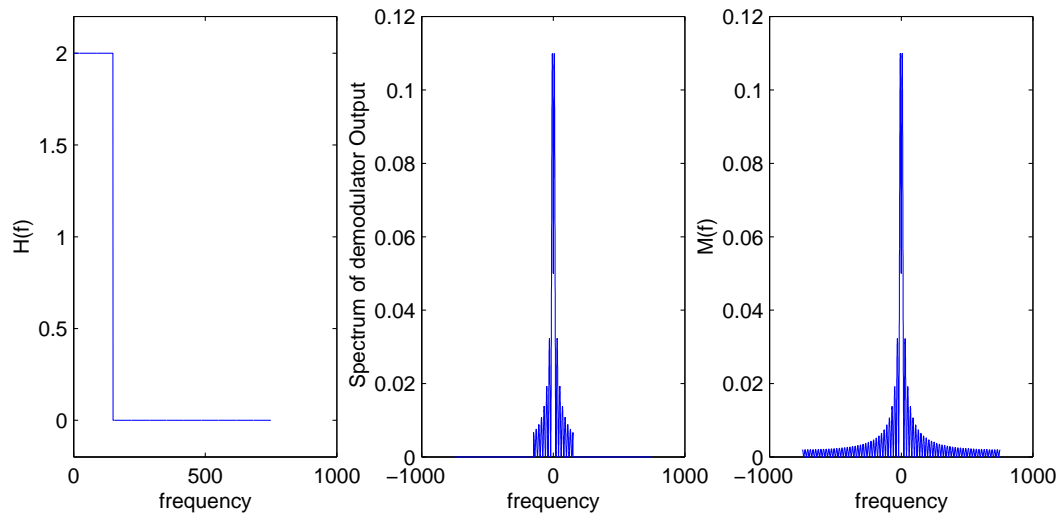


Figure 3: Filter, demodulated output, and message frequency domain.

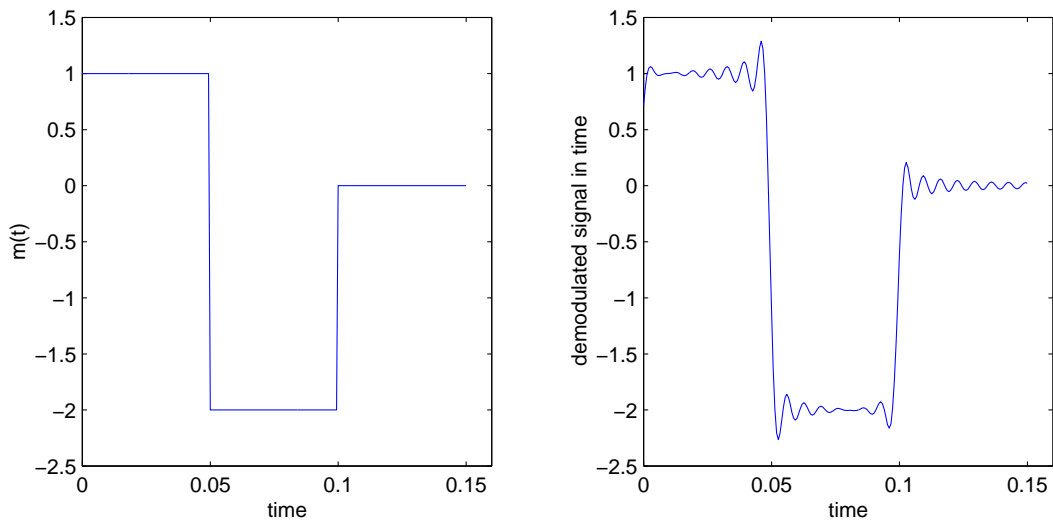


Figure 4: Original signal and demodulated version.