Queuing Network Models for Delay Analysis of Multihop Wireless Ad Hoc Networks

Nabhendra Bisnik and Alhussein Abouzeid
Rensselaer Polytechnic Institute
Troy, NY
bisnin@rpi.edu, abouzeid@ecse.rpi.edu

Important Questions

- Important questions:
  - How throughput scales with network size?
  - How delay scales with network size?
  - Relation between delay and throughput?
  - What are the tradeoffs?
- We developed queuing network models to analyze delay and throughput of multihop wireless ad hoc networks

Outline

- Introduction
- Queuing Network Model
- Main Results
- Simulation Results

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Delay in Multihop Wireless Networks

- End-to-end delay is sum of queuing and transmission delays at intermediate nodes
- Queuing delay depends on
  - ***Packet arrival process*** – how much traffic is handled by network?
  - ***Node density*** – how many interferers are there?
  - ***MAC protocol*** – how the channel is shared?
  - ***Traffic pattern*** – how many times a packet is transmitted before it reaches destination
- Modeling all the factors is quite challenging

Throughput in Multihop Wireless Networks

- *Maximum achievable per node throughput* of a network is the maximum rate at which the nodes of a network may generate traffic while keeping delay finite
- Maximum achievable throughput is inversely proportional to
  - Average time a node takes to serve a packet
  - Average number of flows served by a node

Related Work

- Gupta and Kumar "Capacity of Wireless Networks"
  - Under optimal scheduling, per node throughput scales as $\Theta(\frac{W}{\sqrt{n \log n}})$
- E.Gamal et al "Throughput Delay Trade-off in Wireless Networks"
  - $D(n) = \Theta(n\, T(n))$
  - Assuming that:
    - Packet size scales with throughput
    - Infinite backlog at source
    - Centralized and deterministic scheduling
  - Delay is proportional to number of hops traversed

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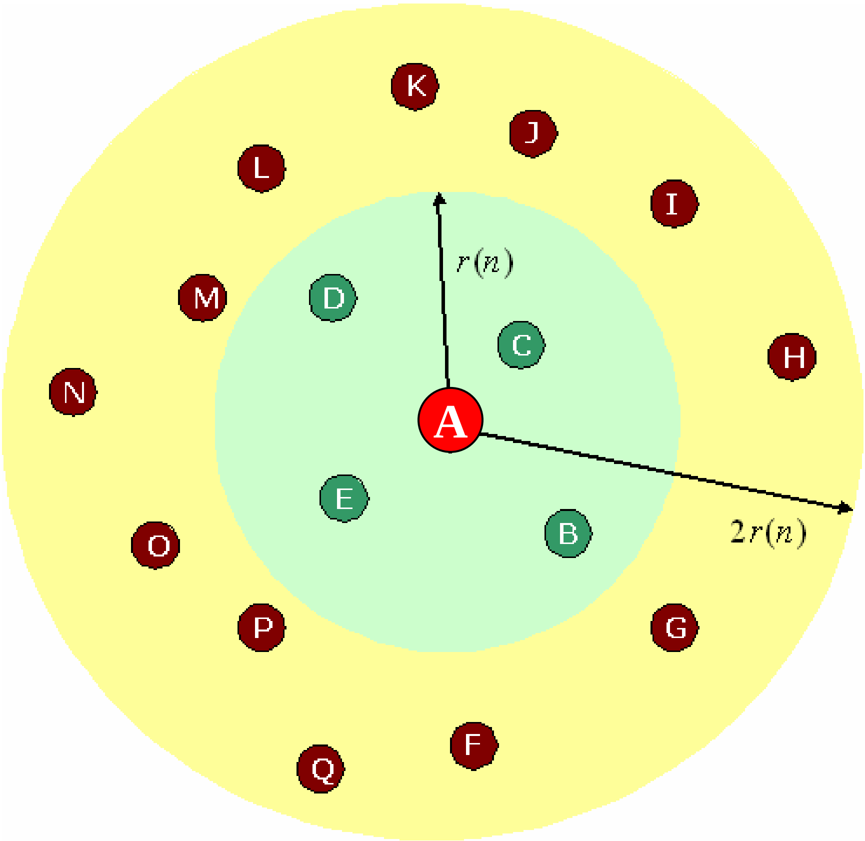
Network and Interference Model

- Network consists of n nodes that are distributed uniformly and independently distributed over a unit torus
- Transmission rate of each node = W bits/sec
- *Interference Model*: node i can successfully forward a packet to node j only if
  - $r_{ij} \leq r(n)$
  - $r_{jk} > r(n)$ $\forall$ nodes k transmitting simultaneously with i



- (green circle) : Neighbors of A – All nodes within distance *r(n)* of A
- (red circle) + (green circle) : Interfering neighbors of A – All nodes within distance 2r(n) of A

Transmission of A is guaranteed to be successful if none of the interfering neighbors of A transmit simultaneously

MAC Model

- Before transmitting a packet each node counts down a random timer
- The duration of the timer is exponentially distributed with mean $1/\xi$
- Once the timer of a node expires it starts transmitting and at the same instant the timers of all interfering nodes is frozen

The MAC model captures the collision avoidance mechanism of IEEE 802.11 and is still mathematically tractable

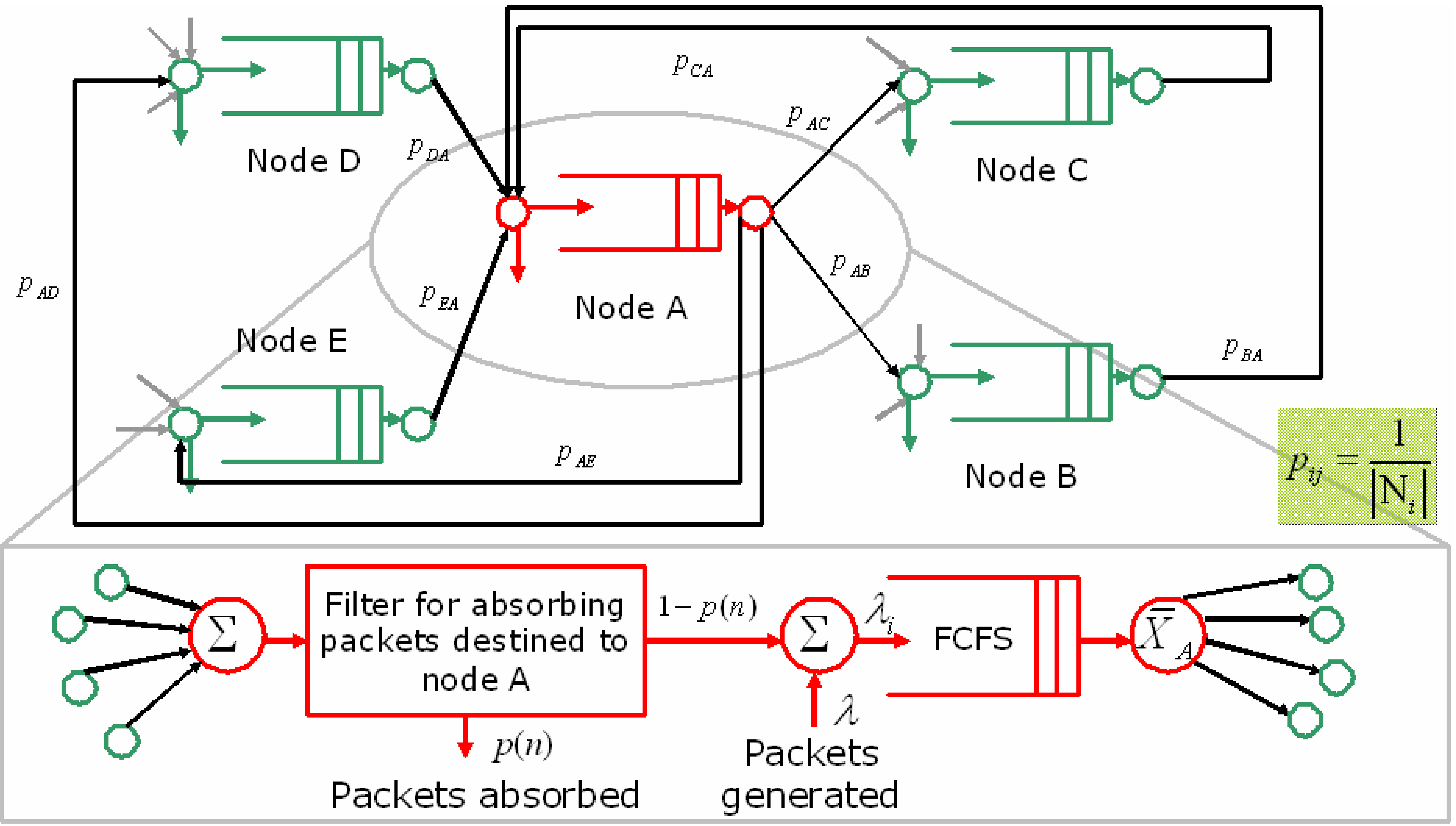
Traffic Model

- Each node is source, destination and relay of traffic
- Size of each packet is fixed and equals L bits
- Each node generates packets at rate λ packets/sec
- When a node receives a packet from its neighbor:
  - The packet is absorbed by the node with probability $p(n)$ (absorption probability)
  - The packet is forwarded to a randomly chosen neighbor with probability $1-p(n)$
- In other words, the fraction of packets received by a node that are destined to it equals $p(n)$

$p(n)$ characterizes the degree of locality of traffic – Low $p(n)$ average hops between a source destination pair is large

Queuing Network Model

- In order to characterize delay, ad hoc network modeled as G/G/1 queuing network
- Each node of the network is a station of queuing network
- Incorporate queuing delays at source and relay in delay analysis
- Diffusion approximation used to analyze the resulting queuing network



The queuing network

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Main Results

- Mean service time ($\overline{X_i}$)– Average time it takes for a node to serve a packet

$$\overline{X_i} = \frac{1/\xi + L/W}{1 - 4nA(n)\lambda_i(L/W)}$$

Service time in absence of interference

Term introduced by interfering neighbors

Where,

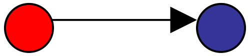
$A(n) = \pi r(n)^2$: Communication area of a node

$\lambda_i = \frac{\lambda}{p(n)}$: Overall arrival rate at a node

$\frac{L}{W}$: Time required to transmit a packet

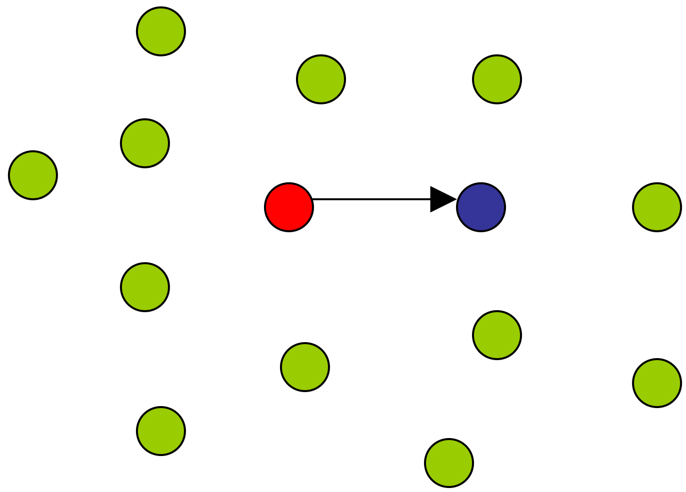
$\frac{1}{\xi}$: Mean backoff duration

Interpreting the Service Time Result



- Transmitter and receiver, in absence of interference
- Service time = Wait for timer to expire + transmission time = $1/\xi + L/W$

Interpreting the Service Time Result



- Now suppose there are *k* interferes, each with packet arrival rate α
- Fraction of time for which the channel is occupied by the interferers = $k\alpha L/W$
- The fraction of time the channel is available to the transmitter = $1 - k\alpha L/W$
- In our model $\alpha = \lambda_i$ and k = 4nA(n), therefore $\overline{X_i} = \frac{1/\xi + L/W}{1-4nA(n)\lambda_i(L/W)}$

Main Results

- Average end-to-end delay ($\overline{D}$) – Average time in which packet reaches the destination after being generated at source

$$\overline{D} = \frac{\rho}{\lambda(1-\hat{\rho})}$$

Where,

$\rho = \lambda_i \overline{X_i}$: Utilization factor of a node

$\hat{\rho} = \exp\left(-\frac{2(1-\rho)}{c_A^2 \rho - c_B^2} \right)$

c_A^2: Squared coefficient of variance (SCV) of inter-arrival time

c_B^2: SCV of service time

The value of end-to-end delay is governed by ρ and SCVs of service and inter-arrival times.

Main Results

- Maximum achievable throughput (λ_{max})

$$\lambda_{max} = \frac{p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}} \quad \text{or, } \lambda_{max} = o\left(\frac{W}{\overline{s}(4nA(n))L}\right)$$

Where,

$\overline{s}$: Average number of hops traversed by a packet between source and destination

As expected, MAT varies inversely with mean path length, node density and communication radius of nodes

Comparison with Kumar-Gupta Results

- When parameters of our model are comparable to that of Kumar-Gupta model i.e. $p(n) = \sqrt{\frac{\log(n)}{n}}$ and $A(n) = \frac{\log(n)}{n}$

$$\lambda_{max} = \frac{\frac{1}{4\pi}\frac{W}{\sqrt{n\log nL}}}{1+\frac{1/\xi+L/W}{4\pi\log nL/W}} \quad \text{or} \quad \lambda_{max} = o(\frac{W}{\sqrt{n\log n}})$$

The bound is similar to Gupta-Kumar bound but is not achievable. This is expected as channel capacity is wasted due to random access.

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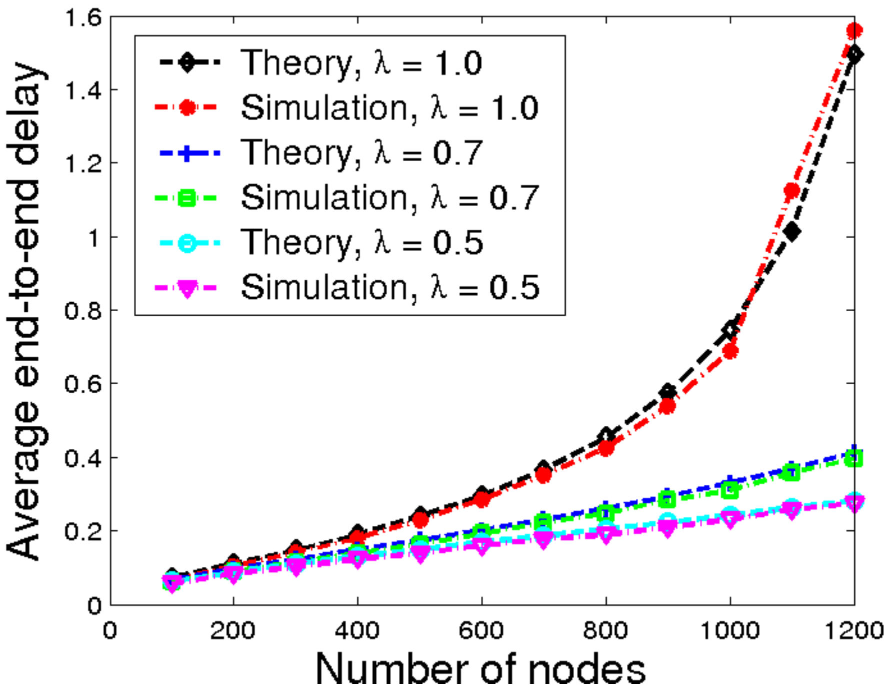
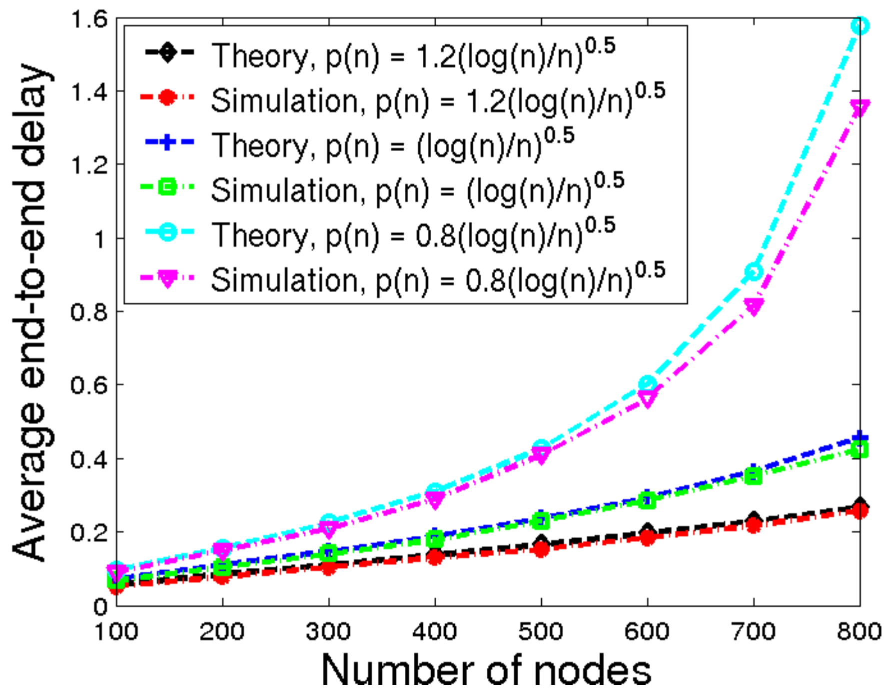
Simulation Results





Comparison of theoretical and simulation results

Conclusion and Future Work

- Developed queuing network models for multihop wireless ad hoc networks
- Used diffusion approximation to evaluate average delay and maximum achievable per-node throughput
- *Future work*: extend analysis to many to one cases, taking deterministic routing into account

Thanks!