Canonical Forms and Logic Miniminization

Today:

- **First Hour: Canonical Forms**
  - **Section 2.2.2 of Katz's Textbook**
  - **In-class Activity #1**
- **Second Hour: Incomplete Functions, Introduction to Logic Minimization**
  - **Section 2.2.4 and 2.2.1 of Katz's Textbook**
  - **In-class Activity #2**

minterms

#	A	B	C	minterms
0	0	0	0	A' B' C' = m_0
1	0	0	1	A' B' C = m_1
2	0	1	0	A' B C' = m_2
3	0	1	1	A' B C = m_3
4	1	0	0	A B' C' = m_4
5	1	0	1	A B' C = m_5
6	1	1	0	A B C' = m_6
7	1	1	1	A B C = m_7

Shorthand notation for minterms of 3 variables

SOP Shorthand

Shorthand notation for SOP expressions

F(A,B,C) = A' B C + A B' C' + A B' C + A B C'
+ A B C

= ∑ m(3,4,5,6,7)

F(A,B,C)' = A' B' C' + A' B' C + A' B C'

= ∑ m(0,1,2)

POS Shorthand

Shorthand notation for POS expressions

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

F(A.B.C) = (A + B +C)
• (A + B + C') (A + B' + C)

= Π M(0,1,2)

F(A,B,C)' = (A + B' +C')
• (A' + B + C) (A' + B +C')
• (A' + B' + C) (A' + B' + C')

= Π M(3,4,5,6,7)

Do Activity #1 Now

- **Reference: Section 2.2.2 of Katz's Textbook**
- **Minterms: (Shorthand)**
  - **m0, m1, … etc**
  - **∑ m(0,1,...)**
- **Minterms: (Canonical)**
  - **A'B'C', A'B'C … etc**
  - **A'B'C' + A'B'C+ … etc**
- **Maxterms: (Shorthand)**
  - **M0, M1, … etc**
  - **Π M(0,1,..)**
- **Maxterms: (Canonical)**
  - **(A'+B'+C'), (A'+B'+C), … etc**
  - **(A'+B'+C')• (A'+B'+C) • … etc**

Don't Cares & Canonical Forms

Shorthand Representation

Representation of "D" in BCD

D(A,B,C,D) = m_1 + m_3 + m_5 + m_7 + m_9
+ d_{11} + d_{13} + d_{15}

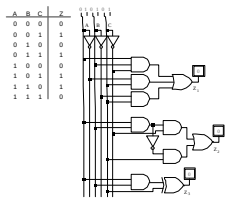
= ∑[m(1,3,5,7,9) + d(11,13,15)]

D(A,B,C,D) = M_0 • M_2 • M_4 • M_6 • M_8
• D_{10} • D_{12} • D_{14}

= Π[M(0,2,4,6,8) • D(10,12,14)]

Alternative Gate Realizations

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



Two-Level Realization (inverters don't count)

Multi-Level Realization Advantage: Reduced Gate Fan-ins

Complex Gate: XOR Advantage: Fewest Gates

Simplifying Equations

Example: full adder's carry out function

Cout = A' B Cin + A B' Cin + A B Cin' + A B Cin

Alternative

	A' B Cin + A B' Cin + A B Cin' + A B Cin	[3]
[9]	(A' + A) B Cin + A B' Cin + A B Cin' + A B Cin	[8]
	(1) B Cin + A B' Cin + A B Cin' + A B Cin	[5]
	B Cin + A B' Cin + A B Cin' + A B Cin	[1D] [3]
[9]	B Cin + A (B' + B) Cin + A B Cin' + A B Cin	[8]
	B Cin + A (1) Cin + A B Cin' + A B Cin	[5]
[9]	B Cin + A Cin + A B (Cin' + Cin)	[1D] [8]
	B Cin + A Cin + A B (1)	[5]
	B Cin + A Cin + A B	[1D]

NOTE: Minimization is just an exercise in applying laws of Boolean Algebra learnt earlier!

Do Activity #2 Now

Due: End of Class Today

RETAIN THE LAST PAGE (#3)!!

For Next Class:

- **Bring Randy Katz Textbook**
- **Required Reading:**
  - **Sec 2.2.1 & 2.3 (omit 2.3.6) of Katz**
- **This reading is necessary for getting points in the Studio Activity!**