Optical Coherence Tomography

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- **Optical imaging in turbid media**
- **Coherence and interferometry**
- **Optical coherence tomography**
- **Functional Optical Coherence Tomography**

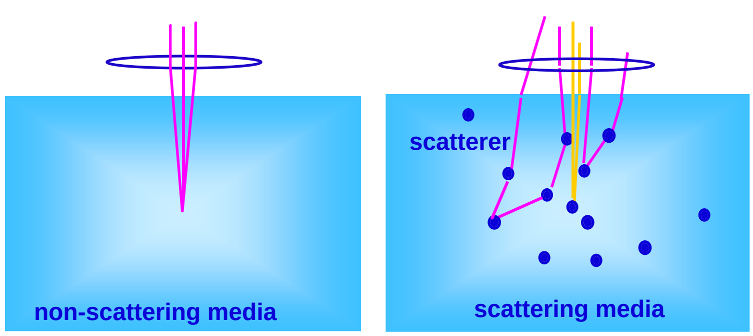
Hecht Chapter 7, 9, 12

Optical Tomographic Imaging of Tissue Structure and Physiology

Challenge: ***Scattering of photon destroy localization***

Mean free scattering path:

Skin tissue: $1/\mu_s$~ 50 µm **Blood: $1/\mu_s$**~ 8 µm

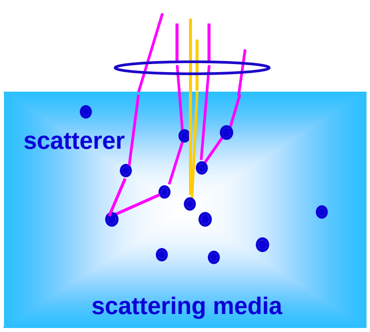


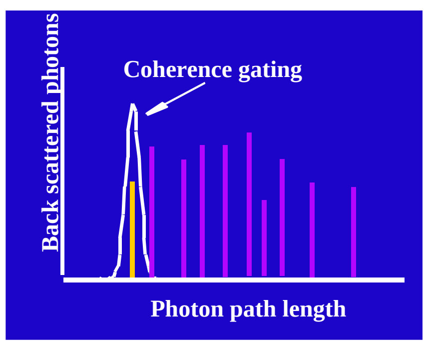
Optical Tomographic Imaging of Tissue Structure and Physiology

Technology:

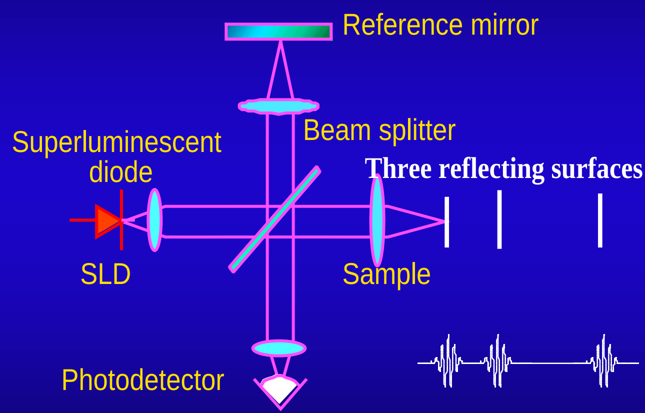
- *Time of flight (only ballistic photons or minimally scattered photons are selected)*
- *Photon migration (amplitude and phase of photon density wave are measured)*
- *Optical coherence tomography (coherence gating are used to select minimally scattered photons)*

Optical Coherence Tomography: Coherence Gating

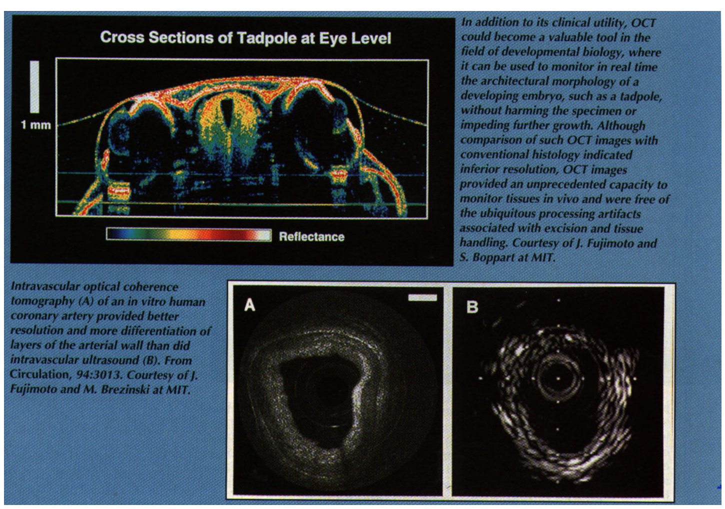




Optical Coherence Tomography



Optical Coherence Tomography



Interference of monochromatic light

Electromagnetic wave:

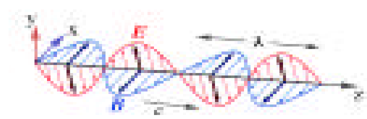
$E = A\cos(\omega t + \varphi)$ A: amplitude φ: phase

Interference: Superposition of waves

$E = E_1 + E_2 = A_1 \cos(\omega t + \varphi_1) + A_2 \cos(\omega t + \varphi_2)$

Phase difference:

$\varphi = \varphi_2 - \varphi_1$

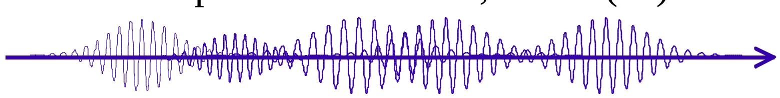


Detection of light waves:

$I \quad \langle E^2 \rangle$

$c = 3\times10^8$ m/s, $\nu = 5\times10^{14}$ Hz, $T = 2\times10^{-15}$ sec,

Detector response time ~10^{-9} s,-> $\langle \sin(\omega t) \rangle = 0$



Interference of monochromatic light

Detection of light waves:

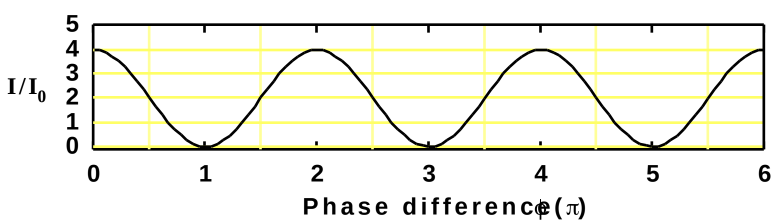
$$I \propto \langle E^2 \rangle = \langle (A_1 \cos(\omega t + \varphi_1) + A_2 \cos(\omega t + \varphi_2))^2 \rangle$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi) \qquad \varphi = \varphi_2 - \varphi_1$$

If $I_1 = I_2 = I_0$ $I = 2I_o(1 + \cos(\phi))$

In phase $\phi = 0, 2\pi, 4\pi, \ldots$ $I = 4I_o$

Out of phase $\phi = \pi, 3\pi, 5\pi \ldots$ $I = 0$



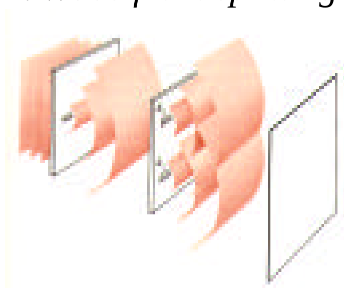
Coherent Sources

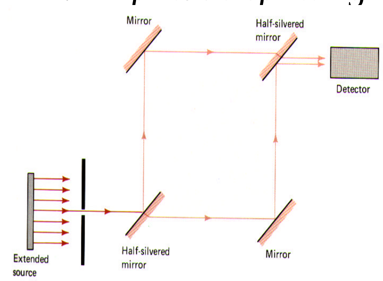
- ***Monochromatic***
- ***Definite and constant phase relation***

$E = E_1 + E_2 = A_1 \cos(\omega t+\varphi_1) + A_2 \cos(\omega t+\varphi_2)$

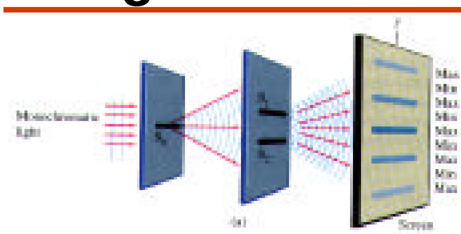
Methods to obtain two coherent sources:

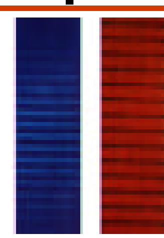
I. Wave front splitting *II. Amplitude splitting*

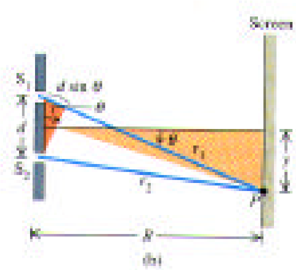




Young's Interference Experiment

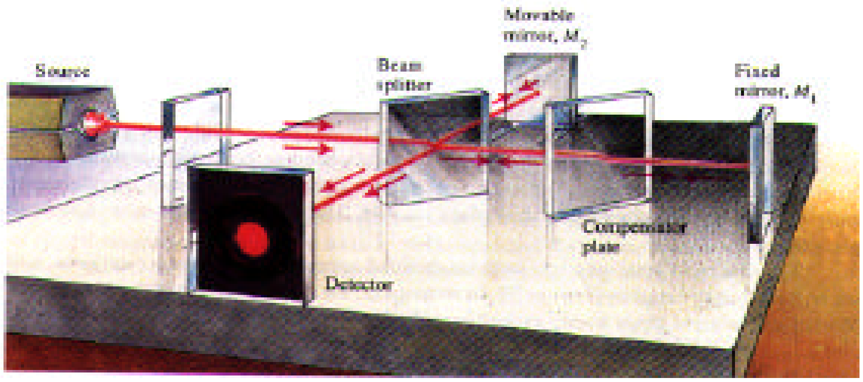




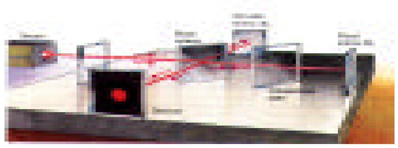


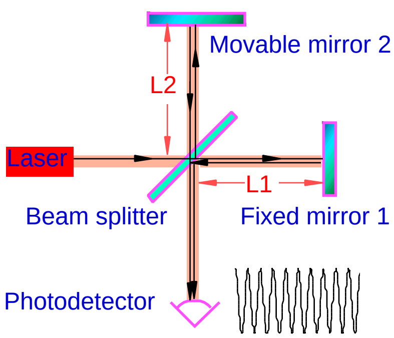
- **Optical path length difference:** $\Delta L = d\sin\theta$
- **Phase difference:** $\phi = 2\pi\Delta L/\lambda$
- **Constructive interference:**
  $2\pi d\sin\theta/\lambda = 2m\pi$ -> $\sin\theta_m = m\lambda/d$
  $m=0,1,2,.....$
- **Destructive interference:**
  $2\pi d\sin\theta/\lambda = (2m+1)\pi$ -> $\sin\theta_m = (m+1/2)\lambda/d$
  $m=0,1,2,.....$

Michelson interferometer



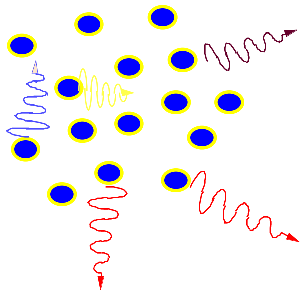
Michelson interferometer

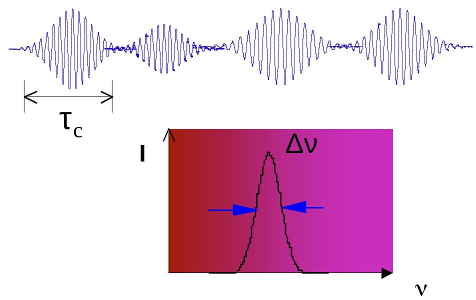




- **Optical path length difference:**
  $\Delta L = 2(L_2 - L_1)$
- **Phase difference:** $\phi = 2\pi\Delta L/\lambda$
- **Detected Light Intensity:**
  $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi)$
- **Constructive interference:**
  $2\pi\Delta L/\lambda = 2m\pi$
  $\Delta L = m\lambda$
  $m = 0, 1, 2, \ldots$
- **Destructive interference:**
  $2\pi\Delta L/\lambda = (2m+1)\pi$
  $\Delta L = (m+1/2)\lambda$
  $m = 0, 1, 2, 3, \ldots$

Photon sources





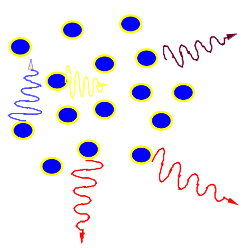
Atoms or molecules radiate wavetrains of finite length

- **More than one wavelength (spectral bandwidth)**
- **Fixed phase relation only within individual wavetrain**

Coherence

Correlation of light wave at two points in space-time:

$$\Gamma(\mathbf{r_1},\mathbf{t_1};\mathbf{r_2},\mathbf{t_2}) = <\mathbf{E}(\mathbf{r_1},\mathbf{t_1})\mathbf{E}(\mathbf{r_2},\mathbf{t_2})>$$

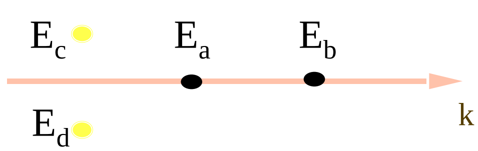


Temporal Coherence (longitudinal)

$$\Gamma=<\mathbf{E_a(t)E^*_b(t)}>$$

Spatial Coherence (lateral)

$$\Gamma=<\mathbf{E_c(t)E^*_d(t)}>$$

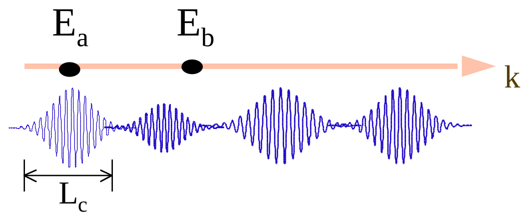


Temporal Coherence

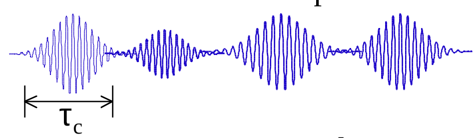
Correlation of light wave along the light propagation direction

$\Gamma = \langle E_a(t) E^*_b(t) \rangle$

$= \langle E(t+t_{ba}) E^*(t) \rangle$



Coherence length:
The length of the wavetrain where there is definite phase relation.



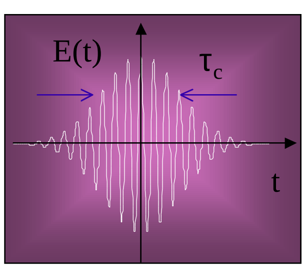
Coherence time:
The time for the elementary wavetrain to pass a single point

$L_c = c\,\tau_c$

Temporal Coherence

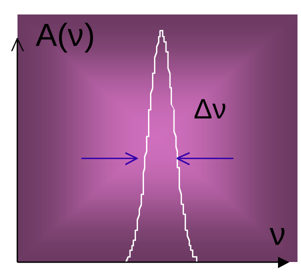
Temporal coherence is a measure of spectral bandwidth

A high (good) temporal coherence gives a narrow spectral bandwidth ("pure" light of single wavelength (color))



Fourier transform pair

$$A(\nu) = \int_{-\infty}^{\infty} E(t) e^{-i2\pi\nu t} dt$$

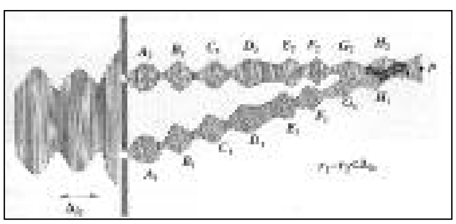


$\tau_c \quad 1/\Delta\nu$

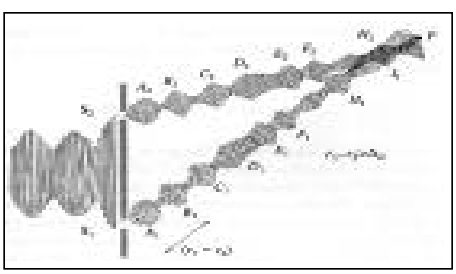
Coherence lengths of light sources

Source	Mean Wavelength $\bar{\lambda}$(nm)	Linewidth* $\Delta\lambda$ (nm)	Coherence Length Δl_c
Thermal IR (8000–12 000 nm)	10 000	≈4000	≈25 000 nm = $2.5\bar{\lambda}$
Mid-IR (3000–5000 nm)	4000	≈2000	≈8000 nm = $2\bar{\lambda}$
White light	550	≈300	≈900 nm = $1.6\bar{\lambda}$
Mercury arc	546.1	≈1.0	≤ 0.03 cm
Kr^{86} discharge lamp	605.6	1.2×10^{-3}	0.3 m
Stabilized He-Ne laser	632.8	$\approx 10^{-6}$	≤ 400 m
Special He-Ne laser	1153	8.9×10^{-11}	15×10^{6} m

The effect of finite coherence length



Path length difference
r_2-r_1 << L_c
same wavetrain overlap
Interference fringe observable



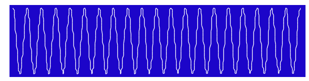
Path length difference
r_2-r_1 >> L_c
Different wavetrain overlap
No interference fringe observable

Partially Coherent Sources

Coherent source:

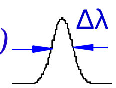
•Monochromatic: same wavelength

•Constant phase relation

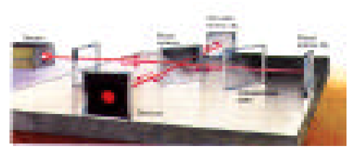


Incoherent source:

•Broad spectrum band $P(\nu)$

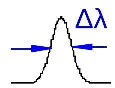


•Random Phase



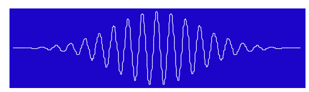
Partially coherent source:

•Broad spectrum band ($\Delta\lambda$=10~100 nm), $P(\nu)$

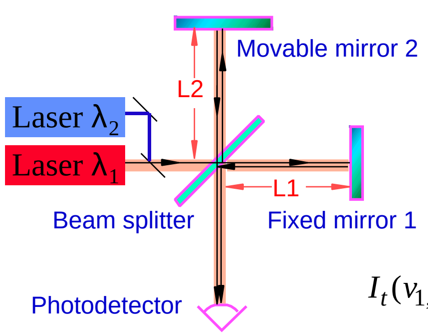


•Definite phase relation within coherence length L_c (2~15 μm)

- *If* $\Delta L < L_c$, *Interference observed*
- *If* $\Delta L >> L_c$, *Interference disappeared*



Interference with Partial Coherence Light Source



$$I_1(\nu_1)=2I_0(\nu_1)[1+\cos(2\pi\Delta L\nu_1)]$$

$$I_2(\nu_2)=2I_0(\nu_2)[1+\cos(2\pi\Delta L\nu_2)]$$

$$I_t(\nu_1,\nu_2)=\sum_1^2 I_i(\nu_i)$$

$$I_t(\nu_1,\nu_2)=2\sum_1^2 I_0(\nu_i)+2\sum_1^2 I_0(\nu_i)\cos(2\pi\Delta L\nu_i)$$

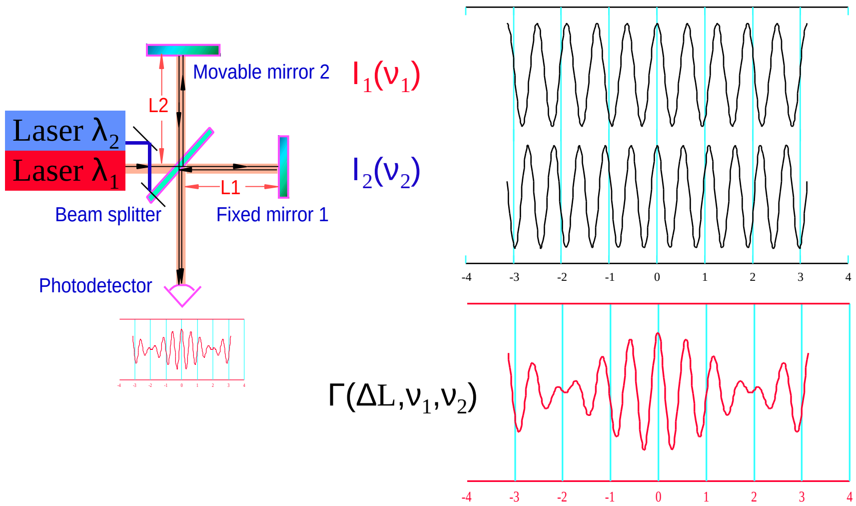
Phase change: $\phi=2\pi\Delta L\nu$

$\nu=1/\lambda$

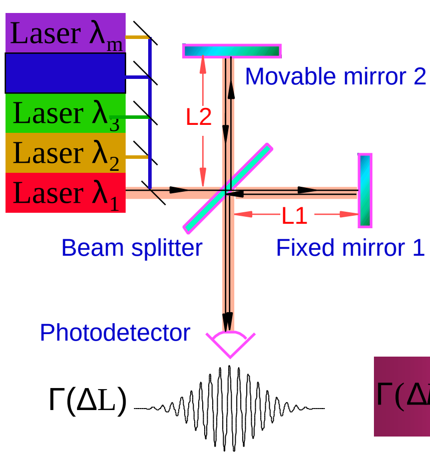
Interference terms Γ

$$\Gamma(\Delta L,\nu_1,\nu_2)=2\sum_1^2 I_0(\nu_i)\cos(2\pi\Delta L\nu_i)$$

Interference with two light sources of different frequency



Interference with Partial Coherence Light Source



$I_1(\nu_1)=2\ I_0(\nu_1)[1+\cos(2\pi\Delta L\nu_1)]$

$I_2(\nu_2)=2\ I_0(\nu_2)[1+\cos(2\pi\Delta L\nu_2)]$

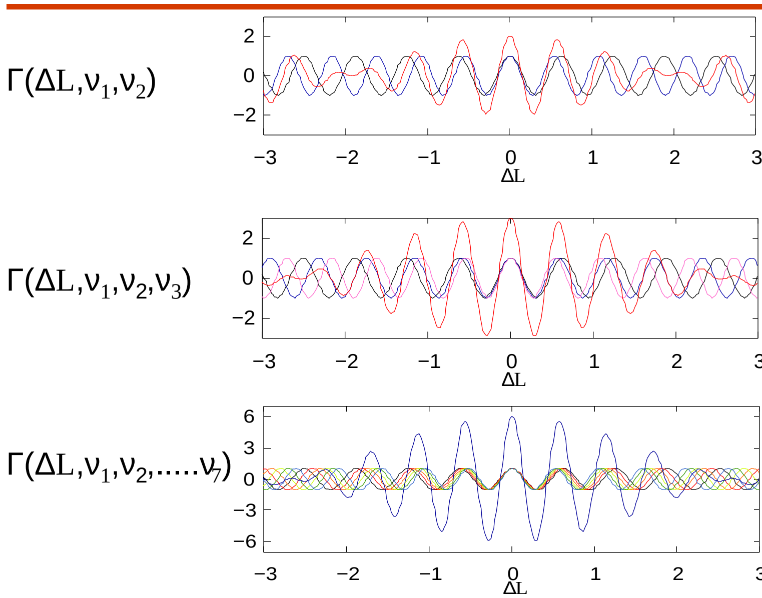
$I_3(\nu_3)=2\ I_0(\nu_3)[1+\cos(2\pi\Delta L\nu_3)]$

....

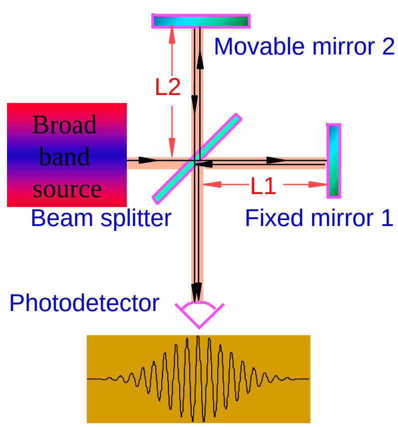
$I_m(\nu_2)=2\ I_0(\nu_2)[1+\cos(2\pi\Delta L\nu_m)]$

$$\Gamma(\Delta L,\nu_1,\nu_2..\nu_m)=2\sum_{1}^{m} I_0(\nu_i)\cos(2\pi\Delta L\nu_i)$$

Interference with Partial Coherence Light Source



Interference with partial coherence light source

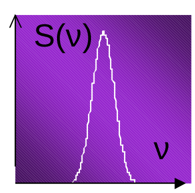


For discrete light with different wavelength

$$\Gamma(\Delta L, v_1, v_2 .. v_m) = 2\sum_{1}^{m} I_0(v_i)\cos(2\pi \Delta L v_i)$$

For continuous spectra with spectral density of S(ν):

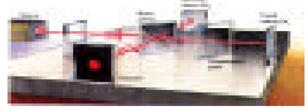
$$\Gamma(\Delta L) \approx 2 I_0 \int_{0}^{\infty} S(v)\cos(2\pi \Delta L v) dv$$



Interference of partially coherent light

Assuming the electrical fields from the partial coherent source light coupled into the interferometer is written as an harmonic superposition

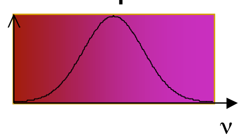
$$E(t) = \int_{-\infty}^{\infty} A(\nu) e^{2\pi i \nu t} d\nu$$



Where: E(t) is electrical field amplitude emitted by a low coherent light source; A(ν) is the corresponding spectral amplitude at optical frequency ν.

Because phase in each spectral component are random and independent, cross spectral density of A(ν) satisfies,

Source spectrum



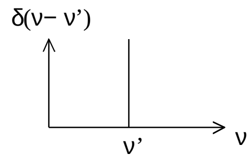
$$< A^*(\nu) A(\nu') >= S(\nu)\delta(\nu - \nu')$$

Where: $S_o(\nu)$ is the source power spectral density [W/Hz]; δ(ν− ν') is the Dirac delta function satisfying

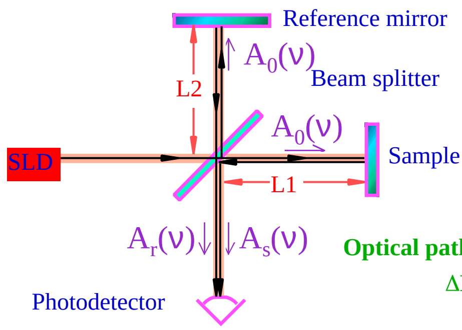
$$\delta(\nu - \nu') = 0 \quad if \quad \nu \neq \nu'$$

and

$$\int_{-\infty}^{\infty} f(\nu)\delta(\nu - \nu') d\nu = f(\nu')$$



Interference of partially coherent light



Optical path length difference:

$\Delta L = 2(L_2 - L_1)$

Assume light coupled equally into reference arm and sample arm with spectral amplitude of $A_o(\nu)$. The light coupled back to the detect from the sample and reference arm is given by:

$$A_r(\nu) = e^{i 2\pi\nu L_r} K_r A_o(\nu)$$

$$A_s(\nu) = e^{i 2\pi\nu L_s} K_s A_o(\nu)$$

Interference of partially coherent light

If the time delay (τ) between light in reference and sample paths is changed by translating the reference mirror, total power detected at the interferometer output is given by a time-average of the squared light amplitude

$$I_t(\tau) = \left\langle \left| E_r(t) + E_s(t) \right|^2 \right\rangle = I_r + I_s + \Gamma_{oct}(\Delta L)$$

$$\Gamma_{oct}(\Delta L) = 2\int_0^\infty K_r K_s S(\nu)\cos(2\pi\Delta L\nu)d\nu$$

Assuming that there is no spectral modulation in the reflectivity of both the sample and reference arms

$$\Gamma_{oct}(\Delta L) = 2K_r K_s \int_0^\infty S(\nu)\cos(2\pi\Delta L\nu)d\nu$$

If the source spectral distribution is a Gaussian function

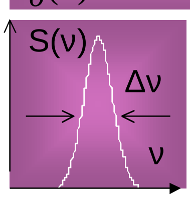
$$S_o(\nu) \quad e^{-4\ln 2\left(\frac{\nu-\nu_o}{\Delta\nu}\right)^2}$$

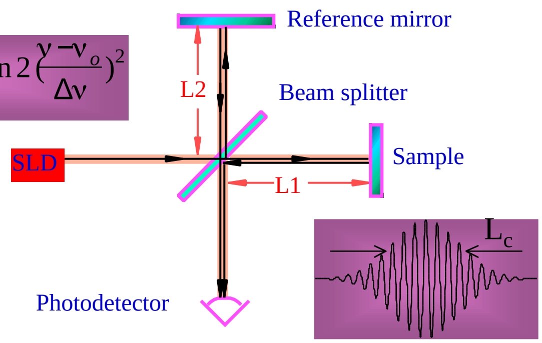
$$\Gamma_{oct}(\tau) \quad e^{-4\ln 2\left(\frac{\Delta L}{L_c}\right)^2} \cos(2\pi\Delta L\nu)$$

Where L_c is the coherence length of the partial coherence source given by

Optical Coherence Tomography

$$S_o(\nu) \quad e^{-4\ln 2\left(\frac{\nu-\nu_o}{\Delta\nu}\right)^2}$$





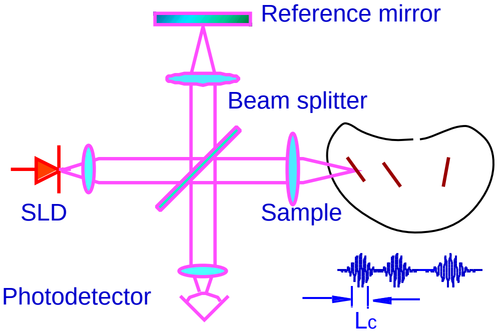
$L_c = 0.44\lambda^2/\Delta\lambda$

$$\Gamma_{oct}(\Delta L) \quad e^{-4\ln 2\left(\frac{\Delta L}{L_c}\right)^2} \cos(2\pi\Delta L\nu)$$

Interference fringes observed only when optical path lengths are matched within coherence length of the source

Optical Coherence Tomography

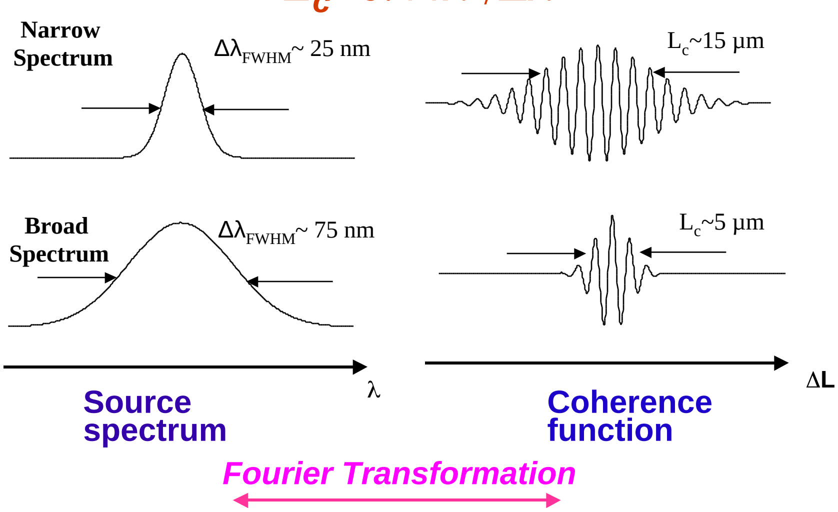
—— *Michelson interferometer with a broad band partially coherent source*



Axial spatial resolution: $L_c = 0.44\lambda^2/\Delta\lambda$

Coherent Length

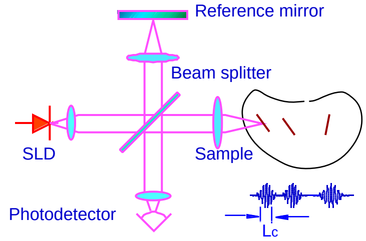
$$L_c = 0.44\lambda^2/\Delta\lambda$$

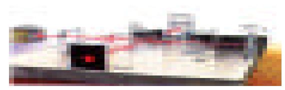


Optical Coherence Tomography

—— Michelson interferometer with a broad band partial coherent source

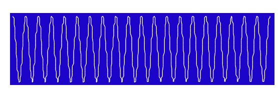
- ***Fringe amplitude proportional to backscattered light***
- ***Longitudinal (depth) resolution: L_c***
- ***Coherence length: L_c=0.44$\lambda^2/\Delta\lambda$, (2~15 μm)***
- ***Lateral resolution by focusing optics (1~10 μm)***
- ***Probing depth: $1/\mu'_s \sim 5/\mu'_s$***



Interference 

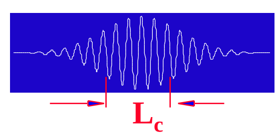
- ***Coherence sources***

$$I \propto \langle E^2 \rangle \quad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2\pi\Delta L/\lambda)$$



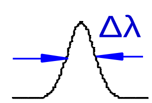
- ***Partially coherence sources***

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, e^{-4\ln 2\left(\frac{\Delta L}{L_c}\right)^2} \cos(2\pi\Delta L/\lambda)$$



- ***Source power spectrum***

$$P(\lambda) = e^{-4\ln 2\left(\frac{\lambda-\lambda_0}{\Delta\lambda}\right)^2}$$

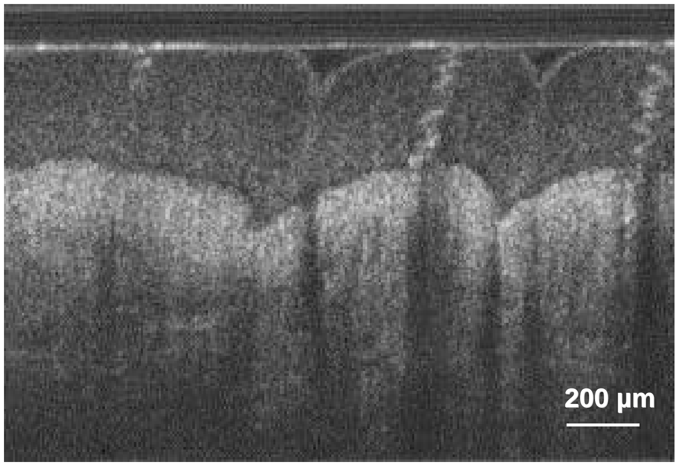


- ***Coherence function***

$$\Gamma(\Delta L) \quad e^{-4\ln 2\left(\frac{\Delta L}{L_c}\right)^2} \cos(2\pi\Delta L\nu)$$

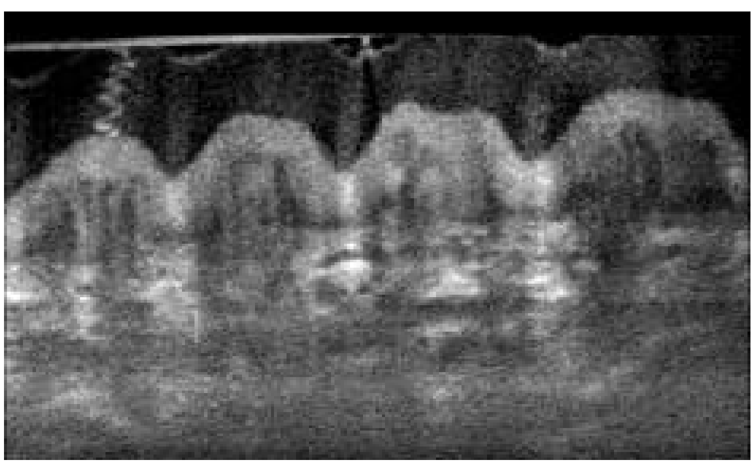
L_c=0.44$\lambda^2/\Delta\lambda$

Optical Biopsy



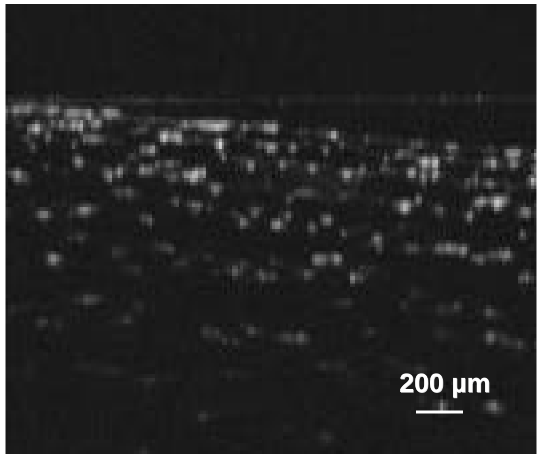
OCT *in vivo* image of a human hand

Optical biopsy: Speckle averaged OCT image



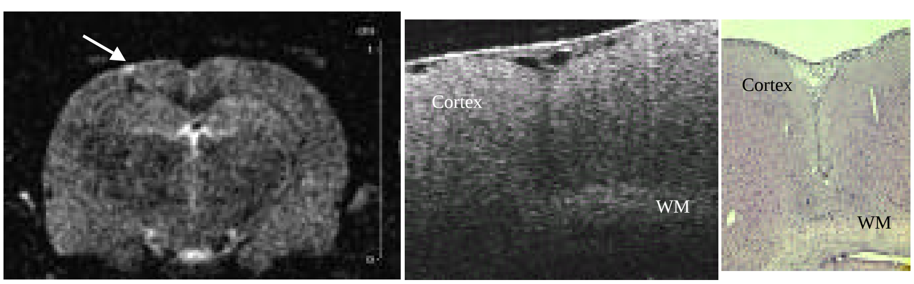
Xiang et. al.

High Resolution OCT



Y. Zhao, Z. Chen et al., SPIE, 2001

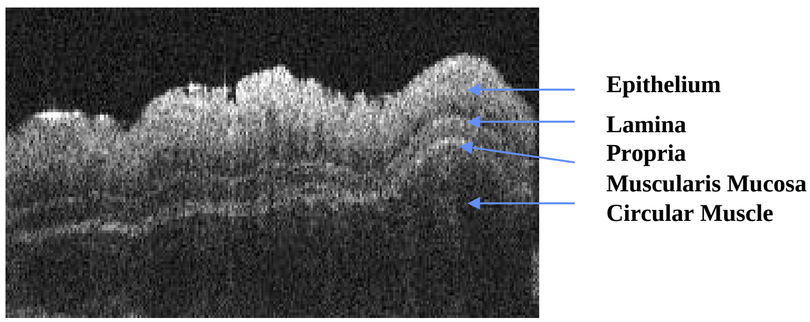
Visualization of neonatal freeze lesion
Investigating epilepsy in animal model



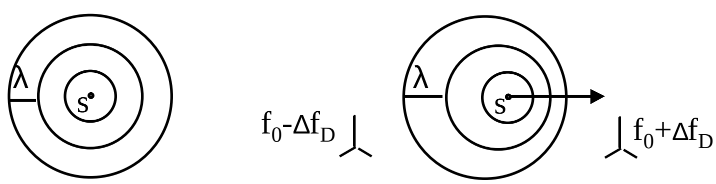
4.7T MRI (1.8 x 1.3 cm) **OCT (2 x 1.8mm)** **Histology**

R. D. Pearlstein, Z. Chen, et al.

Optical biopsy: OCT image of rat esophagus



Optical Doppler Tomography

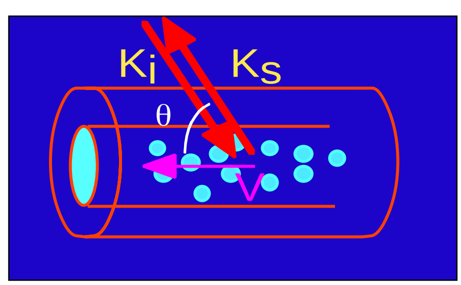


Doppler frequency shift:

$$\Delta f = \frac{1}{2\pi}(\vec{k}_s - \vec{k}_i)\ \vec{V}$$

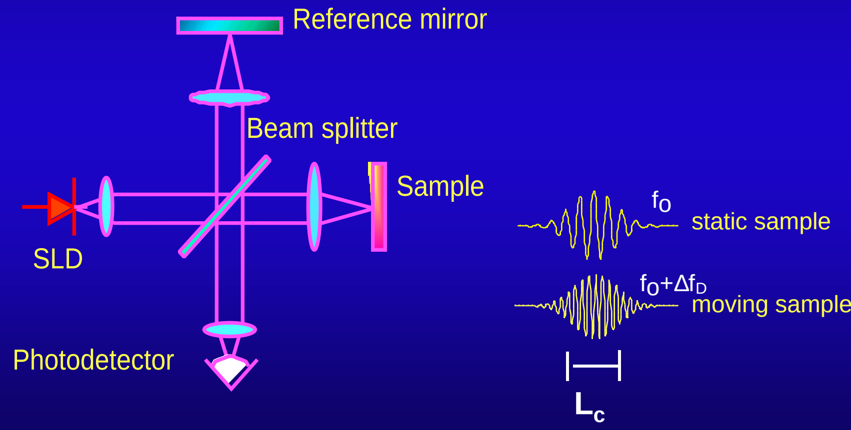
Velocity:

$$V = \Delta f_D \lambda / (2\cos(\theta))$$



Optical Doppler Tomography

Combining Doppler velocimetry with optical sectioning capability of OCT



Optical Doppler Tomography