ECSE-4962
Introduction to Subsurface Sensing and Imaging Systems

Lecture 8: More on Waves, Their Interactions

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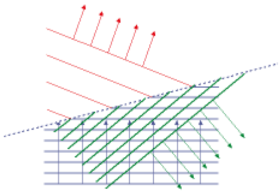
GE Global Research

Outline of Course Topics

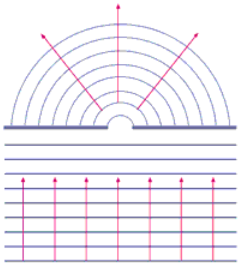
- THE BIG PICTURE
  - What is subsurface sensing & imaging?
  - Why a course on this topic?
- EXAMPLES: THROUGH TRANSMISSION SENSING
  - X-Ray Imaging
  - Computer Tomography
  - Intro into Optical Imaging
- COMMON FUNDAMENTALS
  - propagation of waves
  - interaction of waves with targets of interest
- PULSE ECHO METHODS
  - Examples
- MRI
  - A different sensing modality from the others
  - Basics of MRI
- MOLECULAR IMAGING
  - What is it?
  - PET & Radionuclide Imaging
- IMAGE PROCESSING & CAD

Waves at Interfaces

- Reflection & Transmission
- Refraction
  - At interfaces of media with differing propagation speeds.
  - Can you have reflection between media w. a common propagation speed?
- Diffraction
  - Sommerfeld's (1894) definition:
    - "… any deviation of light rays from rectilinear paths which cannot be interpreted as reflection or refraction."



Reflection & Refraction



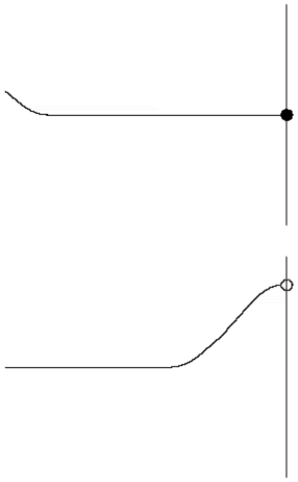
Diffraction

http://lectureonline.cl.msu.edu/~mmp/kap13/cd372.htm

A bit more on reflections

- We can think of reflections as being "hard" or "soft".
- These correspond to cases purely kinetic energy reflections & purely potential energy reflections.

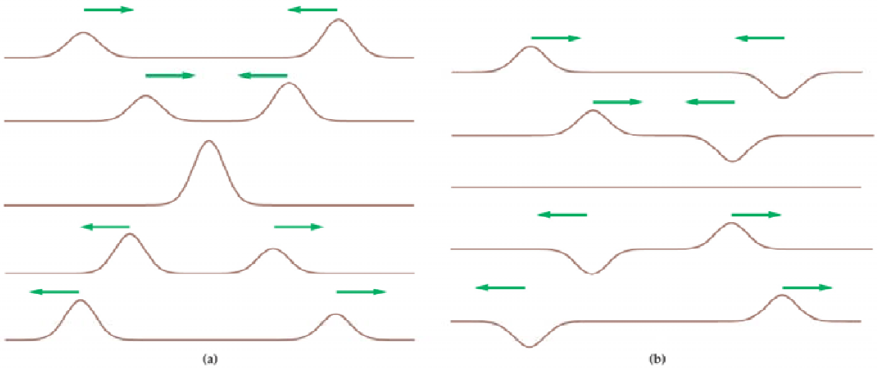
$$R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$



Traveling and Standing Waves

- Traveling waves
  - Launched waves which propagate in a given direction.
- Standing waves
  - Wave pattern caused by interference of two traveling waves.

Superposition of oppositely traveling wave pulses.



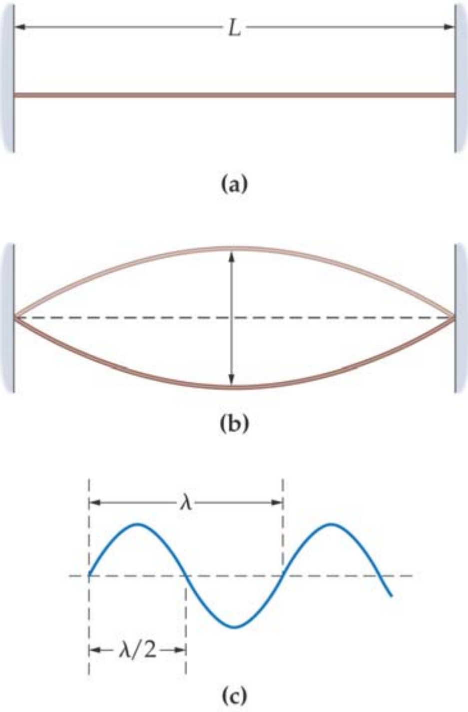
Constructive Interference

Destructive Interference

Superposition of 2 traveling harmonic waves, as a function of time.

- The period and wavelength are exactly the same.
- One wave travels to the right, one to the left.

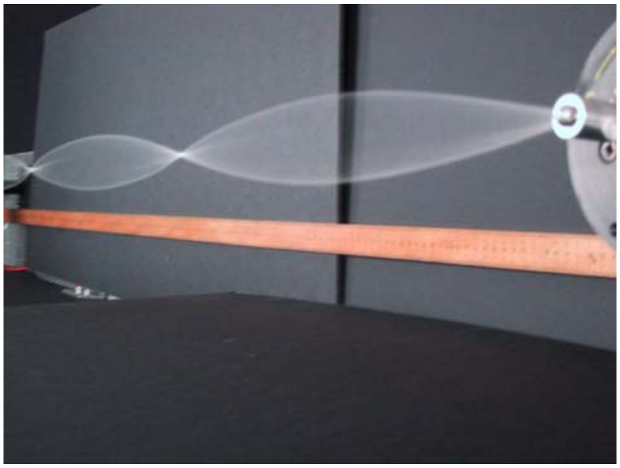
Standing Wave



- Plucking the string in the middle, it will vibrate.
- Note: wavelength in the picture is twice the string length:

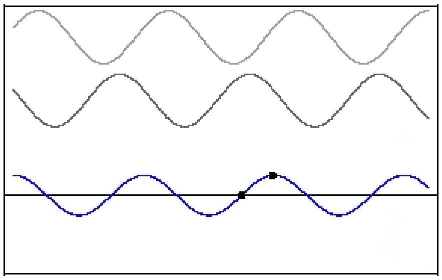
  λ=2L

Standing waves: adding waves traveling in opposite directions.



Standing Waves

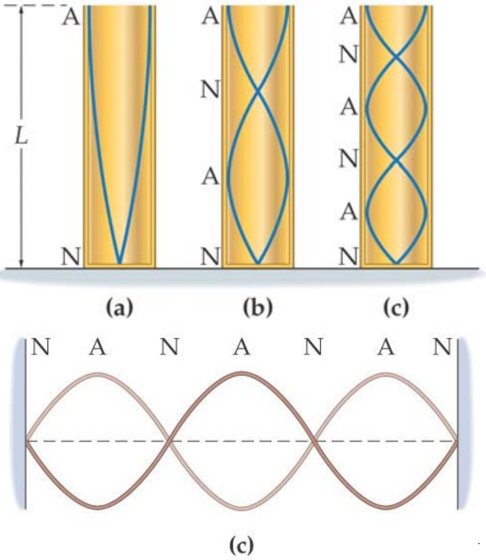
- Two key ingredients for a standing wave:
  - Waves traveling in opposite directions
  - Waves of common frequency
- Key characteristics:
  - Nodes
  - Anti-nodes



www.kettering.edu/~drussell/Demos.html

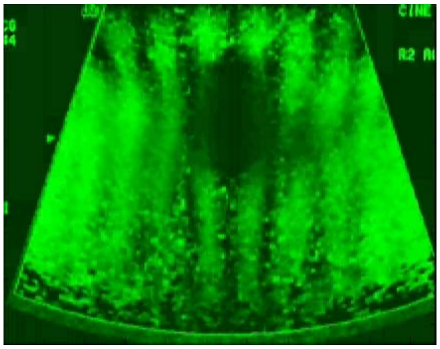
Standing waves and harmonics

- Major role:
  - the basis for musical tones



Clinical Application of Standing Waves

- NIH Grant Application w.
  - RPI: Dr. Joyce McLaughlin of Math Dept.
  - GE Global Research: Dr. Kai E Thomenius
  - U. of Rochester: Drs. D. Rubens & K. Parker
- Study of prostate cancer using “crawling waves”.
  - These waves are actually traveling beat waves.

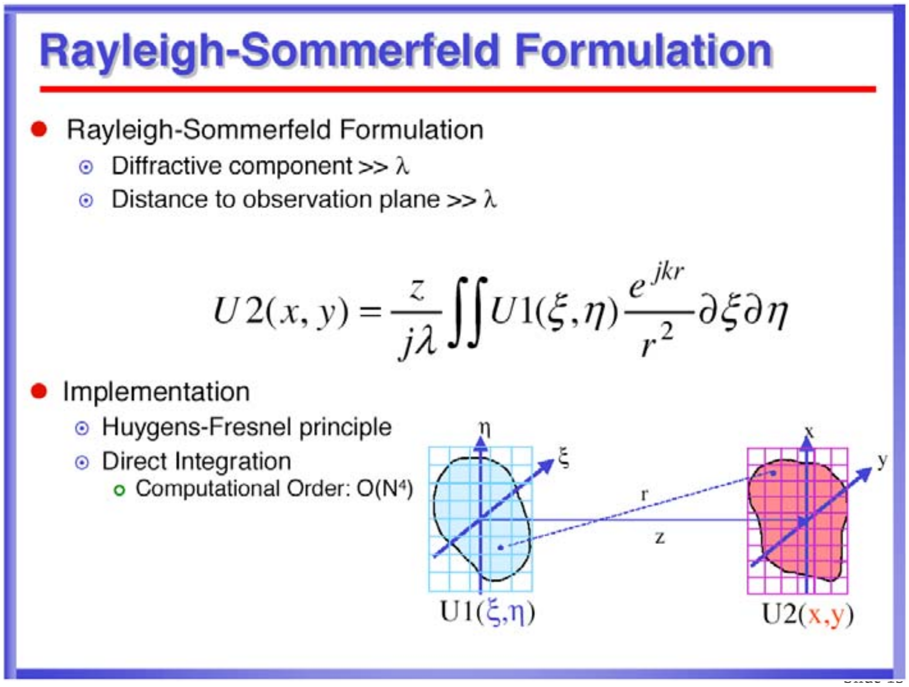


What do we do with the Wave Equation?

- Develop different propagation models
  - These involve an aperture of some sort which define an incoming field.
  - Such models permit the calculation of the field some distance from the aperture.
- An aperture in this context can be:
  - A slit in an opaque screen.
  - A transmitting radar antenna
  - A sound source such as a transmitting sonar transducer.
- Goal of Propagation Models: Given a known field at the aperture, determine what happens to our probe as it travels in the medium of interest.

Example

- How to determine the field generated from an aperture?
  - Start with a general solution to the wave equation.
  - Use appropriate approximations to achieve the desired field descriptor.
  - Account for propagation related effects (e.g. attenuation)
  - Test out the result.



Rayleigh-Sommerfeld Formulation

- Rayleigh-Sommerfeld Formulation
  - Diffractive component >> λ
  - Distance to observation plane >> λ

$$U2(x,y) = \frac{z}{j\lambda} \iint U1(\xi,\eta) \frac{e^{jkr}}{r^2} \partial\xi \partial\eta$$

- Implementation
  - Huygens-Fresnel principle
  - Direct Integration
    - Computational Order: O(N⁴)

Simplifications, Example from Radar

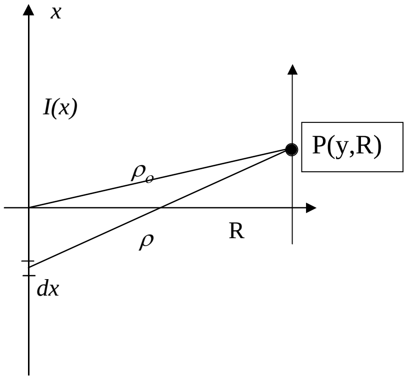
- We will work with the Rayleigh-Sommerfeld solution:

$$E(y,R) = K \int_{line} \frac{I(x)\exp(jk\rho)}{\rho} dx$$

$$where \quad \rho = \sqrt{R^2 + (y-x)^2}$$

$$= \rho_o \sqrt{1 - \frac{2yx}{\rho_o^2} + \frac{x^2}{\rho_o^2}}$$

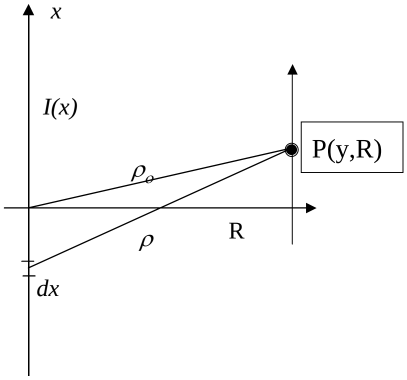
- In many applications, ρ_o is greater than the aperture, esp. if R >> aperture.
- In such cases the denominator in the integral varies much more slowly than the numerator



Based on Steinberg: Principles of Aperture & Array System Design

Simplifications

$$E(y,R) = K\int_{line} \frac{I(x)\exp(jk\rho)}{\rho}\,dx$$



- If we expand the ρ in the phase term in a binomial series and keeping the dominant terms, we get

$$\rho = \rho_o\left(1 - \frac{2yx}{\rho_o^2} + \frac{x^2}{2\rho_o^2}\right)$$

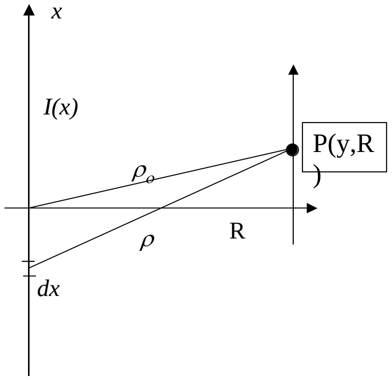
$$= \rho_o - \frac{yx}{\rho_o} + \frac{x^2}{2\rho_o}$$

- Now the complex field strength is

$$E(y,R) = \frac{K\exp(-jk\rho_o)}{\rho_o}\int_{line} I(x)\exp\left[jk\left(x\sin\theta + \frac{x^2}{2\rho_o}\right)\right]dx$$

Simplifications

$$E(y,R) = \frac{K\exp(-jk\rho_o)}{\rho_o}\int_{line} I(x)\exp\left[jk\left(x\sin\theta + \frac{x^2}{2\rho_o}\right)\right]dx$$



- Normalizing this expression to the value at $\theta = 0$ and setting $u = \sin\theta$ gives us:

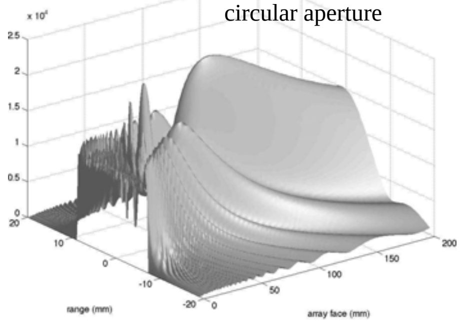
$$f(u) = \int i(x)\exp\left[jk\left(xu - \frac{x^2}{2\rho_o}\right)\right]dx$$

- The first term in the integrand is the Fourier kernel exp(*jkxu*).
  - This is associated with the **Fraunhofer** or **far-field zone** of diffraction theory.
- The second term in the integrand is the Fresnel kernel exp($-jkx^2/(2\rho_o)$).
  - This is associated with the **Fresnel** or **near-field zone** of diffraction theory.

Far-field or Fraunhofer Zone

$$f(u) = \int_{line} i(x)\exp\left[jk\left(xu - \frac{x^2}{2\rho_o}\right)\right]dx$$

Field due to a circular aperture



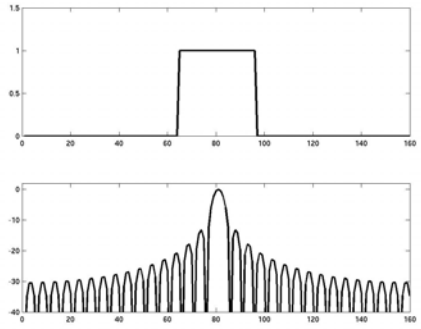
- If ρ_o is so large that the variation of quadratic term is <<1 over the aperture, that term will have little effect on the field integral.
- If that is the case, we can ignore the quadratic term.
- Our field expression now becomes a Fourier integral:

$$\boxed{f(u) = \int_{line} i(x)\exp[jkxu]dx}$$

Far-field or Fraunhofer Zone

$$\boxed{f(u) = \int_{line} i(x)\exp[jkxu]dx}$$

- A good rule of thumb for the transition to far field is a distance of $D^2/(4\lambda)$.
- For analyses beyond this point, the above Fourier expression is accurate.
- This is highly desirable, consider a uniform line source.

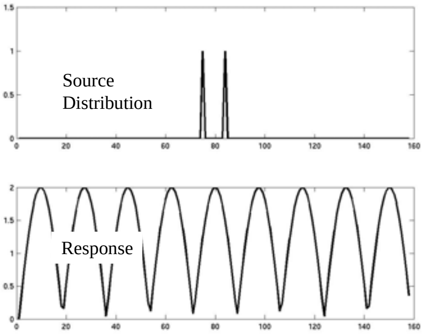


- In this example, the aperture function is a rectangle.
- The far-field response is its Fourier Transform, the sinc-function.

Far-field Response & Fourier Transforms

- This is, indeed, a powerful result.
- Everything we know about Fourier transforms can be applied:
  - Linearity - add two sources
  - Delay Theorem - shift source
  - Etc.
- What is the response of a point source?
- What is the far-field response of a sinusoidal transmit pattern?

Interferometer



When faced with a new aperture (array or some other aperture), think of its Fourier Transform to get an idea of the likely field.

Focused Designs

- We often work with curved radiations (parabolic dishes, lenses, focused transducers).
- In 1949, O’Neil published a very nice theory for determining the field strength along the axis of a spherical source.
- While the derivation is beyond what we want to cover in this course, the result is extremely useful.

O’Neil, HT, “Theory of Focusing Radiators”, JASA vol. 21:5, pp. 516 - 527, 1949

O'Neil's Formula

- O'Neil developed the following expression for a focusing radiation:

$$p = \rho c u_o P j \ \exp\left(j(\omega t - kM)\right)$$

where

$$P = E \sin k\delta / 2, \quad E = \frac{2}{1 - x/A},$$

$$\delta = B - x, \quad M = \frac{B + x}{2},$$

$$B = \sqrt{(x-h)^2 + a^2} = \sqrt{x^2 - 2xh + b^2}$$

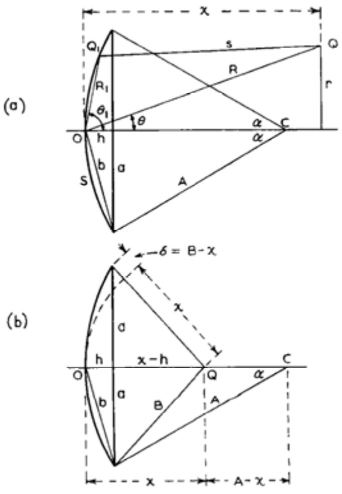


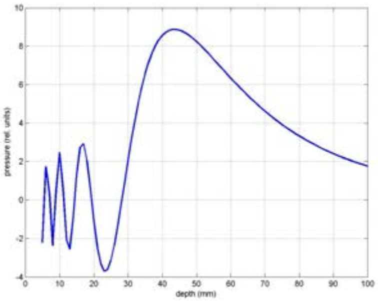
FIG. 1. Dimensions and coordinates.

M-file for O'Neil

```
function p = oneils(lambda,a,R,z)

% O'Neil's expression for axial pressure profile
% called by p = oneils(lambda,a,R,z).  This function returns
% the value of capital P as given by Eq. 3.1 in O'Neil's
% paper.  To get the pressure amplitude one has to multiply P
% by rho * c * u0.
%   where  lambda is the wavelength
%          a is the radius of the aperture
%           R is the radius of curvature
%          z is the vector of distance along the axis
k = 2 * pi / lambda;
del_z = z(1,2) - z(1,1);
h = R - sqrt(R.^2 - a^2);
in = find(R == z);
if isempty(in) == 0,
  z(in) = z(in) + 1.0e-4;
end
E = 2 * R ./ (R - z);
delta = sqrt((z - h).^2 + a^2) - z;
p = E .* sin(k * delta / 2);

if isempty(in) == 0,
  z1 = z(in) - del_z / 2;
  E = 2 * R ./ (R - z1);
  delta = sqrt((z1 - h).^2 + a^2) - z1;
  p1 = E .* sin(k * delta / 2);

  z2 = z(in) + del_z / 2;
  E = 2 * R ./ (R - z2);
  delta = sqrt((z2 - h).^2 + a^2) - z2;
  p2 = E .* sin(k * delta / 2);
  p(in) = (p1 + p2) / 2;
  disp('calculated new value for p at roc');
end
```

O’Neil Formulation

- Graph shows a typical result:
  - Aperture radius = 10 mm
  - Wavelength = 0.77 mm
  - Radius of curvature = 50 mm
- Notice the location of the peak response – it does not coincide with the focal location. Why?



Recap of the Lecture

- Overview of scalar diffraction theory w. materials from optics, acoustics.
  - Relation of the solutions to actual problems encountered.
- Working with the R-S formulation, derived highly simplified expression as the Fourier transform of the aperture.
- Demonstrated a closed form expression for focused circular apertures.

Homework

1. Determine the near-to-far field transition for the following cases:
   a) Planar laser wave at 800 nm wavelength that travels through a 1 mm opening in an opaque screen.
   b) A one THz electromagnetic radiation coming from a 5 mm antenna.
   c) A 20 mm ultrasound transducer operating at a frequency of 10 MHz.
2. Enter the oneils m-file
   a. Keeping aperture size constant for transducer in Slide 25, change wavelength from 0.5 to 1.0 in 0.1 mm steps. Graphically show the impact of the change in wavelength.
   b. Describe qualitatively the changes in the axial profile. How will this affect image data acquisition in terms of sensitivity and the depths at which the imager is likely to work well?

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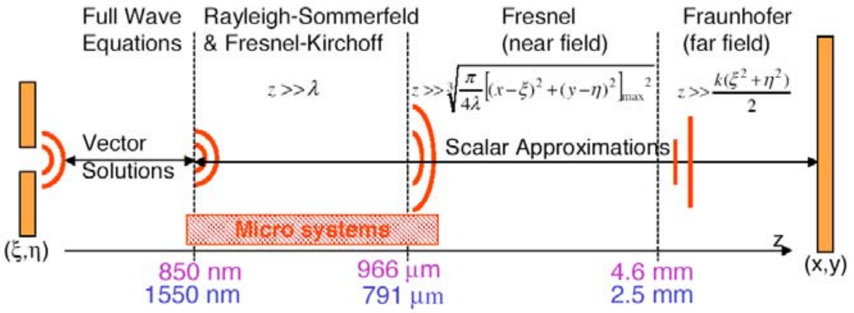
Backup Slides

Validity of Scalar Models



Examples: 50 μm Aperture, 200 μm Observation, λ=850 nm, λ=1550 nm

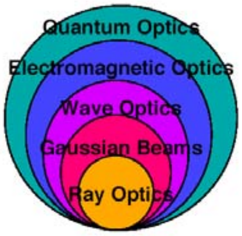
- Fraunhofer Approximation - Assume planar wavefronts
- Fresnel Approximation - Assume parabolic wavefronts
- Rayleigh-Sommerfeld Formulation - Spherical wavefronts

Optical Propagation Models

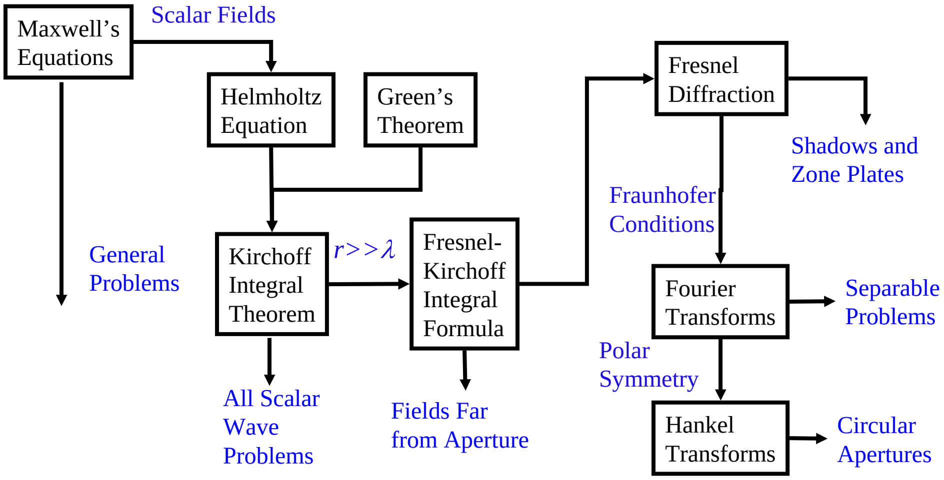
Increased Computation Speed

Increased Accuracy

- Geometric Ray Propagation
  - Positions and angles
  - No optical signal characterization
- Gaussian Propagation
  - 8 scalar parameters - z0, x, y, λ, etc.
  - Fast computation - no explicit integration at each component
  - Limited diffraction compensation
- Scalar Diffraction Propagation
  - Fraunhofer, Fresnel, Rayleigh-Sommerfeld
  - 2D complex wavefront
  - Propagation by summation of wavefronts
- Full-Wave Propagation
  - Finite Difference, Finite Element, Rigorous Coupled Wave, Boundary Integral
  - Computationally intensive



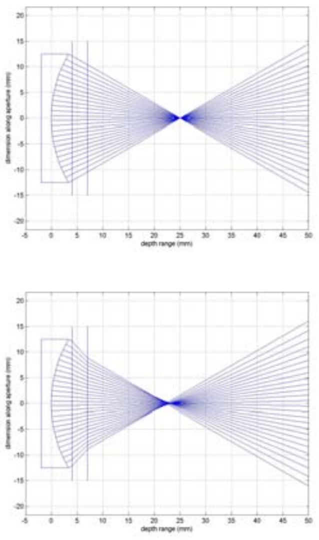
Evolution of Wave Equation Solutions



For more details, a great reference is Ch. 3 of Goodman's **Fourier Optics**, McGraw-Hill

Ray Tracing

- Assumes waves propagate in straight lines – ray optics.
- Number of software packages of varying degrees of sophistication available.
- In example, what is the impact of a plate on our focal characteristics?
  - What happens to the focal point?
  - Do all the lines go through the focus?
- Relatively straightforward.



A Few Words about Green’s Theorem

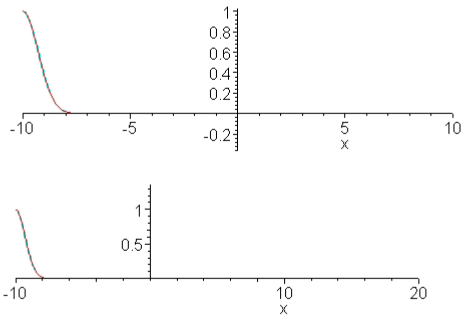
- Key mathematical relationship that relates a volume integral of two functions in space to the associated surface integral.
- Goodman: “prime foundation of scalar diffraction theory”.
- Important aspects to its application:
  - Careful choice of the “Green’s function”, *G* below. This can be simply an expanding spherical wave from a point source.
  - Careful choice of the closed surface S below.
- Some of the integral theorems are derived using these assumptions.

$$\iiint\limits_{V}\left(G\nabla^2 U - U\nabla^2 G\right)dv = \iint\limits_{S}\left(G\frac{\partial U}{\partial n} - U\frac{\partial G}{\partial n}\right)ds$$

Reflections at Interfaces

- Some points on reflection:
  - If $Z_2 = Z_1$, there is, obviously, no reflection.
    - Hence, an impedance mismatch is needed to get a reflection.
  - If $Z_2 > Z_1$, there is no polarity change, however, if $Z_2 < Z_1$, the echo will be inverted.
    - This is necessary for there to be continuity in values at the boundary.
  - Upper video clip: Z1 / Z2 = 0.5
  - Lower video clip: Z1 / Z2 = 2.0
- For a nice discussion of this, check: http://physics.usask.ca/~hirose/ep225/animation/reflection/anim-reflection.htm

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



Transmission at Interfaces

- Some points on transmission:
  - If $Z_2 = Z_1$, there is100% transmission.
    - Hence, an impedance mismatch is needed to get a reflection.
  - If $Z_2 > Z_1$, there is no polarity change, however, if $Z_2 < Z_1$, the echo will be inverted.
    - This is necessary for there to be continuity in values at the boundary.
  - Upper video clip: Z1 / Z2 = 0.5
  - Lower video clip: Z1 / Z2 = 2.0
- For a nice discussion of this, check: http://physics.usask.ca/~hirose/ep225/animation/reflection/anim-reflection.htm

$$T = \frac{2Z_1}{Z_2 + Z_1}$$