Routing in Ad Hoc Networks: A Theoretical Framework with Practical Implications

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Objective: Basic Limits on Routing Overhead

- Considering a variable topology network, such as for example a mobile ad hoc network;
  - can we characterize the variability of a network topology?
  - and relate it to the (minimum) routing overhead?
  - and hence discover some basic limits on routing?
  - ...and maybe these limits could be used as reference curves (similar to Shannon’s Capacity for error free communication)
Related work: Theory of Computing

- In TOC, routing typically refers to building distributed memory message passing multiprocessor systems for computing applications, where the interest primarily is in finding the computational cost (complexity) of a certain message passing i.e. routing algorithm (e.g. [1]).

- The hypercube is one of the most popular (and robust) topologies used for building and studying routing in this context (e.g. [2]).


Related work: Optimization

- Saha-Mukherjee [3] proposes an optimization approach to find the optimal number of clusters that minimize the total route computation cost, assuming the route computation cost per hierarchical level is some known constant $\alpha_i$, and a fixed traffic matrix.

- Kleinrock-Kamoun [4] proposes an optimization approach for finding the number of clusters that minimize the size of the routing table in a variable network topology.


Gallager [5] proposes an information theoretic approach to find basic limits on protocol overhead for maintaining the start and stop time of messages between pairs or nodes in a communication network.

In this set-up, the network is the “source” and the protocol is thus merely a “source encoder”.

Proposed Information-Theoretic Framework

- Analyze the changes of topology as random process
  - Define the topology itself as random variable

- Apply information-theoretic principles to quantify the minimum amount of overhead:
  - **Routing message** overhead (bits / unit time).
  - **Routing memory** overhead (bits)
Theoretical Framework (Cont’d)

- Minimum amount of information needed to describe a change in the network topology? *Entropy; Minimum Expected Codeword Length (MCL)*

- Minimum amount of overhead needed to inform the cluster head of that change? *Send the MCL over the shortest paths to cluster heads*

- Memory is needed to support the information exchange? *Topology + topology change info*
Hierarchical Proactive Routing Protocol Model

- Bounded area; N nodes; M sub-regions; Connected network.
- Each node maintains link status info by periodic hello messages at periodic intervals $\tau_i$;
- regular nodes inform cluster head about link changes.
- Whenever there is a change, a regular node receives the new path information from the cluster head.
- Whenever cluster membership changes, the cluster head announces this change to all other clusters at periodic intervals $\tau_e$. 
Mobility and Link Status models

State transition diagram of node movement between clusters. \( q_0 \) is the probability that a node stays in the same cluster.

State transition diagram of the status of an arbitrary link; \( P_{00} \) (\( P_{11} \)) is the probability that a link is down (up) at next time step if the link was down (up) at previous time step.
Analysis Outline

- Topology Granularities
  - Global Ownership Topology
  - Local Ownership Topology
  - Local Detailed Topology

- Analysis – for each topology level
  - MCL based on Cardinality (i.e. all topologies equally likely)
  - MCL based on Topology Stationary Probability Distribution
  - MCL on Prediction Using Previous Topology Knowledge (given a certain mobility and link status change models)
Summary of Results

- **Routing Overhead**
  - $R_e$: Exterior Routing Overhead (exchanging local ownership topologies)
  - $R_i$: Interior Routing Overhead
    - $R_h$: overhead associated with hello messages
    - $R_d$: Notification of link status change to cluster head
    - $R_p$: Notification of new path to the regular node

- **Memory Requirement**
  - **Cluster Head**: $M_c$
    - Global Ownership Topology: $M_{cg}$
    - Local Detailed Topology: $M_{cd}$
  - **Regular Node**: $M_r$
    - Shortest path to cluster head
Application of the results: Network Scaling

- Given the various expressions for routing overhead, there are different methods for scaling the network (increasing N)
  - **Model1**: Increase N but keep $g$ constant (hence decrease $d_0$, the coverage radius of each node)
  - **Model2**: Increase N but keep $g = \Theta(\log N)$ at the critical value needed for connectivity
  - **Model3**: Increase N while keeping $d_0$ constant (hence $g$ increases)

Note1: These methods keep other parameters such as $A$ (area) and $M$ (number of clusters) constant

- Also derived the scaling laws with $M$
## Scaling Laws

- **Scaling with N**

<table>
<thead>
<tr>
<th>Overhead</th>
<th>M1: $g$ const</th>
<th>M2: $g_c$</th>
<th>M3: $d_0$ const</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$</td>
<td>$\frac{3}{N^2}$</td>
<td>$\frac{3}{\sqrt{\log N}}$</td>
<td>$N$</td>
</tr>
<tr>
<td>$R_h$</td>
<td>$N \log N$</td>
<td>$N \log N$</td>
<td>$N \log N$</td>
</tr>
<tr>
<td>$R_d$</td>
<td>$\frac{5}{N^2}$</td>
<td>$\frac{5}{\sqrt{\log N}}$</td>
<td>$N^2$</td>
</tr>
<tr>
<td>$R_p$</td>
<td>$N^2 \log N$</td>
<td>$\frac{5}{\sqrt{\log N}}$</td>
<td>$N \log N$</td>
</tr>
</tbody>
</table>
Other Practical Implications

- $M_{\text{optm}}$ minimizing memory requirement of cluster heads
  \[ M_{\text{optm}} = \sqrt{(\ln 2)N} \]

- $M_{\text{ratio}}$ minimizing the ratio of the memories of regular vs. cluster-head node
  \[ 2M_{\text{ratio}}^2(1 + \ln M_{\text{ratio}}) = (\ln 2)N \]

- $M_{\text{optr}}$ minimizing the total routing overhead in a large network (Model1)
  \[ M_{\text{optr}} = \frac{\tau_e(2N + (1 - p_{11})\beta^2 N \log N)}{4\tau_i} \]
Summary of the analysis
Conclusion

- Developed an information theoretic framework for quantifying hierarchical proactive routing overhead
- Derived expressions for the overhead
- Analyzed these expressions to derive scalability results
- Applied these expressions to find the cluster size that asymptotically optimize several different objectives
Future Work

- Reactive routing overhead (by conditioning on the traffic matrix)
- Tradeoffs between routing overhead and topology accuracy
- Extend the results to multiple hierarchies
- Dynamic cluster formation and elimination
Questions
Backup slides
Memory Requirement
Key Aspects of Hierarchical Routing

- Communication
  - Intra-cluster
  - Inter-cluster

- Maintain the topology
  - Cluster head maintains the detailed connectivity relationships (local detailed topology) within a group of nodes
  - Cluster head also maintains aggregate global information (global ownership topology) for routing beyond the limits of the cluster (i.e. inter-cluster routing)
  - Regular node maintains a path (shortest) to its cluster head
$p_I$ – steady state probability that a direct link between two arbitrary nodes exists.
Mobility and Topology Changes

- Mobility induces topology changes
  - Ownership change (nodes move between clusters) → 1) Update local ownership topology; 2) Update global ownership topology

- Connectivity change (links up/down) → 1) Update local detailed topology; 2) Possible need to update shortest path from a regular node to its cluster head