Routing in Ad Hoc Networks: A Theoretical Framework with Practical Implications

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Abstract— **In this paper, information theoretic techniques are used to derive analytic expressions for the minimum expected length of control messages exchanged by proactive routing in a two-level hierarchical ad hoc network. Several entropy measures are introduced and used to bound the memory size necessary for the storage of the routing tables. The entropy rates of the topology sequences are used to bound the communication routing overhead - both the interior routing overhead within a cluster and the exterior routing overhead across clusters. A scalability analysis of the routing overheads with regard to the number of nodes and the cluster size is provided under three different network scaling modes. Finally, practical design issues are studied by providing the optimal cluster sizes that asymptotically minimize (i) the memory requirement for each cluster head; (ii) the total control message routing overhead.**

I. Introduction

In this paper, we consider the class of stateful routing protocols that require to maintain state information about the network topology (e.g. routing tables). We derive lower bounds on the minimum routing overhead (bits per unit time) and memory (bits) associated with a stateful routing protocol in an ad hoc network of mobile nodes as a function of the network parameters.

The topology of an ad hoc network is randomly changing due to the random movement of nodes. This is the primary reason for topology change considered in this paper. To maintain up to date topology information, nodes of a network exchange control messages containing the new topology or topology change information. The memory requirement is related to the amount of state information stored or processed. The routing overhead is related to the product of the message size and the number of hops the message travels.

Clustering is a well known design technique that could potentially reduce routing overheads. But the tradeoffs involved in the design, such as for example the tradeoff between the cluster size and routing overhead, has not been precisely characterized. Several questions could be raised from the network design perspective; What are the criteria to select the cluster size? Given a total number of nodes and the nodes mobility pattern, what is the average optimal cluster size minimizing the average memory requirement or average total routing overhead? In this paper, we attempt to fill in this gap in the literature by providing quantitative analysis and answers for these questions.

In order to derive lower bounds on memory requirement and routing overhead, the key is to find lower bounds on the sizes of exchanged control messages, and then relate this to the rate of exchange through the parameters of the mobility model. The minimum complexity of the control message sizes is intuitively a function of the complexity of the network. We capture this dependence through the information-theoretic measure called the *Minimum Expected Codeword Length (MCL)* (see for example [1] Section 5.4) which is the minimum number of bits required to describe (i.e. encode) a change. Because of the hierarchical (clustered) structure, three different topology granular views are analyzed. For each topology granular view, three methods are used to derive expressions for the MCLs; topology cardinality (Method 1), topology probability distribution (Method 2) and topology prediction (Method 3).

Up to our knowledge, there is no previous work that bounds routing overhead using such information theoretic measure – related analytical work focuses on *modeling* (rather than bounding) routing overhead [2], [3]. Gallager [4] analyzes the protocol overhead in a pure information theoretic manner in data communication networks. His paper uses an entropy measure to determine the basic limitations on the amount of protocol information that must be transmitted in a data communication network to keep track of source and receiver addresses and the starting and stopping of messages. This paper determines lower bounds on the average control message size and memory requirement for routing in a variable topology network to keep track of the topology information in order to maintain paths between nodes in the network.

Gavoille [5] attempts to define routing as a distributed algorithm over a static undirected random graph a representation of a communication network. The interesting aspect of this paper is that it formulates routing problem to be a distributed algorithm over a network (modeled as a random graph), and proposes multiple open research problems based on the structures of these graphs and the quality of services required. One of the interesting open questions is the trade-off

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between the size of memory used to store topology information in the nodes of a network and the accuracy of finding shortest paths between nodes. The problem is somewhat similar to our problem of finding out the trade-off between memory requirement and communication routing overhead.

The main contributions of this paper are (i) to derive analytic expressions of the MCLs for three information theoretic techniques for each topology level; (ii) bound the routing overhead by the entropy rates of the sequences of topologies; (iii) bound the memory requirement; (iv) provide a scalability analysis of memory requirement and routing overhead with the number of nodes and the cluster size for three different modes of scaling; and (v) provide an analysis of the optimal cluster sizes that asymptotically minimizes the memory requirement for cluster head and the total routing overhead.

The paper is organized as follows. Section II introduces the network models (including the routing protocol, topology definitions and mobility model) and the notations used in the remainder of the paper. Section III analyzes the minimum expected codeword lengths (MCLs). Section IV discusses the relationships between the entropy rates, the topology evolution, and the routing overhead. Section V derives lower bounds on the nodes memory requirements, and Section VI derives lower bounds on exterior and interior routing overheads. Section VII studies the scalability of the overheads as a function of the network size. Practical implications of applying the results are presented in Section VIII. Finally, Section IX summarizes our results and outlines several potential applications of this methodology in ad hoc networks and the other areas.

II. Network Model

A. Topology

We consider a fixed number N of distinguishable nodes that move within a bounded region. Each node has a unique identifier - denoted as *NUI* (from 1 to N). The bounded region is divided into fixed number M of sub-regions, labelled by a unique sub-region index, denoted as *RUI* (from 1 to M). There is a maximum of $K > 1$ neighboring sub-regions for any given sub-region. Furthermore, we assume that the mobility patterns of the nodes are statistically independent and identically distributed (i.i.d).

All the nodes within a sub-region form a cluster. A *cluster head* is selected for each non-empty sub-region, which is randomly chosen from the nodes within the sub-region. Let the integer valued random variable $\mathcal{N}$ denote the number of nodes within a given sub-region. The range of $\mathcal{N}$ is $0, 1, \ldots, N$. Let $0 \leq n \leq N$ denote the number of nodes in a given sub-region at some time instant. It is possible that a sub-region becomes empty and thus will not have a cluster or a cluster head. A node that is not a cluster head is called *regular node*. A cluster head *owns* a regular node if the two nodes belong to the same sub-region. Any node that does not belong to the same sub-region is called *exterior node*.

We understand that the current cluster model does not capture the need of supporting dynamic cluster formation and elimination for ad hoc networks. But our work in this paper can be extended to include the dynamic cluster formation and elimination by including the cluster formation policies and dynamic distribution of this information.

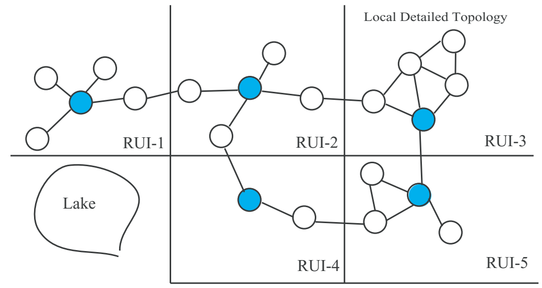

Fig. 1. A snapshot of the network topology. Blue (dark) nodes are cluster heads. There is one cluster head in every sub-region. Sub-regions do not have to be rectangular or identical.

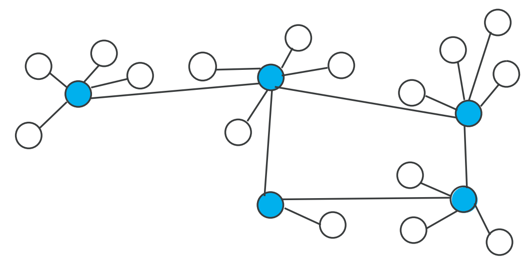

Fig. 2. The corresponding global ownership topology. Blue (dark) nodes are cluster heads.

The underlying physical topology of an ad hoc network is represented by a connected, undirected randomly changing graph, $G = (V, E)$, where V is the set of graph nodes and E is the set of edges. An example of the physical topology is depicted in Figure 1. Each mobile node is represented by a graph node using mobile node's *NUI*. An edge exists between two arbitrary graph nodes if single-hop communication between those two corresponding nodes is possible. In this paper, we assume that the transmission and reception ranges of a node are equal and hence edges are bi-directional.

Three different topology granular views are deduced from the physical topology in this paper. Each topology granular view is represented by a graph. The first topology granular view is *local detailed topology*, which is a sub-graph of the physical topology (see Figure 1, sub-region *RUI*-3). Each local detailed topology only has the nodes of a given sub-region, and it inherits the edges from the physical topology. We have M instances of local detailed topologies (sub-graphs) at any given moment. The second topology granular view is a simplified version of local detailed topology named *local ownership topology* which only specifies the cluster head and the *NUI* list of the regular nodes of the cluster omitting the detailed knowledge of the physical connectivity of the nodes. In a graph representing a local ownership topology, each regular node has an edge to its cluster head. The third topology granular view is *global ownership topology*, which is the aggregation of all local ownership topologies (Figure 2). In a graph representing the global ownership topology, an edge exists between each regular node to its cluster head to reflect the ownership relationship, and an edge exists between two cluster heads of neighboring sub-regions to reflect the neighborhood relationship.

B. Proactive Routing Protocol

The model of routing between two mobile nodes is as follows. Each regular node maintains a shortest path to the cluster head. When a source node needs to communicate to a destination node of the same sub-region, the source node first sends a route query to its cluster head following the shortest path. Upon receiving the route query, the cluster head computes the shortest path between the source and the destination nodes, and sends the shortest path information to the source node. The packets between the source and destination nodes will follow the shortest path. When a source node needs to communicate to a destination node in a remote sub-region, the source node sends packets to its cluster head. The cluster head forwards the packets to the remote cluster head of the remote sub-region where the destination node is located. The remote cluster head further forwards these packets to the destination node.

To support the above routing model, we assume a generic two-level hierarchal proactive routing protocol. Each cluster head maintains the knowledge (i.e. the up to date status) of the *local detailed topology* of its sub-region and the global ownership topology of the whole network. Each regular node maintains the knowledge of a shortest path to its cluster head.

The routing protocol includes a mechanism for detecting, collecting and distributing the network topology changes, which is described as follows. Each node has a clock that is not required to be synchronized. Each node notifies its existence via a periodic transmission of "HELLO" messages to its neighboring nodes, and detects the link changes with its neighboring nodes by listening to the transmission of "HELLO" messages from its neighboring nodes. In addition each regular node periodically sends the link status changes (if any) to its cluster head. In this protocol, we select the two time periods to be the same. The update message of link status change is sent out at discrete constant time intervals, such as for example by incorporating the information in the "HELLO" messages. The time period is the *interior update time* τ_i. We assume that τ_i is a constant for all the nodes. The assumption is helpful for technical reasons. It also matches several routing protocol designs due to practical reasons. For example, in OLSR proactive protocol [6], the parameter "HELLO_INTERVAL" can be viewed as τ_i in our protocol model. The transmission of "HELLO" messages is widely used in the wireless communication protocols to detect the physical/MAC layer link status changes, which is called the "HELLO" protocol [3]. Upon receiving the link status change message from any of its regular nodes, a cluster head updates its local detailed topology. If the cluster head finds the shortest path of a regular node has broken, it will notify the regular node of a new shortest path.

Similarly, each cluster head periodically broadcasts its local ownership topology changes to other cluster heads. The time period is *exterior update time* τ_e. Upon receiving an updated message from another cluster head, a cluster head updates the global ownership topology.

The process of detecting, collecting and distributing the topology changes produces a routing overhead $\mathbb{R}_t$. We separate the routing overhead into two parts. The first part is the *interior routing overhead* $\mathbb{R}_i$ which is the bit rate needed to maintain the local detailed topology of a sub-region. This part includes the overhead of detecting the link status changes by sending "HELLO" message, updating the cluster head knowledge of the link status changes, and maintaining the shortest paths between the regular nodes to their cluster head. The second part is the *exterior routing overhead* $\mathbb{R}_e$ which is the bit rate needed to maintain the global ownership topology, which includes the overheads of distributing local ownership topologies among the cluster heads. Hence $\mathbb{R}_t = \mathbb{R}_i + \mathbb{R}_e$.

In order to support the routing protocol operation, each regular node requires $\mathbb{M}_r$ bits and each cluster head requires $\mathbb{M}_c$ bits of memory.

Finally, we make an important (though well known) observation that aids in the routing overhead analysis. The portion between an intermediate node and the cluster-head along the shortest path between a regular node and the cluster head is also a shortest path between the intermediate node and the cluster head. Hence, we assume in the routing protocol that this portion of the path is used by the intermediate node as the shortest path between itself and the cluster head. Therefore, each node needs only to remember its next neighboring node along its shortest path.

C. Mobility

Nodes are assumed to be capable of moving freely within the region of interest, and hence to leave a cluster and to join another cluster of another sub-region. Furthermore, we assume that if a node leaves a sub-region, it has equal chance to join any cluster of its neighboring sub-regions.

Let the random variable $X_j = \{0, 1\}$ denote the link status between any two nodes - a and b of a sub-region, where 1(0) represents the link exists (does not exist). The index j denotes the discretized time when a sends the update message of the link status change to its cluster head. The sequence of $\{X_1, X_2, ..., X_j, ...\} = \{X_j\}_{j=1}^{\infty}$ forms a random process.

Similarly, let $Y_j = \{0, 1\}$ denote the ownership status of a node for a given sub-region, where 0(1) denotes the node is outside (inside) the sub-region. The index i denotes the discretized time when the cluster head broadcasts its local ownership topology change to other cluster heads. The sequence of ownership status $\{Y_1, Y_2, ..., Y_j, ...\} = \{Y_j\}_{j=1}^{\infty}$ forms another random process.

The random processes $\{X_j\}_{j=1}^{\infty}$ and $\{Y_j\}_{j=1}^{\infty}$ are modeled as Markov chains. The mobility model of the network is specified by the following three parameters: (i) the conditional probability p_{00} that two nodes in the same sub-region that are not directly connected remain unconnected within the

TABLE I

LIST OF NOTATIONS - I

Notation	Description
N	Total number of nodes
M	Total number of sub-regions
K	Maximum number of neighboring sub-regions
$\mathcal{N}$	Number of nodes in a given sub-region
n	Number of nodes in a given sub-region at some time instant
τ_i	*Interior update time*
τ_e	*Exterior update time*
p_{00}	Probability that two nodes remain unconnected within τ_i
p_{11}	Probability that two nodes remain connected within τ_i
q_0	Probability of node staying in same sub-region within τ_e
l_r	Average path length between a regular node and its cluster head
l_c	Average path length between two neighboring cluster heads

TABLE II

LIST OF NOTATIONS - II

Notation	Description
G	Cardinality of Global Ownership Topology
I_G^C	MCL for Global Ownership Topology based on Cardinality
I_G^P	MCL for Global Ownership Topology based on Topology Stationary Probability Distribution
I_G^M	MCL for Global Ownership Topology based on Prediction Using Previous Topology Knowledge
L	Cardinality of Local Ownership Topology
I_L^C	MCL for Local Ownership Topology based on Cardinality
I_L^P	MCL for Local Ownership Topology based on Topology Stationary Probability Distribution
I_L^M	MCL for Local Ownership Topology based on Prediction Using Previous Topology Knowledge
D	Cardinality of Local Detailed Topology
I_D^C	MCL for Local Detailed Topology based on Cardinality
I_D^P	MCL for Local Detailed Topology based on Topology Stationary Probability Distribution
I_D^M	MCL for Local Detailed Topology based on Prediction Using Previous Topology Knowledge
R_i	Interior routing overhead within time interval τ_i
R_e	Exterior routing overhead within time interval τ_e
$\mathbb{R}_i$	Interior routing overhead in bit/second
$\mathbb{R}_e$	Exterior routing overhead in bit/second
$\mathbb{R}_t$	Total routing overhead in bit/second
$\mathbb{M}_r$	Memory required for a regular node
$\mathbb{M}_c$	Memory required for a cluster head

interval of the interior update time τ_i, (ii) the conditional probability p_{11} that a direct link between two nodes of the same sub-region remains connected within the interval τ_i, and (iii) the probability q_0 that a node stays at the same sub-region within a time interval τ_e. We assume that all the above three probabilities are constants for the network (which is true for a homogeneous mobility model).

D. Summary of Notations

Tables I and II summarize the main notations in this paper, where Table I contains the network parameters of the two-level hierarchical network and Table II contains the quantities defined and determined by this research.

In the analysis of the effect of mobility, we will refer to the entropy function $H(\cdot)$ defined as

$$H(p) = -p \log p - (1-p)\log(1-p)\,; 0 < p < 1.$$

III. MINIMUM EXPECTED CODEWORD LENGTH

In this section, we derive the MCLs for the topologies described in Section II-A. The MCL is computed using the entropy as defined by Shannon [7]. In this paper, all logarithms are base 2.

Three different information-theoretic methods are used to deduce the MCLs for each topology granular view, based on (i) the cardinality of the topology granular view without any prior knowledge of the probability distribution of topologies, (ii) the probability distribution of the topologies, and (iii) the conditional probability distribution based on the knowledge of topology at the previous time step and the mobility pattern.

A. Global Ownership Topology

1) Cardinality:

Theorem 3.1: The total number of possible topologies is given by

$$G = \sum_{i=1}^{\min(N,M)} \frac{N!M!}{i!(M-i)!(N-i)!} i^{N-i} \qquad (1)$$

Proof: Due to space constraints, please see [8]. ■

The result can be verified using a simple example with $N = 4$ and $M = 2$, where it is easy to show using manual calculation that the total combination is 56, which is the same as the result from (1).

2) MCL based on Cardinality: Let I_G^C denote the MCL based on cardinality. The interpretation of I_G^C is that it is the information amount needed to describe a specific global ownership topology without any knowledge of the topology probability distribution and topology evolution.

$$I_G^C = \log(G) \qquad (2)$$

It is easy to show after some algebraic manipulations that

$$M^N \le G \le \left\lceil \frac{N}{M} \right\rceil^M M^N \qquad (3)$$

and equivalently

$$N \log M \le I_G^C \le N \log M + M \log \left\lceil \frac{N}{M} \right\rceil \qquad (4)$$

The result (4) can be interpreted as follows. The lower bound in (4) is the minimum number of bits needed to describe the global ownership topology when omitting the information about which nodes are the cluster heads. Equation 4 shows that the introduction of cluster heads at most increases the MCL by $M \log \left\lceil \frac{N}{M} \right\rceil$. For large N,

$$\lim_{N\to\infty} \frac{N \log M + M \log \left\lceil \frac{N}{M} \right\rceil}{N \log M} = 1 \qquad (5)$$

and hence

$$I_G^C \approx N \log M \quad ; N \gg 1 \tag{6}$$

3) MCL based on Topology Stationary Probability Distribution: Based on the assumption that mobile nodes are identical, distinguishable, and able to freely move within the region, it is straightforward to conclude that the global ownership topology has a uniform probability distribution if not considering the cluster heads. For the sake of simplicity, we assume

$$I_G^P = I_G^C \tag{7}$$

i.e. the MCL based on the topology probability distribution is equal to the MCL based on cardinality. One argument is that the information amount to specify the ownership ($N \log M$ is much bigger than the information amount to specify the cluster head $M \log \lceil \frac{N}{M} \rceil$) when N is large.

4) MCL based on Prediction Using Previous Topology Knowledge: Given the global ownership topology at the previous update time instant, the information needed to update the new ownership of a node is

$$i_G = -1\left(q_0 \log q_0 + (1-q_0)\log(\frac{1-q_0}{K})\right) \tag{8}$$

Since nodes mobility patterns are *i.i.d*, the total information required to describe the ownership of all the nodes is Ni_G. Similar to the discussions of (4), the introduction of cluster heads increases the MCL by at most $M \log \lceil \frac{N}{M} \rceil$.

Let I_G^M denote the MCL required to describe the topology based on prediction using previous topology knowledge, then

$$Ni_G \le I_G^M \le Ni_G + M \log \lceil \frac{N}{M} \rceil \tag{9}$$

When $q_0 = 1/(K+1)$, i_G in (8) reaches its maximum value. For this case, the probability of a node staying at the current sub-region is equal to the probability that the node moves to any of the neighboring sub-regions. Therefore,

$$I_G^M \le Ni_G + M \log \lceil \frac{N}{M} \rceil \le N \log(K+1) + M \log \lceil \frac{N}{M} \rceil \tag{10}$$

with equality when $q_0 = \frac{1}{K+1}$

For large N, the second item of the right hand part of the previous equation can be ignored.

From (6) and (10),

$$\frac{I_G^M}{I_G^C} \le \frac{\log(K+1)}{\log M} \le 1 \, ; N >> 1 \tag{11}$$

with equality when $(K+1) = M$

The above result can be explained as follows. In the case of prediction, the number of bits needed to specify the cluster of a node at next time slot scales as $\log(K+1)$; while in non-prediction case, it scales as $\log M$ since the prediction case assumes the knowledge of current cluster and $(K+1)$ possible clusters at the next time slot.

Equation 11 says that the MCL (I_G^M) based on prediction is smaller than MCL (I_G^C) based on cardinality, even if the mobility parameter is chosen to maximize i_G (8) and I_G^M (i.e. when $q_0 = \frac{1}{K+1}$).

For many practical scenarios, even the largest number of neighboring sub-regions is usually less than the total number of sub-regions. In this case, for large N, the above result states that using prediction to update the topology information will result in large savings in the MCL.

B. Local Ownership Topology

1) Cardinality:

Theorem 3.2: The total number of possible local ownership topologies for a given sub-region is

$$L = (2^{N-1})N + 1 \tag{12}$$

Proof: See [8] ■

Note that the number of topologies is more than the simple 2^N when $N > 2$ due to the different ways of selecting a cluster head.

2) MCL based on Cardinality: Let I_L^C denote the MCL based on the cardinality. Then

$$I_L^C = \log(2^{N-1}N + 1) \tag{13}$$

For large N,

$$\lim_{N\to\infty} \frac{\log(2^{N-1}N+1)}{N} = 1 \tag{14}$$

and hence

$$I_L^C \approx N \quad ; N \gg 1 \tag{15}$$

From (6) and (15),

$$\frac{MI_L^C}{I_G^C} \approx \frac{M}{\log M} \quad ; N \gg 1 \tag{16}$$

where the factor "M" comes from the fact that there are M local ownership topologies.

The result of (16) says that the sum of the MCLs required to describe the local ownership topologies individually is larger than the MCL required to describe the global ownership topology based on the cardinalities. The reason is that I_L^C computes the MCL of local ownership topology information independently even though the local ownership topologies of different sub-regions are related. This is the overhead due to the fragmentation of a global ownership topology into multiple independent local ownership topologies; and the overhead (16) is increasing with M.

3) MCL based on Topology Stationary Probability Distribution: Each node is equally likely to belong to any of the sub-regions with probability $\frac{1}{M}$. For each sub-region, the maximum information required to specify a cluster head is $\log N$. Let I_L^P denote the MCL based on the stationary probability distribution.

$$I_L^P \le -N\left(\frac{1}{M}\log\frac{1}{M} + (1-\frac{1}{M})\log(1-\frac{1}{M})\right) + \log N \tag{17}$$

For large M,

$$\lim_{M\to\infty} \frac{\log M + (M-1)\log M - (M-1)\log(M-1)}{\log M} = 1 \tag{18}$$

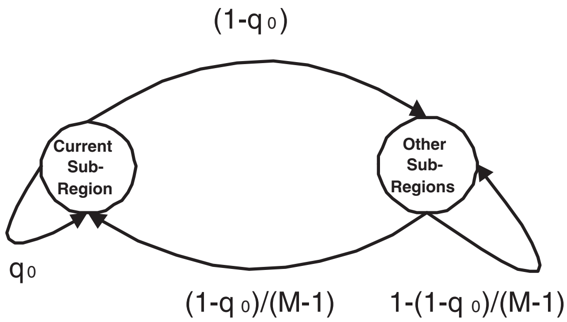

Fig. 3. The state transition diagram of a node moving into and out of a given sub-region.

and hence,

$$I_L^P \leq \frac{N \log M + M \log N}{M} \; ; M \gg 1 \tag{19}$$

From (15) and (19),

$$\frac{I_L^P}{I_L^C} \leq \frac{N \log M + M \log N}{NM} \; ; M \gg 1 \; ; N \gg 1 \tag{20}$$

4) MCL based on Prediction Using Previous Topology Knowledge: Assuming that each node is equally likely to move to any of its neighboring sub-regions, the probability that a node moves into a given sub-region within τ_e is $\frac{1-q_0}{M-1}$. Figure 3 depicts the state transition diagram of a node ownership for a given sub-region. The ownership of a node is defined as the status of whether the node belongs to the sub-region or not. Given that the node is owned by the sub-region, the information amount to describe ownership change at the next time step is $H(q_0)$. Similarly, given that the node is not owned by the sub-region, the information amount to describe ownership change at the next time step is $H(\frac{1-q_0}{M-1})$.

The events that could change the local ownership topology of a given sub-region are: 1) the change of the ownership of some nodes due to nodes joining from other sub-regions or moving out of the sub-region; and/or 2) the possible need of re-selecting a cluster head due to the departure of the existing cluster head or the change of an empty sub-region to a non-empty sub-region.

For a sub-region with n nodes, the first part requires

$$i_1(n) = nH(q_0) + (N-n)H(\frac{1-q_0}{M-1}) \tag{21}$$

bits. The probability that a new cluster head needs to be selected at next time step is $(1-q_0)$ if $n > 0$, or 1 if $n = 0$. If there is a need for selecting a cluster head, the information amount to specify a new cluster head is $\log N$. Therefore the second part of information amount is $(1-q_0)\log N$ bits. Finally, the information amount to describe the topology change is

$$i(n) = nH(q_0) + (N-n)H\left(\frac{1-q_0}{M-1}\right) + (1-q_0)\log N \tag{22}$$

The probability distribution of $\mathcal{N}$ (the number of nodes in a sub-region) is binomial with parameter $\frac{1}{M}$. Let I_L^M denote the MCL based on previous local ownership topology knowledge. Taking the average over $P_{\mathcal{N}}(n)$,

$$\begin{aligned} I_L^M &= (1-q_0)\log N + \\ &\sum_{n=0}^{N} \left[\tfrac{N!}{n!(N-n)!}\left(1-\tfrac{1}{M}\right)^{(N-n)}\left(\tfrac{1}{M}\right)^n\right] i(n) \end{aligned} \tag{23}$$

Define

$$i_L^M \triangleq \frac{1}{M}H(q_0) + \frac{(M-1)}{M}H(\frac{1-q_0}{M-1}) \tag{24}$$

i_L^M can be viewed as the MCL for a node ownership status change.

Theorem 3.3:

$$i_L^M \leq H(\frac{1}{M}) \leq 1 \tag{25}$$

Proof: See [8]. ∎

It is easy to show after some algebraic manipulations that

$$\begin{aligned} I_L^M &= \tfrac{N}{M}H(q_0) + \tfrac{(M-1)N}{M}H(\tfrac{1-q_0}{M-1}) + (1-q_0)\log N \\ &= Ni_L^M + (1-q_0)\log N \end{aligned} \tag{26}$$

$I_L^M = 0$ if $q_0 = 1$ (i.e. if the node does not leave the current sub-region).

C. Local Detailed Topology

1) Cardinality:

Theorem 3.4: The total number of possible local detailed topologies in a sub-region with n nodes is

$$D = n2^{\frac{n(n-1)}{2}} \tag{27}$$

Proof: This can be easily deduced from the adjacency matrix of the graph representation of local ownership topology. The matrix of any instance of this random graph is an $(n \times n)$ symmetric matrix with the diagonal element values equal to 1. There are only $\frac{n(n-1)}{2}$ independent elements. Each element reflects a link status between two nodes of the sub-region. We multiply the quantity by n because there are n different ways of selecting the cluster head. ∎

2) MCL Based on Cardinality: Let I_D^C denote the MCL based on cardinality.

$$I_D^C = \frac{n(n-1)}{2} \tag{28}$$

The MCL can be viewed as a binary bit string with length $\frac{n(n-1)}{2}$. Each bit is either 1 or 0 to reflect whether the direct link exists or not. I_D^C scales as $\Theta(n^2)$. Here, we remove $\log(n)$ from the expression of I_D^C. $\log(n)$ is the information to specify the cluster head. We do not consider the cluster head change when addressing the evolution of local detailed topology.

3) MCL Based on Topology Stationary Probability Distribution: As addressed in Section II-C, the link status of two nodes of a given sub-region is a random variable. Let p_1 denote the probability that the random variable equals to 1, i.e. the two given nodes have a direct link. Then $H(p_1)$ is the MCL. There is a total of $n(n-1)/2$ possible links for the sub-region.

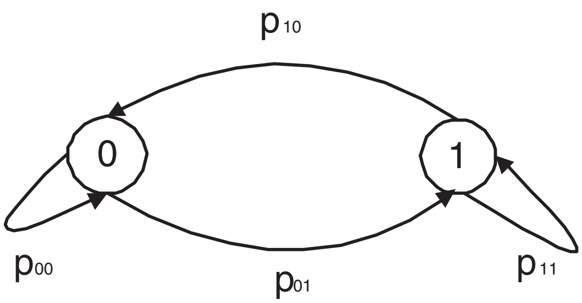

Fig. 4. The state transition diagram of a Markov process for the status of a given link.

Therefore, the MCL I_D^P based on the probability distribution is

$$\begin{aligned} I_D^P &= \tfrac{n(n-1)}{2}(-p_1 \log p_1 - (1-p_1)\log(1-p_1)) \\ &= \tfrac{n(n-1)}{2} H(p_1) \end{aligned} \quad (29)$$

I_D^P also scales as $\Theta(n^2)$. Compared to (28), I_D^P is less than I_D^C. The ratio of the two MCLs is $H(p_1)$. In the next section, we provide an expression for p_1 as a function of p_{00} and p_{11}.

4) MCL Based on Prediction Using Previous Topology Knowledge: As addressed in Section II-C, the change of an individual link is modeled as a Markov process depicted in Figure 4. When the Markov process reaches its steady state,

$$\begin{pmatrix} 1-p_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} p_{00} & 1-p_{11} \\ 1-p_{00} & p_{11} \end{pmatrix} \begin{pmatrix} 1-p_1 \\ p_1 \end{pmatrix} \quad (30)$$

then,

$$p_1 = \frac{1-p_{00}}{2-p_{11}-p_{00}} \quad (31)$$

Given that two nodes are not directly connected, the information amount to describe the link status at the next time step is $H(p_{00})$. Similarly, given that two given nodes are directly connected, the information amount to describe the link status at the next time step is $H(p_{11})$.

Let s denote the total number of possible direct links of a sub-region, then $s = \frac{n(n-1)}{2}$.

For a given topology with l $(0 \le l \le s)$ directly connected links, the information amount to describe the change of the topology is

$$i(l) = (s-l)H(p_{00}) + lH(p_{11}) \quad (32)$$

In our analytic derivations, we assume that the random variables representing the link status are independent and identically distributed (i.i.d). The argument is as follows. Let us randomly select two possible links, say X denotes the link status between node a and node b and variable Y denotes the link status between node c and node d. If all four nodes are distinct, clearly X and Y are independent of each other. The other situation is that one of the nodes is common, say a and d are the same. From the assumption that nodes have an i.i.d mobility pattern, we can infer that the nodes also have i.i.d mobility patterns if observed from node a. Therefore, X and Y are still independent from each other. Let Z denote the link status between node b and node c. Unfortunately, Z is not independent from X and Y. For example, if both X and Y equal 1, i.e. there is a direct link between a and b and between a and c, then there is a higher probability that there is direct link between b and c. This conditional probability enters into the equations with a factor of p_1^2 since X and Y are independent form each other. Since in typical situations, p_1 itself is a relatively small number, the effect of this conditional probability is small.

From the above discussion, the probability distribution of L, the number of links in a local detailed topology is

$$p_L(l) = \binom{s}{l} p_1^l (1-p_1)^{(s-l)} \; ; 0 \le l \le s \quad (33)$$

Taking the expectation of $i(l)$ using (33) and after some algebraic manipulations

$$\begin{aligned} I_D^M &= \tfrac{n(n-1)}{2}((1-p_1)H(p_{00}) + p_1 H(p_{11})) \\ &= \tfrac{n(n-1)}{2} i_D^M \end{aligned} \quad (34)$$

where

$$i_D^M = (1-p_1)H(p_{00}) + p_1 H(p_{11}) \quad (35)$$

I_D^M is the MCL based on the knowledge of the local detailed topology at the previous time instant. Similar to I_D^C and I_D^P, I_D^M also scales as $\Theta(n^2)$. i_D^M can be interpreted as the MCL for each link i_D^M given the previous link status. Compared with (29), the ratio of the two MCLs is $i_D^M / H(p_1)$.

Theorem 3.5: The MCL based on the probability distribution is larger or equal to the MCL based on prediction, i.e.,

$$I_D^M \le I_D^P \le \frac{n(n-1)}{2} \quad (36)$$

and

$$i_D^M \le H(p_1) \le 1 \quad (37)$$

Furthermore, the two MCLs are equal if and only if $(p_{00} + p_{11}) = 1$.

Proof: See [8]. ■

IV. Topology Evolution and Entropy Rate

Entropy rate [1] is defined as the rate of increase of the entropy of a sequence of random variables as the length of the sequence n increases. For $n \to \infty$, this becomes the entropy rate of a stochastic process. The entropy rate E of a stochastic process R_i is given by

$$E = \lim_{n\to\infty} \frac{1}{n} H(R_1, R_2, ..., R_n) \quad (38)$$

For a stationary Markov sequence $(R_1, R_2, ..., R_n, ...)$ with m states,

$$H = -\sum_{i=0}^{m} p_i \sum_{j=1}^{m} p_{j|i} \log p_{j|i} \quad (39)$$

where $(p_1, ..., p_m)$ is the steady state distribution, and $p_{j|i}$ is the state transition probability from state i to state j. This is called the "Markov Entropy."

In the following, we analyze two random processes. The first is the sequence of the local ownership topologies at the time instants that are multiples of τ_e. The second is the sequence of local detailed topologies at time instants that are multiple of τ_i. We demonstrate that both of the random processes are Markov. Finally, the MCLs of (26) and (34) are used to estimate the entropy rates for the above two random processes. The entropy rates are functions of the parameters of the mobility model q_0, p_{00} and p_{11}.

A. Local Ownership Topology

The sequence of ownership status $\{Y_j\}_{j=1}^{\infty}$ forms a Markov process. Let V_i denote a vectored random variable formed by the N random variables of ownership status of all the nodes for the sub-region. The sequence $\{V_j\}_{j=1}^{\infty}$ forms a random process. Clearly, $\{V_j\}_{j=1}^{\infty}$ is also a Markov process. Let the real-valued random variable Z_j denote the *RUI* of the cluster head of the sub-region. Let $L_j = (V_j, Z_j)$. L_j represents the local ownership topology at time step i. Because the probability distribution of Z_{j+1} (next cluster head) at next time step $(j+1)$ is uniquely determined by V_j and Z_j, the sequence of $\{L_j\}_{j=1}^{\infty}$ also forms a Markov process $\mathbb{L}$.

Let E_L denote the entropy rate of the random process $\mathbb{L}$. From (39), the entropy rate is the average conditional entropy given the stationary probability distribution of the random variable. Since I_L^M is the MCL based on the previous local ownership topology, then

$$E_L = I_L^M = \tfrac{N}{M} H(q_0) + \tfrac{(M-1)N}{M} H(\tfrac{1-q_0}{M-1}) + (1-q_0)\log N \quad (40)$$

B. Local Detailed Topology

Similarly, the evolution of the local detailed topology for a given sub-region is a Markov process. As discussed in Section III-C, we can use a random graph to represent the local detailed topology. The adjacency matrix of the random graph has $\frac{n(n-1)}{2}$ independent elements. The value of each element is a random variable representing the status of a link between a pair of nodes in the sub-region. The link status evolution is a Markov process. The $\frac{n(n-1)}{2}$ random variables are independent of each other. Let D_j denote the joint $\frac{n(n-1)}{2}$-tuple random variables. D_j represents the local detailed topology at the time step i. The sequence $\{D_j\}_{j=1}^{\infty}$ forms a Markov process $\mathbb{D}$. Let E_D denote the entropy rate of this random process, then,

$$E_D = I_D^M = \frac{n(n-1)}{2}((1-p_1)H(p_{00}) + p_1 H(p_{11})). \quad (41)$$

V. Memory Requirement Analysis

The method of deriving the memory requirement is as follows. We first determine the topology granular view that needs to be stored based on the role of a node (as cluster head or regular node). For each topology granular view, we separate the memory requirement into two portions, a permanent portion to store the current topology, and a temporary portion to store the topology change. For each topology, the entropy of the stationary probability distribution is the lower bound on the memory required; for topology change, the entropy rate (information amount based on the knowledge of previous topology) is the lower bound on the temporary (additional) memory. Notice that, when there is a change of topology, both the current topology and the topology change information are used to infer the topology at the next time slot. The methodology of finding the memory requirement is summarized in Figure 5.

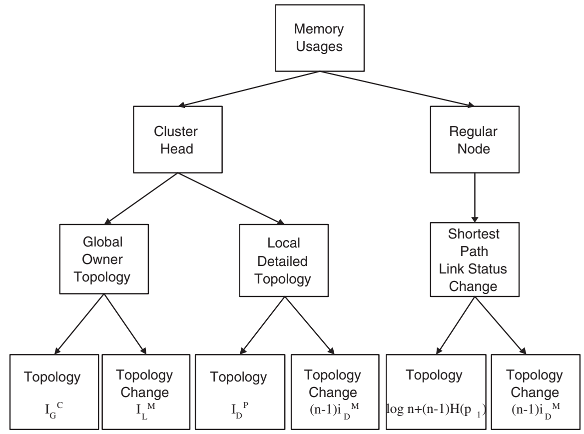

Fig. 5. Memory requirement for cluster head and regular node.

1) Regular Node: A regular node needs to collect the link status changes and sends it to its cluster head, which requires that the regular node store the link status ($H(p_1)$) and link status change (i_D^M) with other nodes of the same cluster. In addition, the regular node needs to maintain a shortest path to its cluster head by remembering the next node along the path. Therefore, the total memory requirement becomes

$$\mathbb{M}_r = \log n + (n-1)\left(H(p_1) + i_D^M\right) \quad (42)$$

Here $\log n$ is the lower bound on the amount of memory that a regular node needs to remember the shortest path to its cluster head (or $\log N$ if the *NUI* of the next node is used). The $(n-1)H(p_1)$ is the permanent portion of memory requirement; and $(n-1)i_D^M$ is the temporary portion of memory requirement. The factor $(n-1)$ comes from the fact that there are $(n-1)$ possible links for a regular node with other nodes.

2) Cluster Head: A cluster head needs to store both the local detailed topology and the global ownership topology. For local detailed topology, the permanent portion of memory requirement is I_D^P, and the temporary portion of memory requirement is $(n-1)i_D^M$. For global ownership topology, the permanent portion of memory requirement is I_G^P, and the temporary portion of memory requirement is I_L^M.

Hence, the memory requirement for maintaining local detailed topology is

$$\mathbb{M}_{cd} = I_D^P + (n-1)i_D^M \quad (43)$$

and, the memory requirement for maintaining global ownership topology is

$$\mathbb{M}_{cg} = I_G^P + I_L^M = I_G^C + I_L^M \tag{44}$$

Here, we use $I_G^P = I_G^C$ of (29). Finally, the total memory requirement for cluster head is

$$\mathbb{M}_c = \mathbb{M}_{cg} + \mathbb{M}_{cd} = I_G^C + I_L^M + I_D^P + (n-1)i_D^M \tag{45}$$

The expressions I_D^P, I_G^P, I_L^M and i_D^M are given in (29),(2), (23), and (35).

VI. Routing Overhead Analysis

In this section, we derive lower bounds on exterior and interior routing overheads. The total routing overhead is the sum of two parts.

A. Exterior Routing

The exterior routing overhead arises from the need of maintaining the global ownership topology. Each cluster head distributes its local ownership topology or its change to other cluster heads. The lower bound on the exterior routing overhead is computed as the product of routing message length (i.e. the MCL) and the average number of hops the message travels (assuming shortest paths).

We assume that each cluster forms and remembers a spanning tree (an algorithm for creating the spanning tree is given in [8]) to distribute the routing message in order to avoid redundant message transmissions. l_c is the average path length between two cluster heads of neighboring sub-regions. The number of hops the update message travels is $(M-1)l_c$. Therefore, the minimum average routing overhead (in bits) during a period of time of τ_e is

$$R_e = M(M-1)I_L^M l_c \tag{46}$$

where the factor M comes from the fact that there are M sub-regions.

B. Interior Routing

The interior routing overhead consists of routing overhead components caused by detecting the link status changes, maintaining the *local detailed topology*, and notifying/informing regular nodes of the new shortest paths.

Let R_h denote the number of bits required to detect the link status changes within a time unit τ_i. Then

$$R_h = N \log n \tag{47}$$

where $\log n$ is the number of bits required for a node to identify itself to its neighboring nodes.

Let R_d denote the number of bits required to maintain the local detailed topology within a time period of τ_i. During the time period, each regular node sends one update message of link status change to its cluster head. The cluster head infers the topology change from the update messages from its regular nodes. The lower bound on the length of the update message is $(n-1)i_D^M$ for a regular node. There are $(n-1)$ regular nodes. Therefore,

$$R_d = (n-1)(n-1)l_r i_D^M \approx 2 l_r I_D^M \tag{48}$$

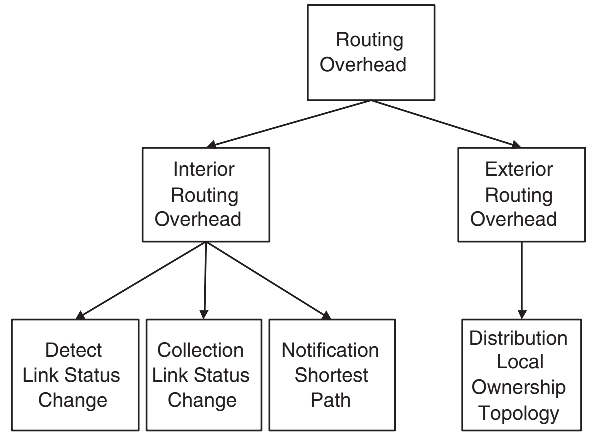

Fig. 6. Routing overhead composition.

where l_r is the average path length from a regular node to its cluster head.

Let R_p denote the number of bits required to send the new shortest paths information (i.e. information about the shortest paths between each node and its cluster head) from the cluster head to the regular nodes at the end of a time slot τ_i. As a consequence of the observation made in Section II-B (last paragraph), an update message for the shortest path for a regular node also updates all the shortest paths of the intermediate nodes along the path. It can be shown (see [8]) that,

$$\begin{aligned} R_p &= (1-p_{11})(n-1)\left(\tfrac{l_r}{2}\right) l_r^2 \log n \\ &\approx \tfrac{(1-p_{11})(n-1)l_r^3 \log n}{2} \end{aligned} \tag{49}$$

Finally, the interior routing overhead is the sum of the three above components, hence,

$$R_i = R_h + R_d + R_p \approx N \log n + 2 I_D^M l_r + R_p \tag{50}$$

C. Total Routing Overhead

Let $\mathbb{R}_h$ denote the minimum bit rate required to detect the link status change, $\mathbb{R}_h = \frac{R_h}{\tau_i}$; $\mathbb{R}_d$ denote the minimum bit rate required to track the changes of the local detailed topology for all the sub-regions, $\mathbb{R}_d = \frac{R_d}{\tau_i}$; and $\mathbb{R}_p$ denote the minimum bit rate required to send the new shortest paths information to the regular nodes for all the sub-regions, $\mathbb{R}_p = \frac{R_p}{\tau_i}$. Finally,

$$\mathbb{R}_i = \mathbb{R}_h + \mathbb{R}_d + \mathbb{R}_p = \frac{N \log n + 2M I_D^M l_r}{\tau_i} + \frac{M R_p}{\tau_i} \tag{51}$$

$$\mathbb{R}_e = \frac{M(M-1)I_L^M l_c}{\tau_e} \tag{52}$$

$$\begin{aligned} \mathbb{R}_t &= \mathbb{R}_e + \mathbb{R}_i \\ &= \tfrac{M(M-1)I_L^M l_c}{\tau_e} + \tfrac{N \log n + M(I_D^M l_r + R_p)}{\tau_i} \end{aligned} \tag{53}$$

The different components of routing overhead are summarized in Fig. 6.

TABLE III

MEMORY REQUIREMENT.

	Topology	Topology Change	Ratio	Scale
$\mathbb{M}_r$	$\log n+$ $(n-1)H(p_1)$	$(n-1)i_D^M$	$\frac{i_D^M}{H(p_1)}$	$O\left(\frac{N}{M}\right)$
$\mathbb{M}_{cd}$	I_D^P	$(n-1)i_D^M$	$\frac{2i_D^M}{nH(p_1)}$	$O\left(\frac{N^2}{M^2}\right)$
$\mathbb{M}_{cg}$	I_G^C	I_L^M	$\frac{i_L^M}{\log M}$	$O(N \log M)$

TABLE IV

SCALABILITY OF ROUTING OVERHEAD WITH N.

	Expression	Case 1	Case 2	Case 3
$\mathbb{R}_e$	$\frac{M(M-1)I_L^M}{\tau_e} l_c$	$N^{\frac{3}{2}}$	$\frac{N^{\frac{3}{2}}}{\log N}$	N
$\mathbb{R}_h$	$\frac{N \log n}{\tau_i}$	$N^{\frac{3}{2}} \log N$	$N^{\frac{3}{2}}$	$N \log N$
$\mathbb{R}_d$	$\frac{Mn(n-1)i_D^M}{\tau_i} l_r$	$N^{\frac{5}{2}}$	$\frac{N^{\frac{5}{2}}}{\log N}$	N^2
$\mathbb{R}_p$	$\frac{M(1-p_{11})(n-1)\log n}{2\tau_i} l_r^3$	$N^{\frac{5}{2}} \log N$	$N^{\frac{5}{2}}$	$N^2 \log N$

VII. SCALABILITY RESULTS

In this section, we analyze the scalability of memory requirement and and routing overhead. For the routing overhead, three models of scalability are considered. Each of the models represents a different method of physical scaling of the network.

To derive the scalability result, we assume that the node distribution within clusters is the same on average, and use $\left(\frac{N}{M}\right)$ for the asymptotic value of n, the number of nodes in a sub-region.

A. Memory Scalability

Table III gives a summary of the memory requirements from Section V. The forth column is the ratio of memory requirements of temporary portion to the permanent portion. The last column is how $\mathbb{M}_r$, $\mathbb{M}_{cd}$ and $\mathbb{M}_{cg}$ scale with N and M.

For $\mathbb{M}_r$, the ratio is $\frac{i_D^M}{H(p_1)}$, which is always less than or equal to 1 based on (37). $\mathbb{M}_r$ scales with the number of nodes n in the sub-region. For $\mathbb{M}_{cd}$, the ratio is $\frac{1}{n}$. $\mathbb{M}_{cd}$ scales with n^2. The major component of $\mathbb{M}_{cd}$ is the memory required to store the current local detailed topology I_D^P. For $\mathbb{M}_{cg}$, the ratio is $I_L^M/I_G^C = \frac{i_L^M}{\log M}$. i_L^M is the node ownership change information and $\log M$ is node ownership information. From (25), the ratio is smaller than $\frac{1}{\log M}$. $\mathbb{M}_{cg}$ scales with $N \log M$. The major component of $\mathbb{M}_{cg}$ is the memory required to store current global ownership topology I_G^C.

B. Routing Overhead Scalability

We derive the routing overhead scalability using the results of Section VI in (46)-(49). These results are summarized in the second column of Table IV. It is worth noting that all the routing overhead components in Table IV are proportional to either l_r or l_c, except $\mathbb{R}_h$ since a "HELLO" message travels over one hop only.

To compute the scalability results from the tables, it is necessary to derive expressions for l_r and l_c. We first make the following approximation,

$$l_c \approx 2l_r \tag{54}$$

The argument is as follows. To send a message from a cluster head to one of its neighbor cluster heads, a message has to travel an average of l_r to reach the boundary of the neighboring cluster and another l_r to reach from the boundary to the head of the neighboring cluster. Next, we relate d_0 to l_r. Here, d_0 denotes the communication radius of a node. Let d_r denote the physical distance between a regular node and its cluster head, and A denote the physical area covered by the network, then $d_r = \Theta\left(\sqrt{\frac{A}{M}}\right)$, and $l_r = \Theta\left(\frac{d_r}{d_0}\right) = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right)$.

Finally, three different cases for selecting d_0 for physically scaling the network are considered. The final results are summarized in Table IV.

1) Model 1: Keep Average Node Degree Constant: g is defined as the average number of nodes within the direct communication radius of a given node, therefore $g = \pi d_0^2(\frac{N}{A})$, and

$$l_r = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right) = \Theta\left(\sqrt{\frac{N}{M}}\right) = \beta\sqrt{\frac{N}{M}} \tag{55}$$

2) Model 2: Keep Network Connected: From [9], g should be $\Theta(\log N)$ to keep the network asymptotically connected. Then $g = \pi d_0^2(\frac{N}{A}) = \Theta(\log N)$, and,

$$l_r = \Theta\left(\sqrt{\frac{N}{M \log N}}\right) \tag{56}$$

3) Model 3: Keep Communication Radius Constant: If d_0 is kept constant,

$$l_r = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right) = \Theta\left(\sqrt{\frac{1}{M}}\right) \tag{57}$$

VIII. PRACTICAL IMPLICATIONS

In this section, we apply the theoretical results of previous sections to answer some ad hoc networks design-related questions. The goal of the section is to provide some general guidelines in selecting encoding techniques and network parameters based on the design priorities.

A. Cluster Size and Memory Requirement

1) The Cluster Size Minimizing the Memory Requirement of Cluster Heads: The memory for current global ownership topology is $N \log M$ (6). The memory requirement for current local detailed topology is $\frac{N^2}{2M^2}H(p_1)$ (29) or $\frac{N^2}{2M^2}$ (28), depending on the encoding technique.

In the following, we use $\frac{N^2}{2M^2}$ instead of $\frac{N^2}{2M^2}H(p_1)$ as the memory requirement to store the interior topology of a cluster (local detailed topology). There are two reasons for doing this. First, p_1 is a parameter related to the mobility, usually it is not a design parameter. Second, in practical scenario, it is more reasonable to use a method not requiring the knowledge of mobility parameter p_1 to encode/decode and store the interior information of a cluster. Now, the total memory requirement for a cluster head is

$$\mathbb{M}_c = N \log M + \frac{N^2}{2M^2} \tag{58}$$

Taking the derivative with respect to M , the cluster size that asymptotically minimizes the memory requirement for each cluster head is

$$M_{optm} = \sqrt{(\ln 2) N} \tag{59}$$

2) The Cluster Size Minimizing the Ratio of Memory Requirements for Cluster Head and Regular Node: Widely different memory requirements for the roles of cluster head and regular node could be a practically unattractive feature. For example, since nodes switch roles, it will be necessary to equip all nodes with memory equal to the larger of the two quantities, and hence incur extra cost of hardware. In the following, we derive the ratio between the memory requirements of a cluster head and a regular node. The lower bound on memory requirement for a regular node is given in (42) using the prediction based encoding technique. Using the same argument as for calculating the memory requirement for cluster head (58), we use the encoding technique based on cardinality to distribute the link status change. The memory requirement becomes $\mathbb{M}_r = \log n + (n-1) \approx \frac{N}{M}$. Here, we neglect the $\log n$ term, and approximate $(n-1)$ by n for simplicity. Finally,

$$r_m = \frac{\mathbb{M}_c}{\mathbb{M}_r} = M \log M + \frac{N}{2M} \tag{60}$$

The ratio is only a function of N and M, and it reaches its minimal value when $M = M_{ratio}$ where M_{ratio} satisfies

$$2M_{ratio}^2 (1 + \ln M_{ratio}) = (\ln 2) N \tag{61}$$

(59) and (61) tell us that in general the optimal cluster size (M_{optm}) for memory requirement and the optimal cluster size (M_{ratio}) for the memory ratio are not necessarily the same. (In fact, they are usually not equal).

B. The Cluster Size Minimizing the Routing Overhead

An important design question is how the cluster size impacts the communication routing overhead. Is there an optimal cluster size M_{optr} that asymptotically minimizes the communication routing overhead?

In the following, we derive the optimal cluster size M_{optr} for the physical scaling model of case 1 (Section VII-B.1) and use encoding technique based on cardinality to distribute routing messages for maintaining both exterior topology and interior topology. The routing overhead for each component is summarized in Table V.

TABLE V
SCALABILITY OF ROUTING OVERHEADS WITH M

	Expression	Scale
$\mathbb{R}_e$	$\frac{2M(M-1)N}{\tau_e}\left(\beta\sqrt{\frac{N}{M}}\right)$	$M^{\frac{3}{2}}$
$\mathbb{R}_h$	$N \log N$	1
$\mathbb{R}_d$	$\frac{N^2}{\tau_i M}\left(\beta\sqrt{\frac{N}{M}}\right)$	$\frac{1}{M^{\frac{3}{2}}}$
$\mathbb{R}_p$	$\frac{(1-p_{11})N \log N}{2\tau_i}\left(\beta\sqrt{\frac{N}{M}}\right)^3$	$\frac{1}{M^{\frac{3}{2}}}$

Taking derivative of the sum of the items in Table V with respect to M and setting the derivative to zero, and after some algebraic manipulations,

$$M_{optr} = \sqrt[3]{\frac{\tau_e \left(2N + (1-p_{11})\beta^2 N \log N\right)}{4\tau_i}} \tag{62}$$

The optimal cluster sizes for other two physical scaling models can also be deduced in a similar manner.

C. Impact of Selection of Encoding Technique

We have presented three encoding techniques for the control messages, based on cardinality, probability distribution (of the topology), or (mobility) prediction. The impact of selection of the encoding technique on the routing overhead is investigated in this section.

1) Impact on Exterior Routing Overhead: First, observe the following. The lower bound on the exterior routing overhead is given in (46) in terms of I_L^M (26) assuming prediction based encoding. The result for the other two techniques can be computed by replacing I_L^M in (46) by I_L^C (from (13)) or I_L^P (from (15)). Comparing these quantities, amounts to comparing N, $NH(\frac{1}{M})$ and Ni_L^M. Theorem 3.3 (25) shows that $i_L^M \leq H(\frac{1}{M}) \leq 1$.

Let us first compare between the prediction based technique, on one hand, versus the other two non-prediction based techniques, on the other hand. For the case that there is not much mobility ($q_0 \approx 1$), using the prediction based encoding technique will be a better choice. This also agrees with the intuition since with high q_0, there is not much benefit of trying to predict the next location of the node.

Now consider the two non-prediction based techniques. The ratio of the overheads of the cardinality-based to the probability distribution-based techniques is determined by $H(\frac{1}{M})$. Hence, it is straightforward to conclude that the probability distribution-based technique is especially beneficial in the case of large M, which is also intuitively reasonable. For example, if there are two clusters, there is not much difference using the two encoding techniques. But when there are four clusters, using the probability distribution encoding technique results in reducing the overhead by 19%, according to the results, and by 46% when there are eight clusters. The percentage is computed using $(1 - H(\frac{1}{M}))$.

2) Impact on Interior Routing Overhead: Interior routing overhead consists of three parts (Table IV), only the $\mathbb{R}_d$ term is related to the encoding technique used. The impact of selection of encoding technique on $\mathbb{R}_d$ can be quantified by comparing

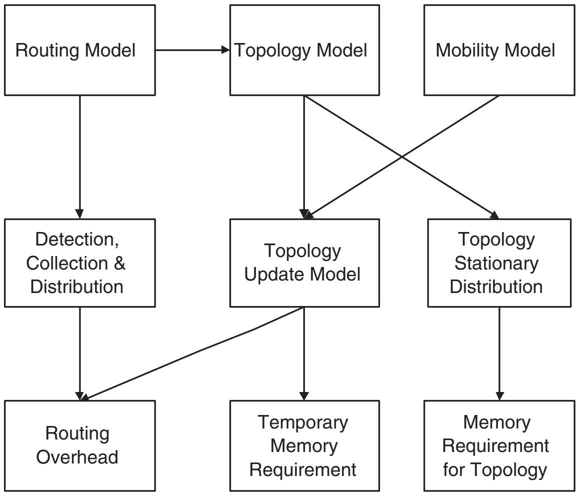

Fig. 7. Methodology of finding lower bounds on memory requirement and routing overhead.

1, $H(p_1)$ and i_D^M. From Theorem 3.5 (36), we have $i_D^M \leq H(p_1) \leq 1$.

The analysis here parallels the one for exterior routing overhead. For the case that there is not much mobility ($p_{00} \approx 1$ and $p_{11} \approx 1$) to change the link status (the change of local detailed topology) for nodes within τ_i, using the prediction based encoding technique will be a better choice. The overhead comparison between cardinality-based and distribution-based techniques is determined by $H(p_1)$. For the extreme two cases where $p_1 \approx 1$ or $p_1 \approx 0$, using the probability distribution-based encoding technique results in dramatic reduction of $\mathbb{R}_d$.

IX. Conclusion and Future Directions

In this paper, we used information theory to build a methodology to analyze the lower bounds on memory requirement and routing overhead for hierarchical proactive routing in mobile ad hoc networks. The procedure of applying this methodology is summarized in Figure 7.

Many avenues of future work can build on this work. First the same methodology could be applied to multiple-level proactive protocols. Second, different routing protocol architectures, in terms of topology information aggregation and routing message distribution schemes could be considered, as a variation of the distributed scheme addressed in this paper. Third, the results could be extended to capture the re-clustering overhead for networks with dynamic cluster size. Forth, we can apply the methodology to analyze reactive routing protocols, where the routing overhead depends on the traffic pattern. Finally, the model can be applied to other models of networks that have variable topology over time, such as computation grids and peer-to-peer overlay networks.

References

[1] T.M. Cover and J.A. Thomas, *Elements of Information Theory*, John Wiley & Sons, 1991.

[2] C. Santivanez, B. McDonald, I. Stavrakakis, and R. Ramanathan, "On the scalability of ad hoc routing protocols," in *Procedings of IEEE INFOCOM*, 2002.

[3] John Sucec and Ivan Marsic, "Clustering overhead for hierarchical routing in mobile ad hoc networks," in *Proceding of IEEE INFOCOM*, 2002.

[4] R. G. Gallager, "Basic Limits on Protocol Information in Data Communication Networks," *IEEE Trans. Inform. Theory*, vol. IT-22, no. 4, pp. 385–398, 1976.

[5] C. Gavoille, "Routing in distributed networks: Overview and open problems," *ACM SIGACT News*, vol. 32, no. 1, pp. 36–52, March 2001.

[6] P. Jacquet, P. Muhlethaler, and A. Qayyum, "Optimized link state routing protocol (OLSR)," INTERNET-DRAFT draft-ietf-manet-olsr-03.txt, IETF MANET Working Group, November 2000.

[7] C. E. Shannon, "The mathematical theory of communication," *Bell System Technical Journal*, pp. 379–423 and 623–656, 1948.

[8] N. Zhou and A. Abouzeid, "Information theoretic bounds on proactive routing overhead in mobile ad hoc networks," Technial report, Rensselaer Polytechnic Institute (RPI), July 2004, http://www.ecse.rpi.edu/homepages/abouzeid/it-routing.pdf.

[9] Feng Xue and P. R. Kumar, "The number of neighbors needed for connectivity of wireless networks," *Wireless Networks*, vol. 10, no. 2, pp. 169–181, 2004.