Throughput Capacity of Hybrid Radio-Frequency and Free-Space-Optical (RF/FSO) Multi-Hop Networks

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***Abstract*— The per-node throughput capacity of hybrid radio frequency and free space optics (RF/FSO) networks is studied and the benefit of using this hybrid network architecture over the pure RF wireless networks is evaluated. The hybrid RF/FSO network consists of an RF ad hoc network of n nodes, m of them (so called super nodes) are equipped with an additional FSO transceiver. Every RF and FSO transceiver is able to transmit at a maximum data rate of W_1 and W_2 bits/sec, respectively. All the super node are connected by the FSO links and thus can form a stand-alone FSO network. With a minimum transmit power objective, an upper bound on the per node capacity of $c_1 W_1 \sqrt{\frac{1}{n \log n}} + c_2 W_2 \frac{\sqrt{m \log m}}{n}$ is derived. In order to prove that this upper bound is achievable, we design a hybrid routing scheme in which the data traffic is divided into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. By properly balancing the load between these two classes of traffic, it is shown that this upper bound is tight when the maximum data rate ratio of FSO and RF transceivers, $\frac{W_2}{W_1}$, grows slower than $\sqrt{n}$. Under such circumstances, the capacity improvement with the support of FSO nodes, as compared with the results for RF wireless networks in [1], is evaluated. A significant capacity gain will be achieved if $\frac{W_2}{W_1} m \log m = \Omega(n)$. The results characterize the number of super nodes and/or the FSO data rate necessary in order to cause a non-trivial increase in the per-node throughput.**

I. Introduction

The capacity of Radio Frequency (RF) wireless networks is constrained by provable limits and does not scale well with the increasing number of nodes in the system due to the interference between concurrent transmission from neighboring nodes [1]. While the RF wireless networks continue to develop and grow in demands, the technology of free space optics (FSO) starts to draw attention from both academia and industry. FSO can provide high data rate and highly directional transmissions using free-space laser beams, whose beam divergence angle is on the order of milli-radians.This perfect directionality in transmission makes the FSO communications nearly immune from interference. However, one of the major limitations of FSO technology is the need for optical links to maintain line-of-sight (LOS), and FSO link availability can be further limited by adverse weather conditions like fogs and heavy snowfalls. The complementary property of RF and FSO motivates us to design hybrid RF/FSO networks, in which the weaknesses of each link type are expected to be mutually mitigated.

It is quite natural to view this RF/FSO combination as a way to solve the capacity scarcity problem in RF wireless networks, or at least turn it around to some extent. Several practical designs of such hybrid networks have been proposed and/or implemented [2], [3], [4], [5] and [6], which may improve the network performance by providing higher throughput and/or better reliability. However, there is still no theoretical work that may give insight on how much the performance can be obtained and how much these improvements can be possibly achieved. Instead of trying to provide new practical solutions to these hybrid RF/FSO networks or to compare the performances of different protocols or algorithms available, this work aims to derive the fundamental limit of the capacity of hybrid RF/FSO networks and show exactly how much this may deviate from the capacity results in the landmark paper by Gupta and Kumar [1].

In this paper we consider a random network scenario, where n nodes are randomly located, i.e., independently and uniformly distributed on a unit area. Each node is equipped with an RF transceiver. Only m of them, which are called *super nodes*, are equipped with an additional FSO transceiver each. Each RF transceiver can omni-directionally transmit or receive at W_1 bits/sec within the communication range r. Each FSO transceiver can directionally transmit within an infinitesimally small angle and omni-directionally receive, all at W_2 bits/sec. The communication range of every FSO transceiver is s. For each node, its destination is randomly chosen and let λ (in bits/sec) denote the maximum data rate at which each source-destination pair is required to transmit and receive. By following the similar definition in [1], we define this λ as the *throughput capacity* of this hybrid RF/FSO network. Here we let this random network satisfy two asymptotic connectivity assumptions: a) All the n nodes are asymptotically connected by RF links; b) All the m super nodes are asymptotically connected by FSO links. This allows us to have two stand-alone networks consisting of n nodes connected only by RF links and m nodes connected only by FSO links, respectively. We need to notice that weather conditions may have different effects on RF and FSO links, as FSO links are highly susceptible to dense fog, smoke and dust particles but relatively less vulnerable to rain conditions and the opposite is true for RF systems. Thus under these asymptotical connectivity assumptions, the random hybrid network can accommodate for adverse weather conditions such as heavy rains or heavy fogs [5]. Although in this paper we do not study the case when RF or FSO links are unreliable, the modelling applied here may reserve possible avenues for the future work.

We analyze the throughput capacity of the hybrid RF/FSO networks under the constraint of minimum FSO transmit power consumption. We minimize the FSO transmit power consumption by choosing the FSO communication range s to be the minimum in maintaining the asymptotic connectivity among super nodes. Although our analysis indicates that the larger the FSO communication range is, the higher the throughput may be achieved, we still have to note that it is impossible to realize a communication system with an arbitrarily large communication range. Then it is reasonable to add certain kind of FSO transmit power constraints when trying to obtain the throughput capacity λ. The constraint of minimum FSO transmit power consumption considered in this paper is certainly a conservative one, and it may be more suitable for the case when energy saving is given high priority in the network design. Since our analysis also show that the RF communication range r is automatically minimized when maximizing the throughput capacity λ, then this power constraint can be rewritten as the constraint of minimum transmit power consumption with no ambiguity.

Our derivation of the throughput capacity can be divided into two parts. First we derive an upper bound on the throughput capacity λ over all routing and transmission strategies. Then we construct a hybrid routing scheme in which the data traffic is divided into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. It is shown that under certain conditions, the network can actually achieve at a throughput on the same order of n and m as the upper bound. We compare the results obtained here with the results on throughput capacity of RF wireless networks evaluated in [1] in order to characterize the capacity improvement contributed by those super nodes. It is observed that a noticeable capacity improvement will be achieved when the number of super nodes m and/or the FSO data rate W_2 are high enough.

The main contributions of this paper are summarized as follows: a) Derive the scaling laws for the throughput capacity of hybrid RF/FSO networks as a function of the total number of nodes n of which $m \leq n$ are super nodes; b) Present a routing and transmission scheme that achieves the derived throughput capacity, which may guide the practical routing protocol design for the hybrid RF/FSO networks; c) Compare with the results against pure RF wireless networks, analyze the capacity improvement of hybrid networks, and characterize the number of super nodes and/or the FSO data rate necessary in order to cause a non-trivial increase in the per node throughput.

The rest of this paper is organized as follows. A brief overview of related work is presented in Section II. Section III outlines the system model that is considered throughout the paper. Section IV derives an upper bound on throughput capacity of RF/FSO networks with minimum transmit power consumption. A constructive lower bound on throughput capacity of RF/FSO networks is presented in section V. Section VI discusses the capacity results obtained, and provides some practical implications. Section VII concludes the paper.

II. Related Work

To the best of our knowledge, there is no prior work in the literature on the capacity analysis of RF/FSO multi-hop networks. However, several attempts have been made to provide capacity improvement by utilizing directional antennas or introducing infrastructure support.

The authors in [7] analyzed the capacity improvement of RF wireless networks using directional antennas in three cases. The capacity gain is shown to be a) $\sqrt{\frac{2\pi}{\alpha}}$ when using directional transmission with divergence angle α and omni reception, b) $\sqrt{\frac{2\pi}{\beta}}$ when using omni transmission and directional reception with divergence angle β, and c) $\frac{2\pi}{\sqrt{\alpha\beta}}$ when both transmission and reception are directional. These capacity gains can only be achievable when the divergence angle is not too small. Thus the results derived in [7] are not applicable to our work since we deal with FSO links with infinitesimally small divergence angle.

[8], [9] and [10] try to improve the capacity by the introduction of infrastructure support (access points or base stations). Authors in [8] depict the infrastructure network as a cellular network, where the base stations are wired by a broadband network and placed at the center of hexagonal cells. They investigate how the number of base stations should scale with the number of ad hoc nodes to achieve significant capacity improvement over the pure RF ad hoc wireless networks. They apply different routing strategies in which the ad hoc nodes are divided into two groups depending on whether they use the cellular network to reach the destination or not. The decision criteria in forming the groups rely on heuristic arguments and may not be the optimal routing strategies. They show that the number of base stations should at least scale with $\sqrt{n}$ to achieve a noticeable gain. The infrastructure network used in [9] consists of randomly located access points which are pre-wired and allocated infinite capacity. The authors in [9] model the node distribution and traffic pattern in the same manner as the random network model used in [1]. They assume that the number of ad hoc nodes per access point is bounded above, and each wireless node is able to transmit at W bits/sec using a fixed transmission range. Under this random network scenario, they show that a per node capacity of $\Theta(\frac{W}{\log n})$ can be achieved. To do this, they specify the upper bound of throughput capacity over all routing and transmission strategies, and then design a specific routing and transmission scheme to achieve this upper bound. The results in [10] extend the work of [9] by allowing nodes to perform power control and properly choosing the number of access points, and further show that it is possible to provide a throughput of $\Theta(1)$ to any fraction $f, 0 < f < 1$, of nodes.

Although overlaps may exist between our work and the prior work such as [8], [9] and [10], there are also major differences that underline the contribution of our work: a) It is more realistic to deploy the super nodes randomly rather than employing a hexagonal cell structure, as compared to [8]; b) We do not impose strong assumptions such as the number of nodes per super node should be bounded or properly chosen, as compared to [9] and [10]; c) We let the maximum data rate of each FSO transceiver, W_2, be some finite value, which is more realistic as compared to the infinite capacity assumptions used in [9] and [10].

III. System Model

The system model includes the settings of node distributions, traffic patterns, and RF/FSO communication models.

A. Node Distributions and Traffic Pattern

In a random scenario, n nodes each equipped with an RF transceiver are randomly located, i.e., independently and uniformly distributed on the surface S^2 of a three-dimensional sphere of area $1m^2$. Only m of them, which are called super nodes, are equipped with an additional FSO transceiver each. Our purpose in studying S^2 is to separate edge effects from other phenomena. Each node has a randomly chosen destination to which it wishes to send λ bits/sec. The destination for each node is independently chosen as the node nearest to a randomly located point, i.e., uniformly and independently distributed. Thus destinations are on the order of $1m$ away on average. This model, except for the FSO capability, is similar to the model in [1].

B. RF Communication Model

Each RF transceiver can omni-directionally transmit or receive at W_1 bits/sec. We use the Protocol Model introduced in [1] as the RF interference model here. All nodes employ a common range r for all their transmissions. Note that all the distances are measured on the surface S^2 of the sphere by segments of great circles connecting two points. Let X_i denote the location of a node; we will also use X_i to refer to the node itself. When node X_i transmits to a node X_j over the RF channel, the data will be successfully received by X_j if

1) The distance between X_i and X_j is no more than r, i.e.,

$$|X_i - X_j| \leq r \tag{1}$$

2) For every other node X_k simultaneously transmitting over the RF channel

$$|X_k - X_j| \geq (1 + \Delta)r \tag{2}$$

The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the RF channel at the same time. It also allows for imprecision in the achieved range of transmissions.

C. FSO Communication Model

Every super node is equipped with an FSO transceiver. Each FSO transceiver can directionally transmit at W_2 bits/sec within an infinitesimally small angle and a common range s. Each FSO transceiver can receive at W_2 bits/sec omni-directionally.

We assume that the orientation of each FSO transmitter can be steered to any possible direction within S^2. Then every super node can transmit data to any other super node within the distance s. In practice, the beam steering functionality has been realized in several ways. Milner et.al. [2] implemented the beam steering using mechanical devices, while Khan et.al. [11] designed a 3-dimensional wide-angle no-moving-parts laser beam steering method. Many other design choices are also available and can be found from [12], [13], [14] and [15]. The omni-directionality assumption for the FSO receiver is also reasonable, as omni-directional FSO receivers are also implemented in [4].

IV. An Upper Bound on Throughput Capacity of RF/FSO Networks with Minimum Power Consumption

In this section, we analyze the throughput capacity of hybrid RF/FSO networks *with minimum total power consumption*. In RF wireless networks, it may be desirable to reduce the transmit power level, or equivalently, to reduce the transmission range r in order to increase network capacity. Kawadia and Kumar [16] argue that the area of the interference is proportional to r^2 whereas the relaying burden, i.e., the number of hops, is inversely proportional to r. Then the area consumed by a packet is thus proportional to r, implying that reducing the transmit power level increases the capacity.

In FSO networks, however, reducing the transmit power level and increasing the capacity are essentially contradicting with each other. Since the area of the FSO interference is actually negligible, the relaying burden is alleviated by simply increasing the FSO transmission range s, which implies that increasing the transmit power level increases the capacity.

Therefore, the objective of minimizing the total power consumption in hybrid RF/FSO networks seems to be contradictory in achieving a higher capacity. However, the capacity results derived here may still be valuable, since reducing the power consumption is always considered to be important when deploying large-scale networks where the resource of energy is limited.

Theorem 4.1:

The throughput capacity of hybrid RF/FSO networks with minimum power consumption is bounded from above by

$$\lambda \leq c_1 W_1 \sqrt{\frac{1}{n \log n}} + c_2 W_2 \frac{\sqrt{m \log m}}{n} \tag{3}$$

where c_1 and c_2 are constant.

Proof:

Each packet in this hybrid network may traverse a number of RF hops and a number of FSO hops. Let $\bar{d}_1$ denote the number of RF hops traversed by a single packet averaged over all packets in the network, and let $\bar{d}_2$ denote the number of FSO hops traversed by a single packet, also averaged over all packets in the network.

From Lemma 5.4 in [1], the number of simultaneous transmissions on an RF channel is no more than $\frac{c_3}{r^2}$. (All the c_i's used above and throughout are constants.) Since each source generates λ bits/sec, there are n sources, and each packet needs to be relayed on the average by $\bar{d}_1$ RF hops, it follows that the total number of bits per second served by the RF part of the entire network is $n\lambda\bar{d}_1$. To ensure that all the required RF traffic is carried, we therefore need

$$n\lambda\bar{d}_1 \leq \frac{c_3 W_1}{r^2} \tag{4}$$

On the other hand, since FSO can operate with negligible interference, the number of simultaneous transmission on FSO channel is no more than m. To ensure that all the required FSO traffic is carried, we therefore need

$$n\lambda\bar{d}_2 \leq W_2 m \tag{5}$$

Now let $\bar{L}$ denote the mean length of a line connecting two independently and uniformly distributed points on S^2. Then we have the following inequality

$$\bar{d}_1 r + \bar{d}_2 s \geq \bar{L} \tag{6}$$

Recall that we make the assumption that all the nodes are asymptotically connected by RF links, and all the super nodes are asymptotically connected by FSO links. To satisfy these two asymptotic connectivity assumptions, [17] suggests that the transmission ranges, r and s, should satisfy

$$r \geq c_4\sqrt{\frac{\log n}{n}} \tag{7}$$

$$s \geq c_4\sqrt{\frac{\log m}{m}} \tag{8}$$

The link equation of an FSO communication system is given by [3]

$$P_r = P_t \cdot \frac{A}{(s\theta)^2} \cdot exp(-\beta s) \tag{9}$$

where P_t is the laser output power, θ is the beam divergence angle (in radians), A is the receiver area, P_r is the received power, and β is the atmospheric attenuation factor.

Compared with the commonly used inverse α^{th} law path loss models, this link equation suggests $\alpha > 2$. It has been shown in [18] that, since $\alpha > 2$, the transmission range should be kept to minimum to minimize the total power consumption in the entire network. Thus we set the FSO transmission range s to be

$$s = c_4\sqrt{\frac{\log m}{m}} \tag{10}$$

From (4), (5), (6), (7) and (10), we have

$$\begin{aligned} n\lambda\bar{L} &\leq n\lambda\bar{d}_1 r + n\lambda\bar{d}_1 s \\ &\leq \frac{c_3 W_1}{r^2}\cdot r + W_2 m \cdot s \\ &= \frac{c_3 W_1}{r} + W_2 m \cdot s \\ &\leq \frac{c_3}{c_4}\cdot W_1\sqrt{\frac{n}{\log n}} + c_4 W_2\sqrt{m\log m} \end{aligned}$$

Then

$$\lambda \leq \frac{1}{\bar{L}}\cdot\left(\frac{c_3}{c_4}\cdot W_1\sqrt{\frac{1}{n\log n}} + c_4 W_2\frac{\sqrt{m\log m}}{n}\right) \tag{11}$$

which yields the result. □

V. A Constructive Lower Bound on Throughput Capacity of RF/FSO Networks with Minimum Power Consumption

To obtain a constructive lower bound on the throughput capacity λ, we need to derive the throughput capacity of random FSO networks with minimum power constraint. This result is given by the following theorem.

Theorem 5.1:

Consider a random FSO network with m nodes randomly and identically distributed on S^2. Every node is equipped with an FSO transceiver. Each FSO transceiver can directionally transmit at W_2 bits/sec within an infinitesimally small angle and a common range s. Each FSO transceiver can receive at W_2 bits/sec omni-directionally. Then the throughput capacity, λ', of this random FSO network with minimum power consumption is given by

$$\lambda' = \Theta\left(W_2\sqrt{\frac{\log m}{m}}\right) \tag{12}$$

Proof:

We derive the throughput capacity of random FSO networks by providing an upper bound on λ', and then show by construction that a throughput of the same order in m can actually be achieved.

To derive the upper bound on λ', we notice that the number of simultaneous transmissions is no more than m, thus the total data rate served by the entire FSO network is no more than mW_2. Now let $\bar{L}$ denote the mean length of a line connecting two independently and uniformly distributed points on S^2. Then the mean length of the path of packets is at least $\bar{L}-o(1)$. Thus the mean number of hops taken by a packet is at least $\frac{\bar{L}-o(1)}{s}$. Then the total number of bits per second served by the entire network needs to be at least $\frac{(\bar{L}-o(1))m\lambda'}{s}$. Then we have the following inequality:

$$\frac{(\bar{L}-o(1))m\lambda'}{s} \leq mW_2 \tag{13}$$

In order to minimize the total transmit power consumption, the FSO communication range s is chosen to be $s =$

$c_4\sqrt{\frac{\log m}{m}}$, then we have the following upper bound on λ':

$$\lambda' \leq c_5 W_2 \sqrt{\frac{\log m}{m}} \tag{14}$$

To show that this upper bound is achievable, we design a routing scheme using similar techniques as in [1]. Let

$$\rho'(m) := \text{radius of a disk of area } 100\log m/m \tag{15}$$

We can construct a Voronoi tessellation $\mathcal{V}_m$ in relation to the number of nodes m and the locations of nodes, in which every Voronoi cell contains a disk of radius $\rho'(m)$ and is contained in a disk of radius $2\rho'(m)$. We choose the range s of each transmission such that

$$s = 8\rho'(m) \tag{16}$$

This range allows direct communication within a cell and between adjacent cells. Every node in a cell is within this distance s from every other node in its own cell or adjacent cell. The routing strategy is to choose the routes of packets to approximate the straight-line which is connecting the source and destination. So the routes actually are the cells that the straight-line intersects. Similar to Lemma 4.14 and Lemma 4.8 in [1], it can be shown that the traffic to be served by each cell is bounded from above, and the number of nodes in each cell is bounded from below. Formally, for any cell V, there is a $\delta'(m) \to 0$ such that

$$\text{Prob}\{\sup_{V\in\mathcal{V}_m} (\text{traffic needing to be carried by cell } V) \leq c_6\lambda'\sqrt{m\log m}\} \geq 1-\delta'(m) \tag{17}$$

And for every disk D of area $100\log m/m$ in S^2, there is sequence $\delta(m) \to 0$ such that

$$\text{Prob}\{\text{number of nodes in } D \geq 50\log m\} \geq 1-\delta(m) \tag{18}$$

It can be thus noted that each cell is able to transmit at least $(50\log m)W_2$ bits/sec. Then with high probability, the rate $c_6\lambda'\sqrt{m\log m}$ can be accommodated by all cell if

$$c_6\lambda'\sqrt{m\log m} \leq (50\log m)W_2 \tag{19}$$

Then the per node achievable throughput is given by

$$\lambda' = c_6 W_2\sqrt{\frac{\log m}{m}} \tag{20}$$

which proves Theorem 5.1. □

The lower bound on λ is given by the following theorem.

Theorem 5.2:

The throughput capacity of hybrid RF/FSO networks with minimum power consumption, λ, is bounded from below by

1) Case 1:

$$\lambda \geq c_7 W_1\sqrt{\frac{1}{n\log n}} + \left(c_8 - c_9\frac{n-m}{n}\sqrt{\frac{\log m}{m}}\right)W_2\frac{\sqrt{m\log m}}{n} \tag{21}$$

when m, n, W_1 and W_2 satisfy

$$(n-m)\log m = o\left(n\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2}\right) \tag{22}$$

2) Case 2:

$$\lambda \geq c_{10}W_1\frac{1}{n-m}\sqrt{\frac{mn}{\log m\log n}} \tag{23}$$

when m, n, W_1 and W_2 satisfy

$$(n-m)\log m = \omega\left(n\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2}\right) \tag{24}$$

Proof:

In order to show that the lower bounds on λ presented above are achievable, we design a hybrid routing scheme in which the data traffic is divided into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. The lower bounds on λ can thus be derived by properly balancing the load between these two classes of traffic.

The proof is organized as follows: The hybrid routing scheme is proposed in Section V.A, in which we design an RF multi-hop routing strategy and a hierarchical routing strategy for the two classes of traffic. In Section V.B, the maximum throughput achieved by the hybrid routing scheme is derived.

A. A Hybrid Routing Scheme

We employ a hybrid routing scheme by dividing the per node throughput λ into two classes. Each source-destination pair transmits packets with constant data rate λ_1 by purely using RF links, and transmits packets with data rate λ_2 by (fully or partly) using FSO links. Hence we have

$$\lambda = \lambda_1 + \lambda_2 \tag{25}$$

The detailed routing strategies for these two classes of traffic are as follows.

1) An RF multi-hop routing strategy for the traffic corresponding to λ_1 : The routing strategy for the traffic corresponding to λ_1 is the same as the one in [1]. Let

$$\rho(n) := \text{radius of a disk of area } 100\log n/n \tag{26}$$

We can construct a Voronoi tessellation $\mathcal{V}_n$ such that every Voronoi cell contains a disk of radius $\rho(n)$ and is contained in a disk of radius $2\rho(n)$. We choose the range r of each transmission such that

$$r = 8\rho(n) \tag{27}$$

This range allows direct communication within a cell and between adjacent cells. The routing strategy is to choose the routes of packets to approximate the straight-line which is connecting the source and destination. So the routes actually are the cells that the straight-line intersects. According to [1], we have the following lemmas.

Lemma 1: Every cell in $\mathcal{V}_n$ has no more than c_{11} interfering neighbors. Furthermore, there is a schedule for transmitting packets such that in every $(1+c_{11})$ slots, each cell in the $\mathcal{V}_n$ gets one slot in which to transmit, and such that all transmissions are successfully received within the transmission and reception coverage.

Lemma 2: There is a $\delta'(n) \to 0$ such that

$$\text{Prob}\{\sup_{V\in\mathcal{V}_n}(\text{RF traffic corresponding to } \lambda_1 \text{ needing to be carried by cell } V) \le c_6\lambda_1\sqrt{n\log n}\} \ge 1-\delta'(n) \quad (28)$$

2) A hierarchical routing strategy for the traffic corresponding to λ_2: The routing strategy for the packets corresponding to λ_2 is described by the following steps:

- Step 1: The packets originated by a source node are routed through RF links to the super node nearest to the source node, if needed;
- Step 2: Packets are then routed through FSO links to the super node nearest to the destination node;
- Step 3: Packets are then routed through RF node to the destination node, if needed.

Here the RF routing scheme used in steps 1 and 3 is the same as the one used in [1]. The FSO routing scheme used in Step 2 is described in the proof of Theorem 5.1. Note that the routing scheme used here for the the traffic corresponding to λ_2 is similar to hierarchical state routing e.g.[5].

Similar to Lemma 4.8 in [1], it can be proved that every disk D of area $100\log m/m$ contains at least one super node with high probability. Formally, for every disk D of area $100\log m/m$ in S^2, there is $\delta(m)\to 0$ such that

$$\text{Prob}\{D \text{ contains a super node}\} \ge 1-\delta(m) \quad (29)$$

Since every disk of area $100\log m/m$ contains at least one super node with high probability, and each Voronoi cell in $\mathcal{V}_n$ contains a disk of area $100\log n/n$, then the mean number of RF hops traversed by a packet during its first and third step in the routing scheme described above is at most $c_{12}\sqrt{\frac{\log m/m}{\log n/n}}$. Then the total data rate corresponding to λ_2 served by the entire RF wireless network is no more than $c_{12}\lambda_2(n-m)\sqrt{\frac{\log m/m}{\log n/n}}$. Thus the average data rate corresponding to λ_2 served by each Voronoi cell V is no more than

$$\begin{aligned} & c_{13}\lambda_2(n-m)\sqrt{\frac{\log m/m}{\log n/n}}\cdot\frac{\log n}{n} \\ = \quad & c_{13}\lambda_2\frac{n-m}{n}\sqrt{\frac{n\log m\log n}{m}}. \end{aligned} \quad (30)$$

Then by making use of the property that the sequence of straight-line segments $\{L_i\}_{i=1}^n$ is i.i.d, we can exploit uniform convergence in the law of large numbers using the same technique introduced in ([1], Section IV.I), and thus obtain the following lemma:

Lemma 3: There is a $\delta'(n)\to 0$ such that

$$\text{Prob}\{\sup_{V\in\mathcal{V}_n}(\text{RF traffic corresponding to } \lambda_2 \text{ needing to be carried by cell } V) \le c_{14}\lambda_2\frac{n-m}{n}\sqrt{\frac{n\log m\log n}{m}}\} \ge 1-\delta'(n) \quad (31)$$

B. Maximum Throughput Achieved by the Hybrid Routing Scheme

From Lemma 1, Lemma 2 and Lemma 3, we can conclude that the data transmission utilizing RF links can be accommodated by all cells if

$$c_6\lambda_1\sqrt{n\log n}+c_{14}\lambda_2\frac{n-m}{n}\sqrt{\frac{n\log m\log n}{m}} \le \frac{W_1}{1+c_{11}} \quad (32)$$

Since the per node throughput capacity of the stand-alone FSO network is given by Theorem 5.1, the data transmission (partially) utilizing FSO links can be accommodated by the FSO network if

$$n\lambda_2 \le c_6W_2\sqrt{\frac{\log m}{m}}\cdot m \quad (33)$$

Then we can derive the maximum throughput achieved by the proposed routing scheme by solving the following optimization problem:

$$\text{maximize } \lambda$$

subject to

$$\begin{gathered} \lambda_1+\lambda_2=\lambda \\ c_6\lambda_1\sqrt{n\log n}+c_{14}\lambda_2\frac{n-m}{n}\sqrt{\frac{n\log m\log n}{m}} \le \frac{W_1}{1+c_{11}} \\ n\lambda_2 \le c_6W_2\sqrt{\frac{\log m}{m}}\cdot m \end{gathered}$$

The optimal solution falls into the following two mutually exclusive cases:

Case 1: When m, n, W_1 and W_2 satisfy

$$(n-m)\log m < \frac{n}{(1+c_{11})c_6c_{14}}\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2} \quad (34)$$

the maximum throughput λ and the data rates of the two classes of traffic, λ_1 and λ_2, are given by

$$\begin{aligned} \lambda = & \frac{1}{(1+c_{11})c_6}W_1\sqrt{\frac{1}{n\log n}} \\ & +\left(c_6-c_{14}\frac{n-m}{n}\sqrt{\frac{\log m}{m}}\right)W_2\frac{\sqrt{m\log m}}{n} \end{aligned} \quad (35)$$

$$\lambda_1 = \frac{1}{(1+c_{11})c_6}W_1\frac{1}{\sqrt{n\log n}}-c_{14}W_2\frac{n-m}{n^2}\log m \quad (36)$$

$$\lambda_2 = c_6W_2\frac{\sqrt{m\log m}}{n} \quad (37)$$

Case 2: When m, n, W_1 and W_2 satisfy

$$(n-m)\log m \ge \frac{n}{(1+c_{11})c_6c_{14}}\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2} \quad (38)$$

the maximum throughput λ and the data rates of the two classes of traffic, λ_1 and λ_2, are given by

$$\lambda = \frac{W_1}{(1+c_{11})c_{14}} \frac{1}{n-m} \sqrt{\frac{mn}{\log m \log n}} \tag{39}$$

$$\lambda_1 = 0 \tag{40}$$

$$\lambda_2 = \frac{W_1}{(1+c_{11})c_{14}} \frac{1}{n-m} \sqrt{\frac{mn}{\log m \log n}} \tag{41}$$

Hence we have proved Theorem 5.2. □

VI. Discussions

A. The Tightness of the Bounds on λ and The Routing Scheme Selection Criteria

We evaluate the tightness of the capacity bounds on λ by comparing the upper bound results in Theorem 4.1 with the lower bound results under two different cases derived in Section V.

It is clear that the lower bound result for Case 1 in (35) will asymptotically become the form of the upper bound on λ in Theorem 4.1, as m grows. Thus in Case 1, Theorem 4.1 and Theorem 5.2 provide a tight bound on λ when

$$(n-m)\log m < c_{15} n \sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2} \tag{42}$$

For Case 2, which requires m, n, W_1 and W_2 satisfying

$$(n-m)\log m \geq c_{15} n \sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}, \tag{43}$$

the lower bound result in (39) does not match the form of the upper bound result in Theorem 4.1. Although we cannot get a general result like in Case 1, we can evaluate the tightness by considering the extreme case when $W_2 \to \infty$. Under these circumstances, the upper bound result becomes infinity, which is certainly not tight. The lower bound result matches the results provided by [9], where the support of an infinite capacity infrastructure network is considered. This implies that the lower bound is tighter than the upper bound under such circumstances.

We can note that for Case 1 the proposed routing scheme works as a combination of two independent schemes, while under Case 2 it completely degenerates to the hierarchical routing scheme as described in Section V.2. In order to guide the practical routing protocol design, we need to derive clear conditions on m, n, W_1 and W_2 for Case 1 and 2. Although there is generally no closed form solutions when (42) or (43) hold with equality, we are still able to exploit some practical implications by presenting the following protocol design principles:

1) When $\frac{W_2}{W_1} = o(n^{\frac{1}{2}})$, the condition for Case 1 is satisfied. Then it is better to employ the hybrid routing scheme to achieve a higher per node throughput.
2) When $\frac{W_2}{W_1} = \Omega(n^{\frac{3}{2}})$, the condition for Case 2 holds. Thus it is better to use the hierarchical routing scheme to achieve a higher per node throughput.
3) When $\frac{W_2}{W_1} = \Theta(n^k)$, where $0.5 \leq k < 1.5$, there exists two critical numbers $M_1(k)$ and $M_2(k)$, $M_1(k) < M_2(k) < n$, satisfying

$$f(m) := (n-m)\log m = c_{15} n \sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}, \quad \text{for } m = M_1, M_2. \tag{44}$$

Since $f(m)$ is strictly concave over $[1, n]$, hence

$$(n-m)\log m < c_{15} n \sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}, \quad \text{for } m < M_1(k) \text{ or } m > M_2(k) \tag{45}$$

$$(n-m)\log m > c_{15} n \sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}, \quad \text{for } M_1(k) < m < M_2(k) \tag{46}$$

Thus the hybrid routing scheme is more desirable to achieve a higher per node throughput when $m < M_1(k)$ or $m > M_2(k)$. On the other hand, the hierarchical routing scheme is preferred when $M_1(k) < m < M_2(k)$.

B. Capacity Improvement

We define the capacity gain G, as the capacity ratio between hybrid RF/FSO networks and RF wireless networks. We further define the capacity improvement ratio, R, as $R = G - 1$.

First we take Case 1 into consideration. By comparing with the capacity results in [1], we can show that R may diminish to zero as n grows when

$$\frac{W_2}{W_1} m \log m = o(n) \tag{47}$$

On the other hand, we can possibly achieve a non-negligible capacity improvement by adding FSO transceivers to m nodes if and only if

$$\frac{W_2}{W_1} m \log m = \Omega(n) \tag{48}$$

For Case 2, (39) implies that the capacity improvement ratio R will never diminish to zero. Thus a significant capacity improvement can be achieved with high probability under Case 2.

Thus we can generalize the results for Case 1 and 2 by claiming that we can achieve a non-negligible capacity improvement with high probability if

$$\frac{W_2}{W_1} m \log m = \Omega(n) \tag{49}$$

This result characterizes the number of super nodes and/or the FSO data rate needed in order to guarantee a significant capacity improvement as compared to the case of pure RF wireless networks.

VII. CONCLUSION AND FUTURE WORK

In this paper, we have studied the per node throughput capacity of hybrid RF/FSO networks. A hybrid RF/FSO network consists of an RF wireless network with random deployment and connected by RF links, and only a portion of nodes (so called super nodes) are equipped with an additional FSO transceiver. All the super node are connected by the FSO links and thus form a stand-alone FSO network. The objective of this paper is to derive the asymptotic capacity of such hybrid networks, and evaluate the benefit of using this hybrid RF/FSO network architecture over the pure RF wireless networks.

We consider a hybrid RF/FSO network with a total number of n nodes, and m of them are super nodes. Every RF and FSO transceiver is able to transmit at a maximum data rate of W_1 and W_2 bits/sec, respectively. We show that the throughput capacity of hybrid RF/FSO networks with the constraint of minimum power consumption, λ, is bounded from above by $\lambda \leq c_1 W_1 \sqrt{\frac{1}{n \log n}} + c_2 W_2 \frac{\sqrt{m \log m}}{n}$. In order to evaluate the tightness of this upper bound, we design a hybrid routing scheme as we divide the data traffic into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. Under such routing strategies, we have shown that the upper bound is a tight one when the maximum data rate ratio of FSO and RF transceivers, $\frac{W_2}{W_1}$, grows slower than $\sqrt{n}$.

The analysis regarding our proposed routing strategy also yields some practical implications which may guide the routing protocol design for the hybrid RF/FSO networks. Our analysis have shown that if $\frac{W_2}{W_1}$ grows slower than $\sqrt{n}$, high per node throughput can be achieved by properly balancing the load between the two classes of traffic. If $\frac{W_2}{W_1}$ grows no slower than $\sqrt{n^3}$, then a pure hierarchical routing strategy will suffice to achieve high per node throughput, i.e., all the data traffic will be forwarded with the (partial) support of super nodes.

Furthermore, we have also evaluated the capacity improvement with the support of FSO nodes, as compared with the results for RF wireless networks in [1]. We have shown that a significant capacity gain will be achieved if $\frac{W_2}{W_1} m \log m = \Omega(n)$.

Possible avenues for future research still exist, as we have not derived a tight bound on throughput capacity for the case when $\frac{W_2}{W_1}$ grows no slower than $\sqrt{n}$. In addition, we are also interested in extending the capacity results to the case when RF and FSO transceivers may become unreliable (e.g., under adverse weather conditions).

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