An Evaluation of Weak State Mechanism Design for Indirection in Dynamic Networks

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***Abstract*—State signaling and maintenance mechanisms play crucial roles in communication network protocols. State is used to facilitate indirections in protocols such as routing. Design approaches for traditional state signaling mechanisms have been categorized into soft and hard state. In both approaches, the state is deterministic. Hence, we call both as having strong state semantics, or more crisply, refer to them as *strong* state. If the state tracks entities with dynamic nature, strong state rapidly becomes invalidated and needs to be refreshed explicitly through control packets. In this paper, we evaluate the recently proposed *weak state* [1]. Weak state is a generalization of soft state that is characterized by probabilistic semantics and local updates. It is interpreted as a probabilistic hint and not absolute truth. Weak state also contains the *confidence* in the state value, which is a measure of the probability that the state remains valid. The confidence or the state semantics is decayed locally without the need for explicit state update traffic traversing the network. The local updates also help the protocol use better estimates for the state value. We define two metrics, *pure* distortion and *informed* distortion, to evaluate the consistency of the weak state paradigm and compare it against strong state. Pure distortion measures the average gap between the actual value of the state and the value maintained at a remote node. On the other hand, the use of confidence increases the protocol's ability to cope with even large pure distortion. The resulting effective distortion is captured by the informed distortion metric.**

Using mathematical analysis, we compare weak with strong state. Local updates reduce the pure distortion because the protocol uses the best estimate of state value. The informed distortion is also significantly less because the probabilistic confidence value hints the protocol if the state is invalid. The weak state mechanism can be used to build protocols (eg: WSR [1]), which systematically interpret the state information. The state itself can be mostly updated locally, with less frequent explicit update messages over the network (i.e. leading to dramatic reductions in control traffic).

I. Introduction

The operation of a majority of network protocols rely on state information at remote nodes in order to facilitate indirections. State signaling system is one of the most important building blocks of protocol design. State maintenance at the remote nodes is equally important; however, it has not received a comparable attention from the community. In this paper, we generalize the novel *weak state* concept that has been proposed for routing in dynamic mobile ad-hoc networks in [1] and compare/contrast it with the traditional strong state concept.

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Traditionally, the concept of state can be classified into two categories based on the way that the state is signaled to the remote node: hard and soft state approaches. Hard state remains valid until it is explicitly removed using state-teardown messages by the node that installs the state. The installer node refreshes the state at the remote nodes only when the state is updated. Since the state is removed explicitly, reliable communication is essential. On the other hand, soft state, which was originally introduced by Clark [2], *times out* unless it is refreshed within a time-out duration. The state installer node periodically issues a refresh message. Once this message is received by the node maintaining the state, the timer corresponding to the state is rescheduled. If the timer expires, the state times out and removed from the system. Soft state does not require explicit removal messages. As a result, reliable signaling is not required. Refresh message losses can be tolerated since the state is refreshed periodically.

Both hard state and soft state are regarded as absolute truth. We say such a state information has *strong* semantics or it is a *strong state*. If a state with strong semantics contains information about a dynamic entity, it is rapidly invalidated. The state requires to be explicitly refreshed frequently through control message traffic in order to provide up-to-date information. As a remedy to this problem, *weak state* has been recently proposed [1]. Unlike traditional state, weak state is not deterministic; instead, it yields *probabilistic hints*. Weak state has probabilistic semantics and it is more stable. The state information is accompanied with a *confidence* value, a measure of the probability that the state remains valid. The confidence is *weakened/decayed* in time locally. Once the confidence is below a threshold value, the state is removed from the system. Weakening the state corresponds to aging it and is equivalent to a soft timeout. Hence, weak state is a generalization of soft state. A comparison of hard, soft and weak states are given in Fig. 1. Weak state mechanism design is characterized by two properties:

1) Probabilistic semantics: The information that the state yields is not deterministic. Instead, the validity of the information is subject to a probabilistic confidence value.
2) Local updates: The information maintained at remote nodes can be updated without explicit control messages from the sender. With local updates, the remote nodes can estimate the actual value of the state and decay/weaken the state semantics or the confidence in the information.

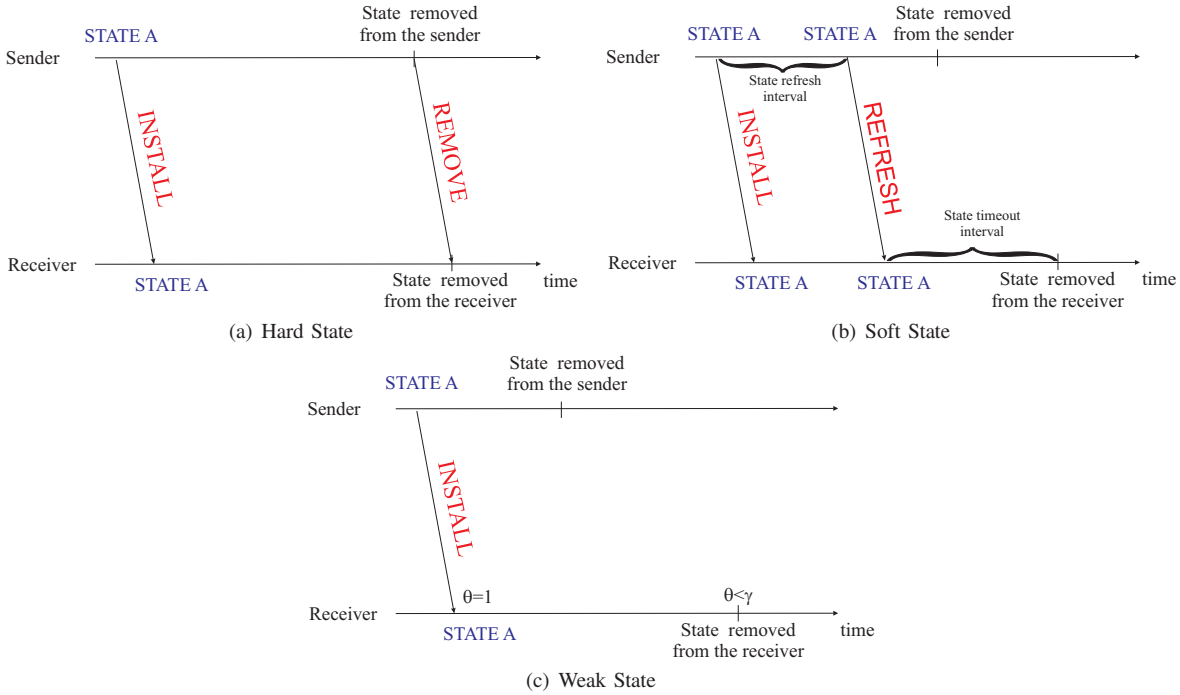

Fig. 1. A comparison summary of hard state, soft state and weak state approaches. Hard state requires explicit control message to be removed. Soft state times out if it is not refreshed within the timeout interval. Weak state is associated with a confidence value θ, which is a decreasing function of time. When the confidence is below a threshold value γ, it is removed.

In this paper, we present an analytical evaluation of weak state mechanism design in terms of its consistency and compare it with strong state. The consistency of the state is characterized by the accuracy of the indirection based on the perceived value maintained at the remote node. One particular value of a state can be interpreted differently for indirections in protocols that utilizes different state mechanism design approaches. In order to evaluate the consistency, we use the *pure* distortion and *informed* distortion metrics. Pure distortion measures the difference between the actual state value and the corresponding perceived state value. If the protocol uses weak state, pure distortion is smaller because the protocol can use an estimate state value instead of a mere last reported value. In addition, using the confidence as a probabilistic hint, increases the protocol's ability to the cope with pure distortion and helps it adapt to dynamism. In other words, it is superior to have a hint with a measure of confidence than having invalid deterministic state which may lead to wrong decisions with probability 1. The effect of the confidence parameter on the consistency of the indirection is captured by the informed distortion metric.

We show that if the state tracks a dynamic entity, weak state causes significantly less distortion than strong state given they are refreshed by the same rate. In other words, weak state mechanism can achieve targeted distortion metrics with a smaller refresh rate and hence less overhead. *This however does not mean that there is no trade off.* The maintained information is not perfect and the protocol may choose different ways to interpret and deal with the confidence. For example, [1] proposed the *Weak State Routing (WSR)* protocol for routing in large scale and dynamic networks. In WSR, a packet is successively biased towards the points yielded by the intermediate nodes that contain increasingly more confident state. The protocol increases the delivery ratio and decreases the overhead significantly at the cost of increasing path length. With the dramatically reduced control traffic, longer paths need not imply longer end-to-end delivery latency since the protocol reduces queueing in the intermediate nodes.

A. Related Work

The weak state concept has been first coined in [1], where it has been used to perform routing in large scale and dynamic mobile ad-hoc networks. However, other realizations are possible such as PROPHET [3] and EDBF [4]. A survey of methods for approximate global state for distributed systems is presented in [5]. The author mentions that such approximate or "weak" state could be a useful primitive for dynamic networks.

The most closely related works to this paper are [6] and [7]. These papers present analytical comparisons of hard state, soft state and the hybrid approaches in terms of consistency. Both model state with strong semantics and focus only on the signaling mechanism. In this paper, we evaluate the states with weak and strong semantics. The results show that weak state is more consistent.

B. Organization of Paper

In Section II, we lay out the foundations of our analysis. In Section III, we extend our analysis to a broader range of scenarios. We model the state information using a variety of stochastic processes in Section IV and evaluate the consistency of strong state and weak state in terms of pure and informed distortion. In Section V, we use experiments to evaluate weak

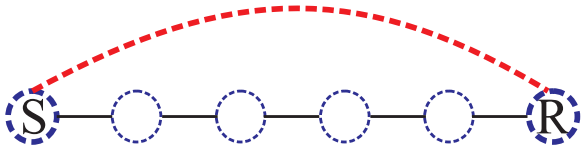

Fig. 2. Single Hop Signaling System. The sender S is connected to the receiver R with a logical link. The logical link can consist of a series of physical links which enables end-to-end signaling

state for a more complex scenario. Finally, we conclude the paper in Section VI.

II. Analysis Framework

A. System and Data Model

In this paper, we adopt the single hop signaling system used in both in [6] and [7]. The system consists of a sender node and a receiver node where the sender installs the state at the remote receiver node and refreshes it. The sender and the receiver are connected to each other via one physical or logical link. The logical link can be composed of a sequence of physical hops (see Fig. 2). `INSTALL`, `REFRESH` and `REMOVE` messages on Fig. 1 are transferred over the logical link. The signaled messages can be lost while being transferred through the logical link. In our analysis, we consider the soft state signaling mechanism.

We assume that the state value, X, is a continuous time stochastic process. The state update corresponds to the changes in the value of the sender's state. The intervals between state updates are independent and exponentially distributed random variables, with parameter λ. The sender periodically refreshes the perceived state value, $\hat{X}$, at the receiver. The value of $\hat{X}$ at any given time is the value of X in the most recently received refresh message. Soft state times out if it is not refreshed within some timeout interval, χ. Weak state, on the other hand, is a tuple $(\hat{X}, \theta)$, where θ is the confidence value, the probability that the state at the receiver remains valid or a measure of this probability. At time t, the confidence is $\theta(t) = P(\zeta > t) = \exp(-\lambda t)$ where ζ denotes the time interval in which the state at the sender remains the same. The confidence of the state is decayed in time locally at the receiver. Once the confidence is below some threshold, γ, the receiver removes the state. The state lifetime is very long in comparison to the average state update interval and state timeout interval; it approaches infinity. In other words, the sender node always maintains a value that corresponds to the state. Table I lists the main variables we use in the paper.

TABLE I
Summary of Notation

Notation	Description
X	State value at the sender
$\hat{X}$	State value at the receiver
ζ	Random variable for state update interval
λ	Average state update rate, i.e. $E[\zeta] = 1/\lambda$
χ	State timeout interval
p	Signaling loss rate
T	State refresh interval
θ	Confidence in state information
γ	Confidence threshold for removing weak state
S	Strong semantics
W	Weak semantics

B. Consistency

We use the term indirection to refer to the decision performed by the protocol. For example, in a routing protocol each entry in the routing table involves indirection from a persistent name (ID) to a locator. The protocol delivers packets using the locator; a next hop, a sequence of hops, etc. The indirection is consistent if the state information at the sender and the receiver is the same. The indirection becomes inconsistent if (i) the state at the sender is updated but the receiver's state is not refreshed, or (ii) the receiver falsely removes the state due to a series of refresh messages being lost in the logical link while the sender still maintains it. The indirection can be also inconsistent if the sender removes the state and the receiver maintains it until the state time-out. We assume that the lifetime of the sender's state is very large and approaches infinity, so this situation is not a factor in our analysis.

Without loss of generality, let's assume that the state is most recently updated at time $t = 0$, $X(0) = \hat{X}(0)$. Let $d(t)$ denote the instantaneous distortion between the state value at the sender and the receiver at time t. If the state has strong semantics[1], the information is deterministic and the instantaneous distortion is defined as

$$d_S(t) = \begin{cases} 0 & \text{if } X(t) = \hat{X}(t) \\ 1 & \text{otherwise.} \end{cases} \tag{1}$$

The state information is probabilistic in weak state. Even if the state at the sender and the receiver are different, the distortion is small if the receiver has small confidence in the state it maintains. If the state at the sender and the receiver are different from each other, the distortion is characterized by the confidence at the receiver, the probability that the state is still valid according to the receiver. On the other hand, weak state can still cause distortion when state at the sender and the state at the receiver are the same. In this case, the amount of the distortion is the probability that the state might be invalid as inferred by the receiver. The resulting instantaneous distortion for weak state is

$$d_W(t) = \begin{cases} 1 - \theta(t) & \text{if } X(t) = \hat{X}(t) \\ \theta(t) & \text{otherwise.} \end{cases} \tag{2}$$

At time t, the probability for the state at the sender changes value is $P\left(X(t) \neq \hat{X}(t)\right) = P(\zeta \leq t) = 1 - \exp(-\lambda t)$. Hence, the expected instantaneous distortion at time t can be modeled as

$$D_S(t) = \begin{cases} 1 - \exp(-\lambda t) & \text{if } t \leq \chi \\ \exp(-\lambda t) & \text{otherwise} \end{cases} \tag{3}$$

$$D_W(t) = \begin{cases} 2\exp(-\lambda t)\left(1 - \exp(-\lambda t)\right) & \text{if } t \leq \chi \\ \exp(-\lambda t) & \text{otherwise.} \end{cases} \tag{4}$$

The second terms in both (3) and (4) are due to false removal of the state at the receiver and the value is the probability that

[1]We denote the semantics of the state as a subscript throughout the paper.

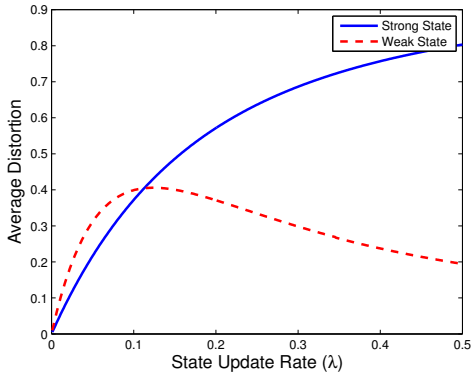

Fig. 3. Comparison against the state update rate λ. A higher λ indicates that the information is more dynamic. For small λ values, the distortion for weak state is larger because the state at the sender is less likely to change and weak state can cause distortion even if $\hat{X} = X$. In this case, the first part of (4) is larger than that of (3). Yet, as the dynamism increases, distortion in strong state grows, but weak state is able figure out that state is not as useful.

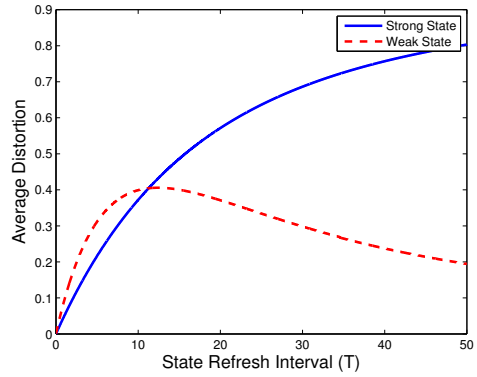

Fig. 4. Comparison of against the state refresh interval, T. If T is small, the state is frequently refreshed by the sender. As a result, strong state is likely to remain valid between two refresh message receptions. However, even if the state values at the sender and the receiver are same, weak state causes distortion according to (2). On the other hand, if T is large, the state probably changes until a new refresh message arrives. Hence, strong state becomes less consistent. Weak state however figures that the maintained information is likely invalidated. Weak state causes the same distortion with less messages.

the state is still valid. Remember that, when the confidence is below the threshold value, the receiver removes the state and the confidence value is taken as 0.

The average overall distortion, which is denoted by $\overline{D}$, within the interval between two state refresh messages received by the receiver is

$$\overline{D} = \sum_{i=1}^{\infty} \frac{1}{iT} p^{i-1}(1-p) \int_0^{iT} D(\tau) d\tau \tag{5}$$

where i is the number of attempts to deliver a refresh message successfully to the receiver and iT is the time between the two consecutive refresh message receptions.

The distortion with respect to the state update rate, λ, is presented in Fig. 3. The figure is obtained with $T = 5$s, $p = 0.02$. For strong state $\chi = 15$s. However, the timeout value for weak state changes with respect to λ. To calculate χ, we use $\theta(\chi) = \gamma = 0.01$. When the update rate is small, the state values at the receiver and the sender are likely to remain the same within the interval the receiver receives two consecutive refresh messages. Given $\hat{X} = X$, the strong state does not cause any distortion. However, the indirection yielded by weak state is probabilistic and it can cause distortion according to (2). Hence, the distortion of weak state is larger than strong state when the dynamism is low. As λ increases, the state becomes more dynamic and changes more quickly. As a result, the state is more likely to change between two consecutive instants the sender refreshes the receiver. Due to its deterministic nature, strong state causes more distortion. In weak state however, local updates of the confidence about the state reduces the distortion. At a high rate value, the state rapidly changes. However, the confidence on the state information also quickly drops, therefore the distortion does not increase.

In Fig. 4, we show how distortion changes with respect to the state refresh interval. Plotting this figure, we used $\lambda = 0.1$ and $p = 0.02$. For strong state $\chi = 3T$ and for weak state χ is calculated as in the previous example. When T is low, the state is frequently updated. As a result, the duration in which $\hat{X} \neq X$ reduces. Consequently, the distortion of strong state is small. Since weak state can make indirection mistakes due to probabilistic decisions, its distortion is slightly higher than the strong state. On the other hand, if T is large, the probability that the state changes within the time interval between the reception of two consecutive state refresh messages is high. Therefore, the distortion increases for strong state. However, in weak state the confidence on the maintained state also decreases with time, therefore distortion does not increase.

The indirections performed with weak state are more accurate in dynamic scenarios since they are performed with probabilistic hints rather than invalid deterministic information that lead to wrong decisions with probability 1. This also implies that the distortion caused by strong state at a refresh rate can be achieved using a larger refresh interval by the weak state. In other words weak state causes less overhead.

III. Extension of Analysis: State with Continuous, Correlated Space

In some scenarios, the information that receiver's state is different than the sender's state may not be sufficient and the amount by which they are apart is also important. For example, if we perform routing using the geographical locations instead of link states, the distance between the exact location of the node and its perceived location matters. To capture this, we extend the analysis in this section. The state at the sender is $\mathbf{x}(t)$ and perceived state at the receiver is $\hat{\mathbf{x}}(t)$ at time t.

Let t_0 and t_1 denote two arbitrary consecutive time instants at which the destination receives state update information, with $\mathbf{x}(t_0) = x_0$. In strong state, the state information perceived by the receiver remains constant until t_1, i.e.

$$\hat{\mathbf{x}}_S(t) = \mathbf{x}(t_0), \quad t_0 \leq t \leq t_1. \tag{6}$$

On the other hand, weak state can be updated locally using the statistical properties of the process[2]. $\hat{\mathbf{x}}_W(t)$ is the expected

[2]Obtaining the information about these properties is beyond the scope of this paper.

value of $\mathbf{x}(t)$, which is the best estimate given $\mathbf{x}(t_0) = x_0$.

$$\hat{\mathbf{x}}_W(t) = E\left[\mathbf{x}(t)|\mathbf{x}(t_0) = x_0\right] \tag{7}$$

Equation (7) captures that weak state is constantly refreshed using previous values and the properties of the process. The confidence of the state at time t, $\theta(t)$ is characterized by the probability that

$$\theta(t) = P\left(|\mathbf{x}(t) - \hat{\mathbf{x}}_W(t)| \leq \rho\right) \tag{8}$$

where difference up to ρ in the state value is tolerable for indirection.

As the difference between the actual value and the maintained value increases, the state at the receiver becomes less valid. (8) yields the probability that the maintained state is close to the actual value. If $\theta(t) \leq \gamma$, the state is removed.

The protocol operating with weak state at remote node knows how much it can trust the maintained value, and hence the effective distortion is lower. However, this does not necessarily suggest how close the value maintained at the receiver is to the state value at the sender. In this section, we differentiate these two senses of distortion.

A. Instantaneous Distortion

As an intermediate step, let $e(t)$ denote the distortion between the state values at the sender and the receiver at time t. We use the mean square error metric and assume that the receiver is aware of the value of the state information at t_0. The instantaneous distortion can be calculated as

$$e(t) = E\left[(\mathbf{x}(t) - \hat{\mathbf{x}}(t))^2|\mathbf{x}(t_0) = x_0\right]. \tag{9}$$

For strong state, $\hat{\mathbf{x}}_S(t) = x_0$.

$$\begin{aligned} e_S(t) &= E\left[(\mathbf{x}(t) - x_0)^2|\mathbf{x}(t_0) = x_0\right] \\ &= E\left[\mathbf{x}(t)^2|\mathbf{x}(t_0) = x_0\right] - 2x_0 E\left[\mathbf{x}(t)|\mathbf{x}(t_0) = x_0\right] + x_0^2 \\ &= \mu_t^2 + \sigma_t^2 - 2x_0\mu_t + x_0^2 \\ &= \sigma_t^2 + (\mu_t - x_0)^2 \end{aligned} \tag{10}$$

where μ_t is the expected value of $\mathbf{x}_t$ given $\mathbf{x}(t_0) = x_0$ and σ_t^2 is the variance under the same condition.

On the other hand, for weak state, we have $\hat{\mathbf{x}}_W(t) = \mu_t$.

$$\begin{aligned} e_W(t) &= E\left[(\mathbf{x}(t) - \mu_t)^2|\mathbf{x}(t_0) = x_0\right] \\ &= \sigma_t^2 \end{aligned} \tag{11}$$

Since $(\mu_t - x_0)^2 \geq 0$ for all t and x_0, we have $e_W \leq e_S$. In other words, the instantaneous distortion for weak state is smaller than that of strong state for the same t.

B. Pure Distortion

We use the term *pure distortion* to describe the average gap between the actual value of the state at the sender and the value maintained at the receiver. Pure distortion is the time average of instantaneous distortion defined in Section III-A. We do not consider the confidence in the state information for the pure distortion. In WSR implementation, the pure distortion corresponds to the distance between the location of a node and the point an intermediate node directs a packet destined to that node.

Similar to the previous section, without loss of generality, let's again assume $t_0 = 0$. Then the average pure distortion within two consecutively received state refresh messages is

$$\overline{e^P} = \sum_{i=1}^{\infty} \frac{1}{iT} p^{i-1}(1-p) \int_0^{iT} e(\tau)d\tau. \tag{12}$$

Since $e_W(t) \leq e_S(t)\ \forall t$, $\overline{e_W^P} \leq \overline{e_S^P}$ for the same T. Therefore, we can use a larger T value for weak state to achieve the same average distortion. Large refresh interval implies lower refresh rate and overhead.

C. Informed Distortion

Other than predicting the current value of the state, local updates associated with weak state are particularly useful for adjusting the confidence in the maintained value. In order to capture the effect of confidence, we introduce the concept of *informed distortion*. Consider a receiver node contains information about a remote node. When asked about the state information, it will reply "The actual state value is within the interval $[\vartheta - \rho, \vartheta + \rho]$ with probability ν." The pure distortion can be a very large value; however, its effect on the accuracy of the indirection will be limited if ν is low. If weak state mechanism design is used, the protocol can adapt itself appropriately instead of making deterministic decisions which lead to invalid indirections with strong state. For example in WSR [1], intermediate nodes do not always bias the packets they receive even if they contain information about the location of the node. If the confidence in the information is low, they relay the packet without using the information. This way, the packets are not forwarded to invalid locations and the information maintained in the intermediate node do not deteriorate the performance even though the pure distortion is high. On the other hand; if the receiver's state is within the interval of ρ of the sender's state the strong state does not cause distortion because such a difference can be tolerable for the protocol. For example, in MANET routing if the difference between the actual location information and the perceived location information of the destination node is within some distance value, the protocol can still deliver the packet. This however may not be true for weak state because the decisions are associated with probabilistic confidence values. In order to capture these effects, we incorporate the confidence into the analysis as following:

$$\epsilon_S(t) = \begin{cases} 0 & \text{if } |\mathbf{x}(t) - \hat{\mathbf{x}}_S(t)| \leq \rho \\ e_S(t) & \text{otherwise} \end{cases}$$

$$\epsilon_W(t) = \begin{cases} (1-\theta(t))e_W(t) & \text{if } |\mathbf{x}(t) - \hat{\mathbf{x}}_W(t)| \leq \rho \\ \theta(t)e_W(t) & \text{otherwise.} \end{cases}$$

The expected values are:

$$e_S^{(I)}(t) = P\left(|\mathbf{x}(t) - \hat{\mathbf{x}}_S(t)| > \rho\right) e_S(t) \tag{13}$$

$$e_W^{(I)}(t) = 2(1-\theta(t))\theta(t)e_W(t). \tag{14}$$

In order to derive an expression for the average informed distortion, we substitute (10) and (11) with (13) and (14) in (12), respectively. Also note that in either case, if $t > \chi$ and $|\mathbf{x}(t) - \hat{\mathbf{x}}(t)| \leq \rho$, there is a distortion involved due to false removal of the state and its expected value is σ^2 in both cases.

IV. Particular Example Processes for State Information

In this section, we use random process models to evaluate weak state and strong state. The analysis of particular forms of these random processes gives insights about the quantitative performance differences resulting from adopting different types of state mechanisms under various dynamic conditions. We use several Gaussian random process models for mathematical tractability. If the random variables $\mathbf{y_1}$ and $\mathbf{y_2}$ are jointly Gaussian, given y_1, $f(y_2|y_1)$ is a normal density with mean $\eta_2 + r\sigma_2(y_1 - \eta_1)/\sigma_1$ and variance $\sigma_2^2(1-r^2)$, where r is the correlation coefficient between $\mathbf{y_1}$ and $\mathbf{y_2}$.

With a Gaussian process, the confidence for a weak state at time t according to (8) is

$$\theta(t) = \mathrm{erf}\left(\frac{\rho}{\sigma_t\sqrt{2}}\right) \tag{15}$$

where $\mathrm{erf}(.)$ is the error function, given that the last refresh message is received at $t = 0$.

A. Wiener Process

This process is characterized by the independent, stationary increments. $\mathbf{x}(t+\tau) - \mathbf{x}(t) \sim \mathcal{N}\left(0, \sigma^2\tau\right)$ for all t and $\tau \geq 0$. The conditional density is $(\mathbf{x}(t+\tau)|(\mathbf{x}(t) = x_0)) \sim \mathcal{N}(x_0, \sigma^2\tau)$.

Wiener process consists of divergence from a reference point. The update in the state information refreshes the starting point for the subsequent process. The process is defined with respect to a reference point and time. The variance of a sample from the process grows linearly with time without bounds. Therefore, we assume that the state information is not removed from the receiver.

In this case, the values of strong and weak states are identical because $x(t)$ is a martingale and hence $\mu_t = x_0$. Given the same refresh interval T, the average pure distortion in both cases, using (12), is

$$\overline{e_W^P} = \overline{e_S^P} = \frac{1}{2(1-p)}\sigma^2 T.$$

We compute the informed distortion numerically. Since $\hat{\mathbf{x}}_S(t) = \hat{\mathbf{x}}_W(t) = \mu(t)$, we have

$$P\left(|\mathbf{x}(t) - \hat{\mathbf{x}}_S(t)| \leq \rho\right) = \theta(t).$$

In Fig 5, we present the pure and informed distortion for strong and weak state with respect to the variance of the state value, σ^2. We take the message loss probability on the logical hop as $p = 0.02$ and the state refresh interval as $T = 5$ seconds. To determine the confidence, we use $\rho = 2$. The state variance yields the dynamism of the state. When the state variance is low, the receiver's state is likely to remain within the tolerable interval of the sender's state. In this case, strong state does not incur any informed distortion; whereas, weak state can still cause informed distortion because of its probabilistic semantics. Even though this happens with low probability, the value in (14) is slightly larger than the one in (13). Consequently, the same trend follows in the time average as well. On the other hand, as the dynamism increases weak state causes much less informed distortion because with probabilistic semantics, it is able to figure that the state is less likely to be valid.

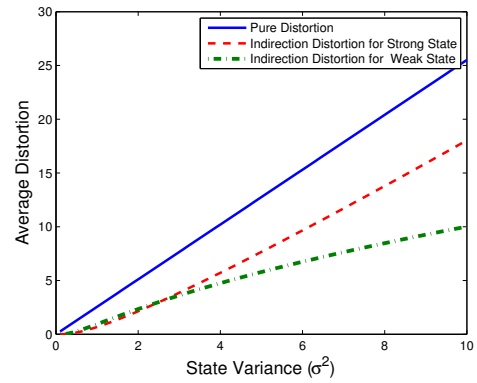

Fig. 5. Pure Distortion and Informed Distortion with respected to state variance (σ^2), which is an indicator of dynamism. Pure distortion is identical for both strong state and weak state because $\mu_t = x_0$ in (10). Because of probabilistic semantics, weak state causes slightly more informed distortion for when the dynamism is low. For highly dynamic state, information distortion is much lower in weak state.

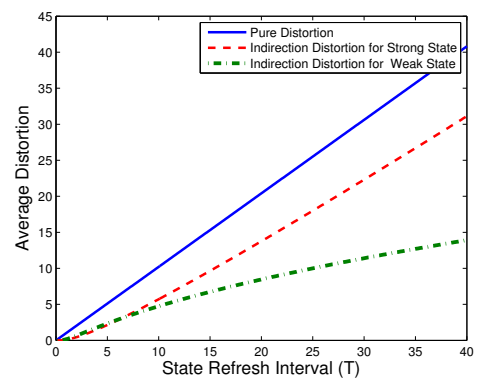

Fig. 6. Pure Distortion and Informed Distortion with respect to the interval between two consecutive refresh messages issued by the sender, T. When T is low, the receiver's state is frequently refreshed and it is more likely to remain within the tolerable interval of sender's state. In this case, strong state does not cause any informed distortion whereas weak state can cause some distortion even though it occurs with low probability. Hence, for small T informed distortion is slightly larger for weak state. When T is large, the informed distortion is much smaller for weak state due to probabilistic semantics.

In Fig 6, we present the resulting pure and informed distortion for strong state and weak state with respect to the refresh interval, T. We use the same parameters as in the previous case with $\sigma^2 = 2$. Pure distortion for strong state and weak state are the same. When the state refresh interval is small, the receiver's state is frequently updated and the probability that it remains within the sender's state's tolerable interval is large. In this case, strong state does not cause any informed distortion. On the other hand, weak state can cause some distortion even though it happens with a small probability. As a result, strong state causes slightly less distortion than the weak state. When T is large, the receiver's state probably does not remain within the tolerable interval. Weak state can infer this due to small confidence. Therefore, as T increases the informed distortion for the weak state becomes significantly

lower than strong state. Even if the pure distortion is very large, the receiver is aware of this and hence the protocol can adapt to prevent any performance loss. In order to achieve the same informed distortion, strong state needs to be updated using a much smaller refresh interval implying more overhead, which in turn causes performance degradation.

B. Ornstein-Uhlenbeck Process

We use Ornstein-Uhlenbeck Process in order to incorporate the state timeout into our analysis. The process is stationary, Gaussian and Markovian. $E[\mathbf{x}(t)] = 0$ as $t \to \infty$. The autocorrelation function is of the form $R(\tau) = \sigma^2 e^{-\alpha|\tau|}$. Given $\mathbf{x}(0) = x_0$, the random variable $\mathbf{x}(t)$ is a Gaussian random variable with mean $\mu_t = x_0 e^{-\alpha t}$ and variance $\sigma_t^2 = \sigma^2(1 - e^{-2\alpha t})$ [8] (chapter 11-1).

Considering the state timeout, the instantaneous distortion values are

$$e_S(t) = \begin{cases} \sigma^2(1 - e^{-2\alpha t}) + (\mu_t - x_0)^2 & t \le \chi \\ \sigma^2 & t > \chi \end{cases} \quad (16)$$

$$e_W(t) = \begin{cases} \sigma^2(1 - e^{-2\alpha t}) & t \le \chi \\ \sigma^2 & t > \chi. \end{cases} \quad (17)$$

The equations are obtained using necessary substitutions in (10) and (11). In both cases, the instantaneous distortion becomes σ^2, the variance of a sample obtained from the process without any conditional information, when $t > \chi$ since the state is removed the receiver does not maintain any value.

The state information maintained at the receiver can be still valid even if the receiver removes the state due to refresh messages that are lost over the logical link. The probability that a state that has been removed is still valid is associated with the confidence given in (15). Considering this, and using (13) and (14) the expected informed distortion values are

$$e_S^{(I)}(t) = \begin{cases} (1 - \Theta(t)) \left(\sigma^2(1 - e^{-2\alpha t}) + (\mu_t - x_0)^2\right) & t \le \chi \\ \Theta(t)\sigma^2 & t > \chi \end{cases}$$

$$e_W^{(I)}(t) = \begin{cases} 2\,(1 - \theta(t))\,\theta(t) \left(\sigma^2(1 - e^{-2\alpha t})\right) & t \le \chi \\ \theta(t)\sigma^2 & t > \chi \end{cases}$$

where $\Theta(t) = P\left(|\mathbf{x}(t) - x_0| \le \rho\right)$ and $\theta(t)$ is given in (15).

For pure distortion, let's introduce an intermediate step

$$E(t_1, t_2) := \int_{t_1}^{t_2} e(\tau) d\tau$$

and $E(t) = E(0, t)$. For $t \le \chi$, we have

$$E_S(t) = \sigma^2 \left(t + \frac{1}{2\alpha} e^{-2\alpha t} - \frac{1}{2\alpha}\right) + x_0^2 \left(t + \frac{2}{\alpha} e^{-\alpha t} - \frac{2}{\alpha} - \frac{1}{2\alpha} e^{-2\alpha t} + \frac{1}{2\alpha}\right) \quad (18)$$

$$E_W(t) = \sigma^2 \left(t + \frac{1}{2\alpha} e^{-2\alpha t} - \frac{1}{2\alpha}\right).$$

The equations are derived by integrating (16) and (17), respectively. Note that (18) is obtained given that x_0 is known. However, x_0 is also a stochastic value. Since $E[x_0^2] = \sigma^2$, in order to find the expected distortion, we plug in σ^2 for x_0^2. Then, the cumulative distortion is

$$E_S(t) = 2\sigma^2 \left(t + \frac{1}{\alpha} e^{-\alpha t} - \frac{1}{\alpha}\right).$$

In order to find the average distortion, we have to divide the time axis into two, before and after timeout, in order to capture (17) and (16). Let's define

$$\kappa = \left\lfloor \frac{\chi}{T} \right\rfloor.$$

The average pure distortion considering the refresh packet losses can be derived as follows:

$$\overline{e^P} = \sum_{i=1}^{\kappa} \frac{1}{iT} p^{i-1}(1-p) E(iT) + \sum_{i=\kappa+1}^{\infty} \frac{1}{iT} p^{i-1}(1-p)\left[E(\chi) + E(\chi, iT)\right].$$

Note that $E(\chi, t) = \sigma^2(t - \chi)$ if $t \ge \chi$.

For weak state,

$$\overline{e_W^P} \approx \sigma^2 \left[1 - \frac{1}{2\alpha T} + \frac{1}{2\alpha T} e^{-2\alpha T}(1-p) \frac{1 - (pe^{-2\alpha T})^\kappa}{1 - pe^{-2\alpha T}} + \frac{1}{\kappa T} p^\kappa \frac{1}{2\alpha} e^{-2\alpha\chi}\right]. \quad (19)$$

(19) is because $\frac{u^i}{i} \approx u^i$ when $u << 1$. Also note that $\frac{u^i}{i} < u^i$. Since u^i converges if $0 < u < 1$, $\frac{u^i}{i}$ converges as well. Similarly for strong state, we have

$$\overline{e_S^P} \approx 2\sigma^2 \left[1 - \frac{1}{\alpha T} + \frac{e^{-\alpha T}}{\alpha T}(1-p) \frac{1 - (pe^{-\alpha T})^\kappa}{1 - pe^{-\alpha T}} + 2p^\kappa \frac{1}{\alpha\kappa T} e^{-\alpha\chi}\right]. \quad (20)$$

We obtain the informed distortion values through numerical analysis. We now compare and contrast the distortion performance of weak and soft state for this particular process. Unless otherwise noted, we use the following default parameters: $\sigma = 1$, $p = 0.02$, $T = 5s$, $\rho = 1/\sqrt{2}$, $\gamma = 0.01$ and $\alpha = 0.1$. For strong state, $\chi = 3T$.

In Fig. 7, we compare strong state and weak state against the state refresh interval. In all cases, more consistent indirection, i.e a smaller distortion, requires larger state refresh rate. The results here are similar to those we present in Fig. 6. The figure shows that all the distortion metrics increase with the refresh interval; however, weak state can achieve a target distortion with larger refresh intervals, and reduces the overhead. Since weak state contains the confidence in the state, the distortion is much smaller in comparison to the strong state which uses deterministic semantics. With weak state, even when the pure distortion is larger, the destination is aware of the probability that the actual state remains within a predefined interval around the value updated locally.

In Fig. 8, we show the effect of the correlation parameter α on the distortion metrics. Consider two samples obtained from the state $\mathbf{x}$ at times t_1 and t_2. The correlation coefficient between $\mathbf{x}(t_1)$ and $\mathbf{x}(t_2)$ is given by $e^{-\alpha|t_1 - t_2|}$. With large

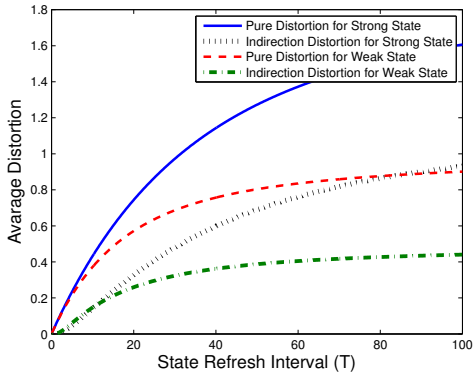

Fig. 7. Pure Distortion and Informed Distortion with respect to the refresh interval, T. With small T, the receiver's state is frequently refreshed and distortion metrics are low. As T grows, distortion metrics increase. Weak state reduces the pure distortion using better estimates. With probabilistic semantics, it is also aware that the perceived state is unlikely valid and hence informed distortion is lower.

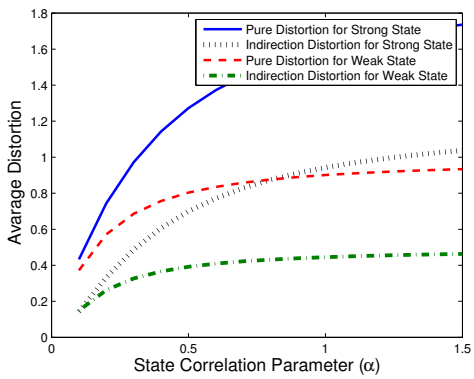

Fig. 8. Distortion metrics with respect to the process correlation parameter, α. Large α indicates that the state value is more dynamic even though the state variance is the same. Weak state reduces the pure distortion by obtaining better estimates about the state value. The probabilistic semantics decreases the informed distortion.

α, the correlation between these two samples decreases. This way, α is similar to λ parameter we used in Section II, the rate at which state is updated.

As α increases, the correlation between the last updated state value and the current value of the state decreases. The strong state approach turns into modeling a random variable with another (almost) uncorrelated random variable that pertains to the same mean and variance, which is the last received update. In this case, the mean square and the pure distortion becomes twice as the variance of the random variables. On the other hand, locally updating the state limits the expected distortion to variance of the state. Since the difference between the

Note that, α indicates how fast the process deviates from previous values. In order to achieve a distortion bound, the state refresh interval should decrease with increasing α. Weak state helps achieving this criterion with larger refresh interval.

V. Experimental Evaluation

In our analysis in previous sections, we assume that the receiver is aware of the properties of the process that the state yields information about. However, this may not always be true. In this section, we perform simulations to evaluate weak state and strong state without such knowledge. We simulate a 500 node network where nodes move according to the Gauss-Markov mobility model [9] in a 2000x2000 m^2 area using ns2. Each node sends its location updates to a fixed base station that is located right in the center of the area through multiple hops using geographical routing. The bit error rate in the wireless channel is 0.001. The refresh messages can also be lost due to MAC layer errors.

In the mobility model we use, the node velocity is modeled as a discrete Ornstein-Uhlenbeck Process, which we have reviewed in the previous section. The node velocity is correlated over time. The node velocity can be represented by the following equation:

$$v_n = \alpha v_{n-1} + (1-\alpha)\bar{\mu} + \sqrt{1-\alpha^2}\omega_{n-1} \tag{21}$$

where α is the correlation parameter, μ is the asymptotic mean velocity vector and ω_n are the independent zero mean gaussian random variables with variance σ^2. v_n is the velocity vector within the time interval n and $n+1$ and it constant in this interval. The location of a node at time n is

$$x_n = x_0 + \sum_{j=0}^{n-1} v_j$$

The central base station maintains the location information of each node. Let $\hat{x}(a)_n$ denote the location entry for node a at time n in the base station. We calculate the distortion metrics numerically. The pure distortion at time n for node a is $|x(a)_n - \hat{x}(a)_n|$, the distance between the actual position and the maintained position. If the receiver were aware about the mobility model as well as the values for $\bar{\mu}$, α and σ, the distance between the actual location of a node and its location perceived by the base station would be a random variable with Rayleigh distribution. With this model, the confidence value at the nth time step would be

$$\theta(n) = 1 - \exp(-\frac{\rho^2}{\sigma_n^2})$$

where

$$\sigma_n = \sqrt{2(1-\alpha^2)} \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} \alpha^{i-j-1}\sigma.$$

However without such knowledge, we cannot use such a probabilistic confidence. Instead, we use a confidence value that exponentially decays with time such that $\theta(n) = q^n$. Moreover, local updates only consist of updating the confidence value rather than obtaining a better estimate, of which advantages are already reported in [9]. In our case, the perceived location for weak state is the last reported location of the node, similar to the strong state.

In Fig 9, we show the average distortion results we obtained from the simulations where the nodes move according to Gauss-Markov model with parameters, $\alpha = 0.75$, $|\bar{\mu}| = 10$

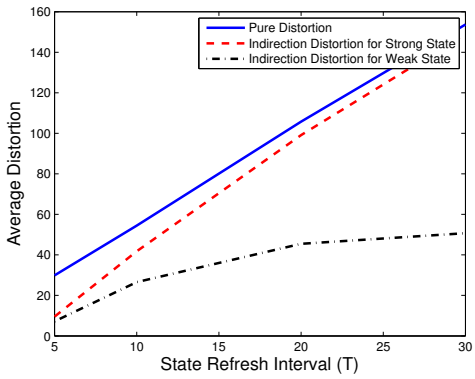

Fig. 9. Distortion with respect to state refresh interval from simulation results where the state corresponds to the positions of nodes where they move according to the Gauss-Markov mobility model but the receiver is not aware of the properties of node mobility. The local updates only consist of weakening state semantics, or the confidence. Hence, pure distortion is equal for each while informed distortion for weak state is smaller.

and $\sigma = 1$. In strong state, state timeout interval is $\chi = 3T$ where T is the state refresh interval. On the other hand, weak state times out at n when $q^n = 0.01$. We used $q = 0.9$ in our evaluation. The pure distortion is identical for both state types since the perceived location information is the same in both cases and the refresh packets are subject to the same loss probability. The difference in the informed distortion stems from the probabilistic semantics of the weak state. The strong state does not cause any informed distortion if the pure distortion is less than ρ, which we took $\rho = 50$ in our calculations. Weak state can cause distortion in this case with probability $1 - q^n$. If pure distortion is larger than ρ, informed distortion is found by weighting the pure distortion by 1 and q^n for strong state and weak state, respectively. The simulation results show that when T is small, the state is frequently updated and the pure distortion is less likely to be larger than ρ. Hence, the informed distortion for strong state is very small. As T increases, the informed distortion for strong state approaches to the pure distortion. On the other hand, weak state can still figure that the state is less likely to remain valid because of its probabilistic semantics. As a result, the informed distortion is significantly lower than strong state if the same refresh rate is used and a target distortion can be achieved using a lower refresh rate and hence less overhead.

VI. Conclusion

In this paper, we have studied the consistency of indirections performed with weak state which is a novel concept that we have proposed in a recent work [1]. Weak state is characterized by local updates and probabilistic semantics unlike traditional strong state where the information is regarded as absolute truth. If the state provides information about a dynamic process, strong state quickly becomes invalidated. On the other hand, weak state is more stable and consistent. In order to evaluate the consistency, we use two metrics: pure distortion and informed distortion. Pure distortion quantifies the average difference between the actual state value and value maintained at the remote node. Protocols that rely on weak state on the other hand also utilize the information about the confidence in the state value. Even if the pure distortion is large, a communication protocol that utilizes weak states can take the necessary steps to improve the performance if the confidence is low. Given the confidence value, the protocol can make informed decisions. We capture this effect using the informed distortion metric.

We present an analysis to compare/contrast weak state and strong state in terms of pure and informed distortion metrics with respect to the parameters of the signaling mechanism as well as the properties of the process that we use to model the state information. Our analysis and simulation results monotonically show that weak state, yielding information about a dynamic process, reduces both pure distortion and informed distortion significantly if the same state refresh rate is used. Even without the information about the process, weak state reduces the informed distortion. In other words, weak state can achieve the a target distortion with a smaller refresh rate and lower overhead. On the other hand, if they contain rather static information, the distortion weak state causes can be larger than that of strong state. Hence, weak state is particularly useful for modeling dynamic information that needs to be refreshed at a rate that is lower than it changes due to network conditions.

In this paper, we do not propose a particular protocol that interprets the confidence in the state value. Even though the information weak state contains is more consistent than that of strong state, it is not exact. The way the protocol chooses the deal with the confidence value lays down tradeoffs in various ways. In the future work, we plan to identify and investigate such tradeoffs. In this paper, we assumed that the lifetime of the state is infinity. With this scenario, state is never removed from the sender and the state at the receiver never models an nonexisting state. In other words, the state does not become *orphaned*. Our plans also include to investigate where this situation can also occur.

References

[1] U. G. Acer, S. Kalyanaraman, and A. A. Abouzeid, "Weak State Routing for Large Scale Dynamic Networks," *MobiCom '07: Proceedings of the 13th Annual ACM International Conference on Mobile Computing and Networking*, pp. 290–301, 2007.

[2] D. D. Clark, "The Design Philosophy of the DARPA Internet Protocols," in *SIGCOMM*, 1988, pp. 106–114.

[3] A. Lindgren, A. Doria, and O. Schelen, "Probabilistic Routing in Intermittently Connected Networks," *Service Assurance with Partial and Intermittent Resources. First International Workshop, SAPIR 2004. Proceedings (Lecture Notes in Comput. Sci. Vol.3126)*, pp. 239 – 54, 2004.

[4] A. Kumar, J. Xu, and E. W. Zegura, "Efficient and Scalable Query Routing for Unstructured Peer-to-Peer Networks," *Proceedings of IEEE INFOCOM*, vol. 2, pp. 1162 – 1173, 2005.

[5] S. Keshav, "Efficient and Decentralized Computation of Approximate Global State," *Computer Communication Review*, vol. 36, no. 1, pp. 69 – 73, 2006.

[6] S. Raman and S. McCanne, "A Model, Analysis, and Protocol Framework for Soft State-based Communication," *Computer Communication Review*, vol. 29, no. 4, pp. 15 – 25, 1999.

[7] P. Ji, Z. Ge, J. Kurose, and D. Towsley, "A Comparison of Hard-State and Soft-State Signaling Protocols," *IEEE/ACM Transactions on Networking*, vol. 15, no. 2, pp. 281 – 294, 2007.

[8] A. Papoulis and U. S. Pillai, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill Science/Engineering/Math, December 2001.

[9] B. Liang and Z. J. Haas, "Predictive distance-based mobility management for multidimensional pcs networks," *IEEE/ACM Transactions on Networking*, vol. 11, no. 5, pp. 718 – 732, 2003.