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Queuing network models for delay analysis of multihop wireless ad hoc networks ☆

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Abstract

In this paper we analyze the average end-to-end delay and maximum achievable per-node throughput in random access multihop wireless ad hoc networks with stationary nodes. We present an analytical model that takes into account the number of nodes, the random packet arrival process, the extent of locality of traffic, and the back off and collision avoidance mechanisms of random access MAC. We model random access multihop wireless networks as open $G/G/1$ queuing networks and use the diffusion approximation in order to evaluate closed form expressions for the average end-to-end delay. The mean service time of nodes is evaluated and used to obtain the maximum achievable per-node throughput. The analytical results obtained here from the queuing network analysis are discussed with regard to similarities and differences from the well established information-theoretic results on throughput and delay scaling laws in ad hoc networks. We also investigate the extent of deviation of delay and throughput in a real world network from the analytical results presented in this paper. We conduct extensive simulations in order to verify the analytical results and also compare them against NS-2 simulations.
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1. Introduction

A multihop wireless ad hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. The transmission power of a node is limited, thus the packets may have to be forwarded by several intermediate nodes before they reach their destinations. Hence each node may be a source, destination and relay. The wireless medium is shared and scarce, therefore ad hoc networks require an efficient MAC protocol [1]. Since ad hoc networks lack infrastructure and centralized control, MAC protocols for ad hoc networks should be distributed, such as random access MAC protocols, e.g. MACA [8] and MACAW [1]. The delay and throughput of wireless ad hoc networks depend on the number of nodes, the transmission range of the nodes, the network traffic pattern and the

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behavior of the MAC protocol. In this paper we investigate how the end-to-end delay and maximum achievable throughput in a random access based MAC multihop wireless network with stationary nodes depend on the number of nodes, transmission range and traffic pattern. We propose a queuing network model for delay analysis of random access multihop wireless ad hoc networks. The queuing network model proposed in this paper is unique in that it allows us to derive *closed form* expressions for the average end-to-end delay and maximum achievable throughput. The packet delay is defined as the time taken by a packet to reach its destination node after it is generated. The average end-to-end delay is the expectation of the packet delay over all packets and all possible network topologies. Our analysis takes into account the queuing delays at source and intermediate nodes. The packets are assumed to have a fixed size and random arrival process. Moreover we also characterize how the average end-to-end delay and maximum achievable throughput vary with the degree of locality of traffic. The primary purpose of this study is not to accurately model the performance of specific standard protocols like IEEE 802.11 MAC (even though the results do provide a good match with NS-2 simulations) but to gain insights into the queuing delays and maximum achievable throughput in random access multihop wireless ad hoc networks.

Several studies have focused on finding the maximum achievable throughput and characterizing capacity-delay tradeoffs in wireless ad hoc networks e.g. [6,7,11,12]. In [7] it is shown that for a wireless network with n stationary nodes, the per-node capacity scales[1] as $\Theta(W/\sqrt{n\log n})$. In [11], the authors use simulations in order to study the dependence of per-node capacity on IEEE 802.11 MAC interactions and traffic pattern for various topologies like single cell, chain, uniform lattice and random network. An estimate of the expressions for one-hop capacity and upper bound of per-node throughput is obtained using the simulation results.

In [6], the authors characterize the delay-throughput tradeoffs in wireless networks with stationary and mobile nodes. It is shown that for a network with stationary nodes, the average delay and throughput are related by $D(n) = \Theta(nT(n))$, where $D(n)$ and $T(n)$ are the average end-to-end delay and throughput, respectively. However the delay is defined as the time taken by a packet to reach the destination *after it has left the source*. Also, according to the network model, the packet size scales with the throughput. Under these assumptions the delay is simply proportional to the average number of hops between a source–destination pair. i.e. *In their model, there is no delay due to queuing*. If, more realistically, the packet size is assumed to be constant and the delay is defined as time taken by a packet to reach the destination *after its arrival at the source*, there would be queuing delays at the source and intermediate nodes.

Several recent studies have proposed queuing models for performance evaluation of the IEEE 802.11 MAC. A finite queuing model is proposed and used in [18] for evaluating the packet blocking probability and MAC queuing delays in a Basic Service Set. A queuing model for performance evaluation of IEEE 802.11 MAC based WLAN in the presence of HTTP traffic is proposed in [13]. In [14] the service time of a node, in IEEE 802.11 MAC based wireless ad hoc network, is modeled as a Markov modulated general arrival process. The resulting M/MMGI/1/K queuing model is used for delay analysis over a single hop in the network. An analytical model for evaluating closed form expression for the average queuing delay over *a single hop* in IEEE 802.11 based wireless networks is presented in [17]. In [15], the authors use queuing theoretic approach in order to calculate the mean packet delay, maximum throughput and collision probability for an elementary four node network with hidden nodes and extend the results to *linear* wireless networks. It is worth noting that none of the prior works [13–15,17,18] extends to a general *two dimensional wireless network*.

We propose a detailed analytic model for multihop wireless ad hoc networks based on open $G/G/1$ queuing networks. We first evaluate the mean and second moment of service time over a single hop by taking into account the back off and collision avoidance mechanisms of IEEE 802.11 MAC. We then use the diffusion approximation for solving open queuing networks in order to derive closed form expression for the average end-to-end packet delay. Using the average service time of the nodes we obtain an expression for the maximum achievable throughput. We present detailed discussions on how the maximum

[1] The asymptotic notations used in this paper have the following meanings:

- $f(n) = \Theta(g(n)) \Rightarrow \exists c_1, c_2, n_o > 0$ s.t. $c_1 g(n) \leqslant f(n) \leqslant c_2 g(n) \forall n \geqslant n_0$.
- $f(n) = O(g(n)) \Rightarrow \exists c, n_0 > 0$ s.t. $0 \leqslant f(n) \leqslant cg(n)\ \forall n \geqslant n_0$.
- $f(n) = o(g(n)) \Rightarrow \exists c, n_0 > 0$ s.t. $0 \leqslant f(n) < cg(n) \forall\ n \geqslant n_0$.
- $f(n) = \omega(g(n)) \Rightarrow \exists c, n_0 > 0$ s.t. $0 \leqslant cg(n) < f(n) \forall n \geqslant n_0$.

achievable throughput obtained from our model compares with the per-node capacity of Gupta–Kumar's model. The main results of this paper are:

1. The average end-to-end packet delay for our model is $D(n) = \frac{\rho}{\lambda \cdot (1-\hat{\rho})}$, where ρ is the utilization factor of a node, λ is the packet arrival rate at the nodes and $\hat{\rho}$ is a variable whose value depends on first and second moments of inter-arrival and service times.
2. The maximum achievable throughput in a multi-hop wireless ad hoc network is $\lambda_{\max} = o\left(\frac{1}{\bar{s}nr(n)^2}\right)$, where $\bar{s}$ is the expected number of hops between a source–destination pair and $r(n)$ is the transmission radius of the nodes.
3. When the parameters of our network model are comparable to the Gupta–Kumar's model [7], $\lambda_{\max} = o\left(\frac{W}{\sqrt{n \log n}}\right)$.

The analytical results are verified against extensive simulations and numerical computations. We also perform NS-2 simulations and discuss how the analytical results compare with the delay results obtained for some of the established wireless protocols.

The rest of the paper is organized as follows. In Section 2 we briefly describe the well known diffusion approximation for solving open $G/G/1$ queuing networks. Section 3 presents a detailed description of the network model. Delay and throughput analysis of multihop wireless networks is presented in Section 4. In Section 5 we present intuitive interpretations of the analytical results and investigate how the results deviate from delay and throughput in realistic networks. The comparison of the analytical and simulation results is presented in Section 6. Finally we present concluding remarks in Section 7.

2. Diffusion approximation method

2.1. About the diffusion approximation and its accuracy

The diffusion approximation was proposed by Kobayashi [9] in 1970s as a technique to solve non-product form queuing networks. The discrete valued queuing process is replaced with a continuous path Markov process with infinitesimal increments. The continuous Markov process is called diffusion process and its probability distribution is described by a *diffusion equation*. In a queuing network, the diffusion equation accounts for the interaction between queuing stations by using variance–covariance matrices for arrival and service processes at each station. Using appropriate boundary conditions, the diffusion equation may be solved for a queuing station and the overall probability distribution of the state of a network is represented by the product of the states of the individual queues. In other words, the diffusion process approximates a non-product form network as a product form network by taking into account the interactions between queuing stations in the diffusion approximation. *This makes the diffusion approximation a powerful tool for evaluating mean closed form results for non-product networks which cannot be accurately analyzed by using other existing analytical techniques.* The fact that the interaction between the queues of the network are taken into account in the form of variance–covariance matrices makes the diffusion approximation a much more accurate approximation than the exponential approximation, where the stations are replaced with a server with exponential arrival and service process of the same means.

The comparison of diffusion approximation with exponential approximation in terms of accuracy is studied in [16]. It is shown that for an open network, the error in diffusion approximation lies between (1.5%, 15%) and (1%, 6%) under light and heavy load conditions. On the other hand, the error of simplistic exponential approximation lies between (30%, 65%) and (30%, 100%) for the same light and heavy conditions. The error of diffusion approximation is much smaller than the exponential approximation and is practically negligible for high load conditions. This is because in heavy load conditions, the approximation of discrete queuing process with continuous Markov process becomes more accurate. Although solving the diffusion equation is quite involved, the end result is not very complex. Thus we get increased accuracy with little increase in complexity. In the rest of this section we present a road map for solving an arbitrary open queuing network using diffusion approximation. The advantage of using the diffusion approximation in this work is that it allows us to obtain closed form expressions for the average end-to-end delay.

2.2. Diffusion approximation steps

In this section we briefly describe how the diffusion approximation is used to solve an open $G/G/1$ queuing network. (Please see [3] for details.)

Consider an open queuing network with n service stations, numbered from 1 to n. The external arrival of a job is a renewal process with an average inter-arrival time of $1/\lambda_e$ and the coefficient of variance of inter-arrival time equals c_A. The mean and coefficient of variance of the service time at a station i are denoted by $1/\mu_i$ and c_{Bi}, respectively.

The *visit ratio* of a station in a queuing network is defined as the average number of times a job is forwarded by (i.e. visits) the station. The visit ratio of station i, denoted by e_i, is given by

$$e_i = p_{0i}(n) + \sum_{j=1}^{j=n} p_{ji}(n) \cdot e_j, \tag{1}$$

where p_{0i} denotes the probability that a job enters the queuing network from station i and p_{ji} denotes the probability that a job is routed to station i after completing its service at station j.

There are two sources of job arrivals at a station: the jobs that are generated at the station and the jobs that are forwarded to the station by other stations. The resulting arrival rate is termed the *effective arrival rate* at a station. The effective arrival rate at the station i, denoted by λ_i is given by

$$\lambda_i = \lambda_e e_i. \tag{2}$$

The *utilization factor* of station i, denoted by ρ_i, is given by

$$\rho_i = \lambda_i/\mu_i. \tag{3}$$

The squared coefficient of variance of the inter-arrival time at a station i, denoted by c^2_{Ai}, is approximated using

$$c^2_{Ai} = 1 + \sum_{j=0}^{n} \left(c^2_{Bj} - 1\right) \cdot p^2_{ji} \cdot e_j \cdot e_i^{-1}, \tag{4}$$

where $c^2_{B0} = c^2_A$.

According to the diffusion approximation, the approximate expression for the probability that the number of jobs at station i equals k, denoted by $\hat{\pi}_i(k)$, is

$$\hat{\pi}_i(k) = \begin{cases} 1 - \rho_i & k = 0, \\ \rho_i(1 - \hat{\rho}_i)\hat{\rho}_i^{k-1} & k > 0, \end{cases} \tag{5}$$

where

$$\hat{\rho}_i = \exp\left(-\frac{2(1 - \rho_i)}{c^2_{Ai} \cdot \rho_i + c^2_{Bi}}\right). \tag{6}$$

The mean number of jobs at a station i, denoted by $\overline{K_i}$, is

$$\overline{K_i} = \rho_i/(1 - \hat{\rho}_i). \tag{7}$$

3. Queuing network model

In this section we present the network model and develop a queuing network model for multihop wireless networks. We also derive expressions for the parameters of the queuing network model.

3.1. The network model

The network consists of $n + 1$ nodes, numbered 1 to $n + 1$, that are distributed uniformly and independently over a torus of unit area. We assume a torus area in order to avoid complications in the analysis caused by edge effects. Each node is assumed to have an equal transmission range, denoted by $r(n)$. Let r_{ij} denote the distance between nodes i and j. Nodes i and j are said to be *neighbors* if they can directly communicate with each other, i.e. if $r_{ij} \leqslant r(n)$. Let $N(i)$ denote the set of nodes that are neighbors of node i. All the neighbors of a node lie on a disc of area $A(n) = \pi r(n)^2$ centered at the node. The area $A(n)$ is termed the *"communication area"* of a node. The communication area is chosen such that the network is connected which ensures that $N(i) \neq \phi\, \forall i$. The transmission rate of each node equals W bits/s.

We consider deployment over unit area so that n also equals the node density of the network. Thus the results of this paper indicate how delay and maximum achievable throughput scale with node density. Characterizing delay and capacity in terms of node density is more apprehensible since it makes the results independent of the area over which the network is deployed. For a network of n nodes, each with communication radius R, deployed over area S, our results can be directly applied by considering an equivalent network deployment of unit area. This can be done by simply setting the parameter $r(n)$ of our model equal to $R/\sqrt{S}$.

We use a special case of the Protocol Model of interference described in [7]. If node i transmits to node j then the transmission will be successful only if (a) $r_{ij} \leqslant r(n)$ and (b) $r_{kj} > r(n)$ for every other node $k \neq i, j$ that transmits simultaneously with node i. In other words, node i can successfully transmit a packet to node j only if i is a neighbor of j and no other neighbor of j is transmitting concurrently with i. (This is equivalent to setting $\Delta = 0$ in the Protocol Model in [7].)

The traffic model for the network may be described as follows. Each node in the network could be a source, destination and/or relay of packets. Each node generates packets with rate λ packets/s. The delay analysis is possible for any packet generation process as long as the mean and SCV of packet inter-arrival time are known. For the sake of simplicity, we assume in our model that the packet generation process at each node is an i.i.d. Poisson process. The size of each packet is constant and equals L bits. When a node receives a packet from any of its neighbors, it either forwards the packet to its neighbors with probability $(1-p(n))$ or absorbs the packet with probability $p(n)$. The probability $p(n)$ is referred to as "absorption probability". In other words, the absorption probability is the probability that a node is the destination of a packet given that the node has received the packet from its neighbors. When a node forwards a packet, each of its neighbors is equally likely to receive the packet. The advantage of such a model is that we can control the locality of the traffic by varying the parameter $p(n)$. The traffic is highly localized if $p(n)$ is large while a small value of $p(n)$ implies unlocalized traffic. This would help us to characterize the effect of the locality of the traffic on the average delay and maximum achievable throughput.

For example, suppose that node j in Fig. 1 receives a packet from i. The probability that node j is the destination of the packet is $p(n)$. The probability that node j forwards the packet to one of its neighbors is $(1-p(n))$. Suppose node j forwards the packet, then the probability that the packet is forwarded to node k is $\frac{1}{|N(j)|}=\frac{1}{4}$.

We assume that each node in the network has infinite buffers which means that no packets are dropped in the network. The packets are served by the nodes on first come first serve basis.

Multihop wireless ad hoc networks can be modeled as a queuing network as shown in Fig. 2a. The stations of the queuing network correspond to the nodes of the wireless network. The forwarding probabilities in the queuing network, denoted by p_{ij}, correspond to the probability that a packet that is transmitted by node i enters the node j's queue. Fig. 2b shows a representation of a node in the ad hoc network as a station in the queuing network.

The end-to-end delay in a wireless network equals the sum of queuing and transmission delays at source and intermediate nodes. We will use the queuing network model shown in Fig. 2a and b in order to mathematically analyze the end-to-end delay.

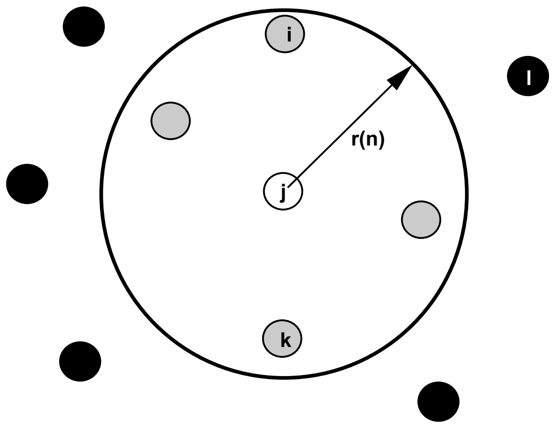

Fig. 1. A portion of a multihop wireless ad hoc network.

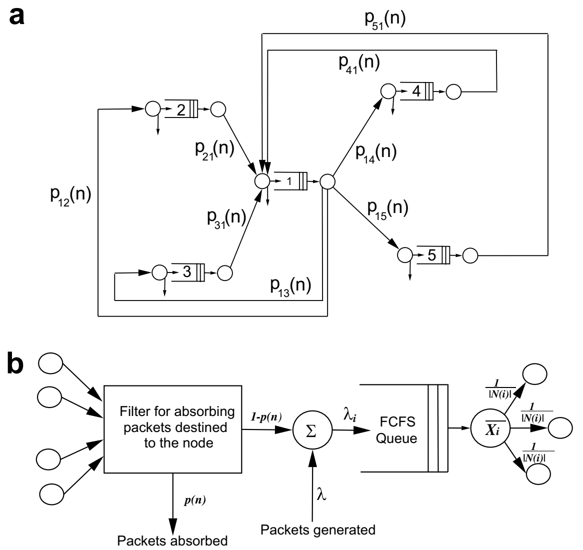

Fig. 2. Queuing network model for multihop wireless ad hoc network. (a) Representation of multihop wireless ad hoc network as a queuing network. (b) Representation of a node of multihop wireless ad hoc network as a station in the queuing network.

3.2. Parameters of the queuing network model

In this section we present expressions for the parameters of the queuing network model of multihop wireless networks. The (relatively straightforward) detailed proofs of the results are presented in [2].

Lemma 1. *The expected probability that a packet is forwarded from node i to node j, denoted by* $\overline{p_{ij}}(n)$, *is*

$$\overline{p_{ij}}(n)=\begin{cases}\frac{1-p(n)}{n}\left(1-(1-A(n))^{n}\right) & i\neq j,\\ 0 & i=j.\end{cases}\tag{8}$$

Lemma 2. *The expected visit ratio of node i, denoted by* $\overline{e_i}$*, is given by*

$$\overline{e_i} = \frac{1}{(n+1)p(n)} \; \forall \, i. \tag{9}$$

Lemmas 1 and 2 indicate that the nodes visit ratio and the forwarding probabilities averaged over all possible instances of the topologies are similar to the visit ratios and forwarding probabilities of an average topology where each node has a number of neighbors equal to the average case. Thus, as a result of these two lemmas, one may derive the remaining set of model parameters (effective packet arrival rate and number of packets traversed) by considering the average case topology. Applying these results in the diffusion model will provide expressions for the average end-to-end delay, defined as the expectation of the packet delay over all packets and all possible networks.

Lemma 3. *The effective packet arrival rate at a node i, denoted by* λ_i*, is*

$$\lambda_i = \lambda / p(n). \tag{10}$$

Lemma 4. *The expected number of hops traversed by a packet between its source and destination, denoted by* $\bar{s}$*, equals* $\frac{1}{p(n)}$.

The average queuing delay depends upon the service time distribution of the nodes. The latter depends on the MAC protocol used by the nodes.

4. Queuing analysis

In this section we first present a model for a random access MAC that accounts for the back off and collision avoidance mechanisms of IEEE 802.11 MAC. We then present the delay analysis of multihop wireless ad hoc networks by integrating the MAC model with the queuing network model developed in Section 3.

4.1. The MAC model

4.1.1. Interfering neighbors

Two nodes are said to be *interfering neighbors* if they lie within a distance of $2r(n)$ of each other (see Fig. 3). The transmission of a node would be successful if none of the interfering neighbors of the node transmits concurrently. Also two nodes may successfully transmit at the same time if they are not interfering neighbors of each other. The definition of interfering neighbors is similar to the definition given in [7].

4.1.2. The random access MAC model

Before transmitting each packet the nodes count down a random back off timer. The duration of the timer is exponentially distributed with mean $1/\xi$. As in IEEE 802.11, the timer of a node freezes each time an interfering neighbor starts transmitting a packet. When the back off timer of a node expires, it starts transmitting the packet and the back off timers of all its interfering neighbors are immediately frozen. The timers of the interfering neighbors are resumed as soon as the transmission of the packet is complete. The time required to transmit a packet from a node to its neighbor is $L/W + T_0$, where T_0 is the time required for the exchange of RTS, CTS and ACK packets. We assume that T_0 is negligible compared to L/W, so in our analysis we assume that the time required to transmit a packet is L/W. The model is mathematically tractable and at the same time captures the behavior of IEEE 802.11 MAC protocol.

4.2. Delay analysis

With the help of the following three lemmas we determine the mean and second moments of the service time of nodes using the random access MAC model. We then present the result for end-to-end delay in multihop wireless networks. The detailed proofs of the results presented in this section are presented in [2].

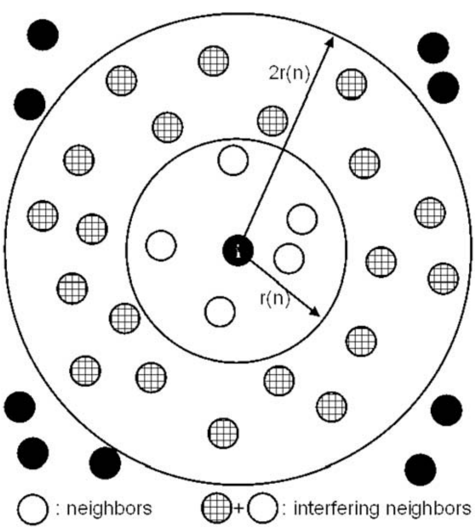

Fig. 3. This figure shows the neighbors and interfering neighbors of node *i* which is in the center of the figure.

Lemma 5. *Let H_i denote the number of interfering neighbors of a node i. Then*

$$E[H_i] = 4nA(n), \tag{11}$$

$$E[H_i^2] = 4nA(n)(1 + 4(n-1)A(n)), \tag{12}$$

where $A(n) = \pi \cdot r(n)^2$.

Lemma 6. *Let M_i denote the number of interfering neighbors of a node i that have at least one packet to transmit. Then under steady state,*

$$E[M_i] = \rho 4nA(n). \tag{13}$$

$$E[M_i^2] = \rho^2 \cdot 4nA(n)(1 + 4(n-1)A(n)) + (1-\rho)\rho 4nA(n). \tag{14}$$

where ρ is the utilization factor of the nodes.

Lemma 7. *Let Z_i denote the number of times the timer of a node i is frozen before its expiration. Then*

$$E[Z_i] = 4 \cdot \rho nA(n). \tag{15}$$

Theorem 1. *Let $\overline{X_i}$ and $\overline{X_i^2}$ denote the mean and second moment of service time required to serve a packet by a node i. Then*

$$\overline{X_i} = E[X_i] = \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i \frac{L}{W}}. \tag{16}$$

$$\overline{X_i^2} = E[X_i^2](1 + 3\bar{m} + 2\overline{m^2})\frac{L^2}{W^2} + 2(2\bar{m}+1)\frac{L}{W}\frac{1}{\xi} + \frac{2}{\xi^2}. \tag{17}$$

where $\bar{m} = E[M_i]$ Eq. (13) *and* $\overline{m^2} = E[M_i^2]$ Eq. (14).

Proof. The time taken by node i to serve a packet, denoted by X_i, is the sum of three terms: (i) the duration of the random back off timer (t_i), (ii) the duration for which the timer remains frozen ($Z_i L/W$), and (iii) the transmission time (L/W). Thus

$$X_i = t_i + Z_i \frac{L}{W} + \frac{L}{W}. \tag{18}$$

Taking expectation of both sides we get,

$$E[X_i] = E[t_i] + E[Z_i] \cdot \frac{L}{W} + \frac{L}{W} = \frac{1}{\xi} + 4\rho nA(n)\frac{L}{W} + \frac{L}{W}. \tag{19}$$

Substituting $\rho = \lambda_i \overline{X_i}$ and by rearranging, we get (16).

Also from (18) we have $X_i^2 = \left(t_i + Z_i + \frac{L}{W}\right)^2$ Given $M_i = m$ and $T_i = t_i, Z_i$ has a Poisson distribution. So $E\left[Z_i^2 | M_i = m, T_i = t_i\right] = m\xi t_i + (m\xi t_i)^2$. Using this and (54), we get

$$E\left[X_i^2 | T_i = t_i, M_i = m\right] = \left(1 + \frac{L^2}{W^2} m^2 \xi^2\right) t_i^2 + \left(\frac{2L}{W} + \frac{3L^2}{W^2} m\xi\right) t_i + \frac{L^2}{W^2}.$$

Taking expectation with respect to t_i we get

$$E\left[X_i^2 | M_i = m\right] = E_{T_i}\left[E\left[X_i^2 | T_i = t_i, M_i = m\right]\right] = (1 + 3m + 2m^2)\frac{L^2}{W^2} + 2(2m+1)\frac{L}{W}\frac{1}{\xi} + \frac{2}{\xi^2}.$$

Taking expectation of the RHS w.r.t m, we get (17). □

Corollary 1. *The standard deviation of service time of a node i, denoted by $\sigma_{X_i}^2$, is given by*

$$\sigma_{X_i}^2 = \frac{L^2}{W^2}\left(\bar{m} + \overline{m^2} + \sigma_m^2\right) + 2(2\bar{m}+1)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2}, \tag{20}$$

where $\sigma_m^2 = \overline{m^2} - \bar{m}^2$.

The squared coefficient of variance of the service time at a node i, denoted by c_{Bi}^2 is given by $\sigma_{X_i}^2 / \overline{X_i}^2$. Using (4), the squared coefficient of variance of the inter-arrival time at a node i, denoted by c_{Ai}^2, is given by

$$c_{Ai}^2 = 1 + \sum_{j=1, j\neq i}^{n+1} \left(c_{Bi}^2 - 1\right)\frac{1 - p(n)}{n} = 1 + \left(c_{Bi}^2 - 1\right)(1 - p(n)).$$

With the knowledge of c_{Ai}^2, c_{Bi}^2 and ρ, we can find the parameter $\hat{\rho}$ as given in (6).

Theorem 2. *For the random access MAC model the average end-to-end delay in a multihop wireless network, denoted by $D(n)$, is given by*

$$D(n) = \frac{\rho_i}{\lambda \cdot (1 - \hat{\rho})}. \tag{21}$$

4.3. Maximum achievable throughput

The *maximum achievable throughput*, denoted by $\lambda_{\max}$, is defined as the maximum packet arrival rate at each node for which the average end-to-end delay remains finite. If the packet arrival rate exceeds $\lambda_{\max}$, the delay would tend to infinity. The following corollary, that follows from Theorem 1, yields a relationship between the maximum achievable throughput and the network parameters.

Corollary 2. *For a multihop wireless network the maximum achievable throughput* $\lambda_{\max}$ *is*

$$\lambda_{\max} = \frac{p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}}. \tag{22}$$

Also from (22), $\lambda_{\max} = o(1/\bar{s}nA(n))$.

The result of Corollary 2 re-emphasizes the importance of carefully choosing the transmission ranges of nodes. $\lambda_{\max}$ increases with decrease in $r(n)$. However if $r(n)$ is too small then the network would become disconnected. According to [10], the network is asymptotically connected for $r(n) = \omega(\sqrt{\log n/n})$. So for a connected network $A(n) = \omega(\log n/n)$ and $\lambda_{\max} = o(\frac{p(n)}{c+4\log(n)(L/W)})$.

Another interesting observation is the dependence of $\lambda_{\max}$ on the traffic pattern. $\lambda_{\max}$ is directly proportional to $p(n)$. From (53) the expected number of hops traversed by a packet equals $1/p(n)$. Thus another way of interpreting the result is that $\lambda_{\max}$ is inversely proportional to the expected number of hops between a source–destination pair.

We further investigate how our result on the maximum achievable throughput compares with the result by Gupta–Kumar on throughput capacity. According to the Gupta–Kumar model, the nodes are distributed uniformly and independently over a sphere of unit surface area and each source chooses a random destination. Therefore the expected distance between a source and the corresponding destination equals the expected distance between two points uniformly and independently distributed on a sphere. Thus the expected distance between a source–destination pair in Gupta–Kumar's model is a constant (i.e. does not vary with n), say $\overline{s_{GK}}$. The transmission range in their model is $\omega(\sqrt{\log n/n})$. Thus the expected number of hops between a source–destination pair in Gupta–Kumar model is $o(\sqrt{n/\log n})$.

In order to compare our results with Gupta–Kumar's results we choose our model parameters such that we have comparable average number of hops between a source–destination pair and comparable transmission range. In our model if we choose $p(n) = \sqrt{\log n/n}$, then the expected number of hops between a source and destination node is $\bar{s} = 1/p(n) = \sqrt{n/\log n}$, which is comparable to the Gupta–Kumar model. Also $r(n) = \sqrt{\log n/n}$ or $A(n) = \pi \log n/n$ makes the transmission range of our model comparable to that of the Gupta–Kumar model. So for the model parameters that are comparable to the Gupta–Kumar model, the maximum achievable throughput is

$$\lambda_{\max} = \frac{\frac{1}{4\pi}\frac{W}{\sqrt{n\log nL}}}{1 + \frac{c}{4\pi\log n(L/W)}}. \tag{23}$$

or $\lambda_{\max} = o(W/\sqrt{n\log n})$.

The above discussion implies that for the similar values of parameters of the network model we get a bound similar to the Gupta–Kumar's bound on throughput capacity, but for our model the bound is not achievable. The reason for the bound not being achievable is that in our model we consider a random access MAC protocol rather than a perfect deterministic scheduling. Thus the bound becomes unachievable because some channel capacity is wasted by the nodes during contention for the channel.

5. Discussions

In this section we present a brief intuitive interpretation of the mean service time and evaluate the maximum achievable throughput for multihop wireless networks. We then discuss how the end-to-end delay scales with number of nodes in the asymptotic case, where the network size tends to infinity and the throughput of nodes tends to the maximum achievable throughput. We also discuss how our analytical results vary from those obtained for a more pragmatic network model.

5.1. Interpretation of mean service time

We present a mathematically non-rigorous, but intuitive, derivation of mean service time of a node for the random access MAC model. This derivation further elucidates the result on service time. Consider a hypothetical *two node network* where one of the nodes transmits packets to the other node. Both nodes use the random access MAC model described in Section 4.1. In this scenario there is no contention for the channel and the average service time of the transmitter would be $\frac{1}{\xi} + \frac{L}{W}$. We refer to $\frac{1}{\xi} + \frac{L}{W}$ as the *uncontended service time*.

Now consider a node (say node 0) with m interfering neighbors, numbered 1 through m. The node and its interfering neighbors use the random access MAC model for collision avoidance. Packets of size L_j bits arrive at a rate of α_j packets/s at neighbor j. From the point of view of node 0, the channel is available when no other interfering neighbor is transmitting. Under steady state, the fraction of time that the channel is available to node 0 is $1 - \sum_{k=1}^{m} \alpha_k (L_k/W)$. So the service time of node 0 would be the uncontended service time scaled by the fraction of time the channel is available to node 0. Hence the service time of node 0 equals $\frac{1/\xi + L_0/W}{1-\sum_{k=1}^{m} \alpha_k (L_k/W)}$. We refer to $\sum_{k=1}^{m} \alpha_k (L_k/W)$ as the *contention term*.

In a multihop wireless network, m is analogous to the number of interfering neighbors and $\alpha_j = \lambda_i, L_j = L \forall j$. The expected value of the contention term (or fraction of time channel is not available to a node) is $4nA(n)\lambda_i \frac{L}{W}$ and therefore the service time of a node equals $\frac{1/\xi + L/W}{1-4nA(n)\lambda_i (L/W)}$.

5.2. Asymptotic scaling of delay

Although the result obtained in Theorem 2 provides a closed form expression of end-to-end delay, it does not provide direct intuition into how delay scales with various network parameters. This is because of the complex dependence of ρ and $\hat{\rho}$ on various network parameters. In this section we simplify the expression of delay for an asymptotic case with network size tends to infinity and the packet generation rate tends to the maximum achievable per-node throughput of the network. We use various approximations to simplify the complex expressions. As a result we obtain a closed form bound on the average end-to-end delay that is easy to interpret. The bound is very tight for the asymptotic case considered in this section. We will first present the assumptions and notations that we use in the analysis. We then consider each of the network parameters whose value we calculated in Section 4 and simplify the expression for the above mentioned asymptotic case.

1. *Network size tends to infinity*, i.e. network size (n) is very large or $n \to \infty$.
2. *High load conditions*, i.e. the probability that a node has a packet to transmit is high ($0 \ll \rho < 1$).
3. *The nodes transmit at minimum power that guarantees asymptotic connectivity*, i.e.
$$r(n) = \sqrt{\frac{\log n}{n}}. \tag{24}$$
4. *The average distance between a source–destination pair is a constant*, i.e.
$$p(n) = \sqrt{\frac{\log(n)}{n}}. \tag{25}$$

We introduce the following new notation for the analysis:

1. We represent the mean duration of back off timer as product of the packet transmission time and a constant $\alpha > 0$, i.e.
$$\frac{1}{\xi} = \alpha \frac{L}{W}, \quad \alpha > 0. \tag{26}$$
2. We represent the packet generation rate (λ) as product of maximum achievable throughput and a constant $\beta > 0$, i.e.
$$\lambda = \beta \frac{W/L}{4\pi\sqrt{n \log n}}, \quad \beta > 0. \tag{27}$$
Since we consider the heavy load conditions, β is assumed to be close to 1.

We now present the analysis for simplification of the delay results. Previously we have obtained the results for the values of the network parameters using the queuing network model. We further simply and approximate the results in order to make them easy to interpret.

5.2.1. Number of active interfering neighbors

Active interfering neighbors are the interfering neighbors of a node that have a packet to send while the node is trying to gain access to the channel. Let M_i represent the number of interfering neighbors of node i. Previously we have shown that the expected value of M_i is given by

$$\overline{M_i} = 4\rho n A(n) = 4\pi\rho n r(n)^2.$$

Substituting $r(n)$ from (24), we get

$$\overline{M_i} = 4\pi\rho \log(n). \tag{28}$$

The second moment of M_i is given by

$$\overline{M_i^2} = \rho^2 4nA(n)(1 + 4(n-1)A(n)) + (1-\rho)\rho 4nA(n).$$

Substituting $r(n)$ from (24), we have

$$\overline{M_i^2} = \rho^2 4\pi \log(n)\left(1 + 4\pi\frac{n-1}{n}\log n\right) + (1-\rho)\rho 4\pi \log n.$$

Since $n \to \infty$, we substitute $\frac{n-1}{n}$ to be approximately equal to 1 and get

$$\overline{M_i^2} \approx \rho^2 4\pi\log n(1+4\pi\log n) + \rho 4\pi\log n - \rho^2 4\pi\log n = \rho 4\pi\log n(1+\rho) + \rho^2 4\pi\log n(4\pi\log n - 1).$$

Again, since we consider the case where $n\to\infty$ therefore we approximate $4\pi\log n - 1 \approx 4\pi\log n$ which yields

$$\overline{M_i^2} \approx \rho 4\pi\log n(1+\rho) + (\rho 4\pi\log n)^2. \tag{29}$$

The standard deviation of M_i is given by

$$\sigma^2_{M_i} = \overline{M_i^2} - \overline{M_i}^2 = \rho 4\pi\log n(1+\rho). \tag{30}$$

5.2.2. Service time

The standard deviation of the service time of node i is given by

$$\sigma^2_{X_i} = \frac{L^2}{W^2}\left(\overline{M_i} + \overline{M_i^2} + \sigma^2_{M_i}\right) + 2(2\overline{M_i}+1)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2}.$$

Substituting $\overline{M_i}, \overline{M_i^2}$, and $\sigma^2_{M_i}$ from (28)–(30), we get

$$\sigma^2_{X_i} = \frac{L^2}{W^2}(4\pi\rho\log n + 8\pi\rho\log n(1+\rho) + (4\pi\rho\log n)^2) + 2(8\pi\rho\log n + 1)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2}. \tag{31}$$

The squared coefficient of variance (SCV) of the service time is given by

$$\begin{aligned} c^2_{Bi} &= \frac{\sigma^2_{X_i}}{\overline{X_i}^2} = \lambda_i^2\frac{\sigma^2_{X_i}}{\rho^2} \\ &= \lambda_i^2\left(\frac{L^2}{W^2}\left(\frac{4\pi\log n}{\rho} + 8\pi\log n\left(\frac{1}{\rho}+1\right) + (4\pi\log n)^2\right) + 2\left(\frac{8\pi\log n}{\rho} + \frac{1}{\rho^2}\right)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2}\frac{1}{\rho^2}\right) \\ &= \lambda_i^2\left(\frac{L}{W}\frac{4\pi\log n}{\rho}\left(\frac{3L}{W} + \frac{4}{\xi}\right) + 8\pi\log n\frac{L^2}{W^2} + \left(4\pi\log n\frac{L}{W}\right)^2 + \frac{1}{\xi\rho^2}\left(\frac{1}{\xi} + \frac{L}{W}\right)\right). \end{aligned} \tag{32}$$

We know that $\lambda_i = \lambda/p(n)$. Substituting $p(n)$ from (25), we get

$$\lambda_i = \lambda\sqrt{\frac{n}{\log n}}. \tag{33}$$

Substituting λ_i from (33) into (32), we get

$$\begin{aligned} c^2_{Bi} &= \frac{\lambda^2 n}{\log n}\left(\frac{L}{W}\frac{4\pi\log n}{\rho}\left(\frac{3L}{W}+\frac{4}{\xi}\right) + 8\pi\log n\frac{L^2}{W^2} + \left(4\pi\log n\frac{L}{W}\right)^2 + \frac{1}{\xi\rho^2}\left(\frac{1}{\xi}+\frac{L}{W}\right)\right) \\ &= \lambda^2\left(\frac{16\pi}{\rho}\frac{L}{W}\left(\frac{L}{W}+\frac{1}{\xi}\right) - \frac{4\pi}{\rho}\frac{L^2}{W^2} + 8\pi\frac{L^2}{W^2} + \log n\left(4\pi\frac{L}{W}\right)^2 + \frac{1}{\xi}\frac{1}{\rho^2\log n}\left(\frac{1}{\xi}+\frac{L}{W}\right)\right). \end{aligned} \tag{34}$$

Substituting $\frac{1}{\xi} = \alpha\frac{L}{W}$ from (26), we get

$$\begin{aligned} c^2_{Bi} &= \lambda^2 n\left(\frac{16\pi}{\rho}\frac{L^2}{W^2}(1+\alpha) - \frac{4\pi}{\rho}\frac{L^2}{W^2} + \log n 16\pi^2\frac{L^2}{W^2} + \frac{\alpha(1+\alpha)}{\rho^2\log n}\frac{L^2}{W^2}\right) \\ &= 4\pi\lambda^2 n\frac{L^2}{W^2}\left(\frac{4(1+\alpha)}{\rho} - \frac{1}{\rho} + 2 + 4\pi\log n + \frac{\alpha(\alpha+1)}{4\pi\rho^2\log n}\right). \end{aligned} \tag{35}$$

Since we are considering the case where $\rho\approx 1, n\to\infty$, and α is a constant, c^2_{Bi} may be approximated as

$$c^2_{Bi} \approx 16\pi^2\lambda^2\frac{L^2}{W^2}n\log n. \tag{36}$$

5.2.3. Arrival process for each node (queue in the queuing network)

The effective packet arrival rate into a queue (λ_i) is given by $\lambda/p(n)$. Substituting $p(n)$ from (25) and λ from (27), we get

$$\lambda_i = \frac{\beta}{4\pi}\frac{W/L}{\log n}. \tag{37}$$

Also according to the diffusion approximation, the SCV of the packet inter-arrival time at node i is given by

$$c^2_{Ai} = 1 + (c^2_{Bi} - 1)(1 - p(n)). \tag{38}$$

5.2.4. Utilization ratio (ρ)

The utilization ratio of a node is equal to $\lambda_i\overline{X_i}$, where λ_i is the effective packet arrival rate at a node and $\overline{X_i}$ is the average service time of the node. The average service time is given by

$$\overline{X_i} = \frac{\frac{1}{\xi}+\frac{L}{W}}{1 - 4nA(n)\lambda_i\frac{L}{W}}.$$

Substituting $r(n), \frac{1}{\xi}$, and λ_i from (24), (26), and (37), we get

$$\overline{X_i} = \frac{\frac{L}{W}(1+\alpha)}{1-\beta}. \tag{39}$$

Using λ_i from (37), ρ is given by

$$\rho = \lambda_i \overline{X_i} = \frac{\frac{\beta(1+\alpha)}{4\pi\log n}}{1-\beta}. \tag{40}$$

5.2.5. The diffusion approximation constant ($\hat{\rho}$)

As previously stated, $\hat{\rho}$ is given by

$$\hat{\rho} = \exp\left(-\frac{2(1-\rho)}{c_{A_i}^2\rho + c_{Bi}^2}\right).$$

Substituting c_{Ai}^2 from (38), we have

$$\hat{\rho} = \exp\left(-\frac{2(1-\rho)}{\rho + c_{Bi}^2\rho(1-p(n)) + c_{Bi}^2 - (1-p(n))}\right).$$

As $n \to \infty, p(n) \to 0$. Thus we approximate $1 - p(n) \approx 1$ and therefore $\hat{\rho}$ may be expressed as

$$\hat{\rho} = \exp\left(-\frac{2(1-\rho)}{c_{Bi}^2(1+\rho) - (1-\rho)}\right). \tag{41}$$

5.2.6. End-to-end delay

As already mentioned in (21), the average end-to-end packet delay is given by

$$\overline{D} = \frac{\rho}{\lambda(1-\hat{\rho})}.$$

We use the following inequality in order to simplify the above relation:

$$\frac{1}{1-e^{-x}} \geqslant \frac{1}{x} \quad \forall x > 0. \tag{42}$$

The relation in the above inequality is close to equality when x is close to 0, i.e. $\frac{1}{1-e^{-x}} \approx \frac{1}{x}$ as $x \to 0$. Using (42), $\overline{D}$ is may be written as

$$\overline{D} \geqslant \frac{\rho}{-\lambda \log \hat{\rho}}.$$

The above relation is close to equality when ρ is close to 1. Substituting $\hat{\rho}$ from (41) in the above relation we have

$$\overline{D} \geqslant \frac{\rho}{2\lambda(1-\rho)}(c_{Bi}^2(1+\rho) - (1-\rho)).$$

Because of high load assumption we approximate $1 - \rho \approx 1$ in the numerator of the above expression. Thus $\overline{D}$ may be expressed as

$$\overline{D} \geqslant \frac{\rho(1+\rho)}{(1-\rho)} \frac{1}{2\lambda} c_{Bi}^2.$$

Substituting ρ from (37), we get

$$\overline{D} \geqslant \frac{\frac{\beta(1+\alpha)}{4\pi\log n}}{1-\beta-\frac{\beta(1+\alpha)}{4\pi\log n}} \frac{1-\beta+\frac{\beta(1+\alpha)}{4\pi\log n}}{1-\beta} \frac{1}{2\lambda} c_{Bi}^2.$$

Since $n \to \infty$, we approximate $1-\beta-\frac{\beta(1+\alpha)}{4\pi\log n} \approx 1-\beta$ and $1-\beta+\frac{\beta(1+\alpha)}{4\pi\log n} \approx 1-\beta$. Thus we get

$$\overline{D} \geqslant \frac{1+\alpha}{4\pi\log n} \frac{\beta}{1-\beta} \frac{1}{2\lambda} c_{Bi}^2. \tag{43}$$

Substituting λ from (27) and c_{Bi}^2 from (36), we get

$$\overline{D} \geqslant \frac{1+\alpha}{4\pi\log n} \frac{\beta}{1-\beta} \frac{1}{2} \frac{\beta W/L}{4\pi\sqrt{n\log n}} 16\pi^2 \frac{L^2}{W^2} n \times \log n. \tag{44}$$

Rearranging we the above relation we get

$$\overline{D} \geqslant \frac{1+\alpha}{2} \frac{L}{W} \frac{\beta}{1-\beta} \sqrt{\frac{n}{\log n}}. \tag{45}$$

Or,

$$\overline{D} = \Omega\left(\frac{\beta}{1-\beta}\sqrt{\frac{n}{\log n}}\right). \tag{46}$$

The above result gives us more valuable insight into how delay scales with various network parameters than the result presented in (21). For fixed network size, the delay scales roughly as $\frac{\lambda}{\lambda_{\max}-\lambda}$ with λ. For a fixed packet generation rate, the end-to-end delay scales as $\sqrt{\frac{n}{\log n}}$.

5.3. Comparison with delay and throughput in real networks

The analytical model in this paper is kept reasonably simple so that it is possible to obtain closed form expressions for delay and throughput. In particular our MAC model does not take into account packet collisions and our routing model is random walk of packets over the network. Thus our model deviates from the real world scenarios where the packets collide due to random access MAC with finite collision windows and the packets are routed along fixed paths dictated by routing protocols. In this subsection we discuss how much the delay and maximum achievable throughput in real world networks deviate from our analytical results.

5.3.1. Effect of packet collisions

Consider a more practical MAC model where a node transmits as soon as its transmit timer expires and the interfering neighbors freeze their timers only when they sense the transmission. For such a MAC, the transmission of node i may collide with transmission of an interfering neighbor if difference between the time instances when the transmit timers of nodes i and that of interfering neighbor expire is less than the propagation delay between the nodes. Let d denote the propagation delay between node i and its interfering neighbor that has a packet to send, then the probability that the transmission of i does not collide with that of the interfering neighbor equals $e^{-2\xi d}$. Since the interfering neighbors are located within two hops of node $i, d \leqslant 2r/c = \delta$, where c is velocity of electromagnetic waves. Thus the probability that the transmission of node i does not collide with an interfering node's transmission is greater than $e^{-2\xi\delta}$. So if node i has I interfering neighbors, then the probability that a transmission is a success is bounded by

$$P[\text{Success}] \geqslant e^{-2\xi\delta I}. \tag{47}$$

Let P_s denote the expected probability of success, averaged over all possible topologies, then

$$P_s \geqslant (1 - (1 - e^{-2\xi\delta})4A(n))^n = P_s^{(L)}. \tag{48}$$

The expected number of times a node transmits a packet before it is received successfully by its neighbor equals $1/P_s$. It is easy to see that the RHS of Eq. (19) is scaled by a factor of $1/P_s$ and mean service time may be evaluated by rearranging the resulting equation. So for the more practical MAC model, in which packet collisions take place, the mean service time is bounded by

$$\frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i L/W} \leqslant \overline{X_i} \leqslant \frac{\frac{1}{\xi} + \frac{L}{W}}{P_s^{(L)} - 4nA(n)\lambda_i L/W}. \tag{49}$$

The maximum achievable throughput, evaluated using $\lambda_i \overline{X_i} < 1$, is bounded by

$$\lambda_{\max}^{(L)} = \frac{P_s^{(L)} p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}} \leqslant \lambda_{\max} \leqslant \frac{p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}} = \lambda_{\max}^{(U)}. \tag{50}$$

The dependence of $\lambda_{\max}^{(L)}$, the lower bound of $\lambda_{\max}$, on the rate of transmit timer, ξ, is particularly interesting. As ξ increases, both $P_s^{(L)}$ and $1/\xi$ term in the denominator decrease. Thus there is a tradeoff in choosing the rate of the transmit timer-a high ξ leads to lower waiting time before transmission but leads to higher probability of packet collision. Let $\xi^\star$ be the optimal value of ξ that maximizes the lower bound of $\lambda_{\max}$. Equating $d\lambda_{\max}^{(L)}/d\xi$ to 0 yields that $\xi^\star$ satisfies the following relation

$$\frac{(b(n)\xi^{\star 2} + \xi^\star)e^{-2\xi^\star\delta}}{(1 - 4A(n)(1 - e^{-2\xi^\star\delta}))} = \frac{1}{8nA(n)\delta}. \tag{51}$$

where $b(n) = L/W + 4nA(n)L/W$. Closed form expression for $\xi^\star$ cannot be evaluated from the above relation. However, by approximating $e^{-2\xi^\star\delta} \approx 1$ (high probability of success) and solving the resulting quadratic equation we get

$$\xi^\star \approx \frac{1}{2L/W}\frac{1}{1 + 4nA(n)}\left(\sqrt{1 + \frac{(1 + 4nA(n))L}{2nA(n)W\delta}} - 1\right). \tag{52}$$

As expected, $\xi^\star$ decreases with increase in the expected number of interfering neighbors, packet transmission time and propagation delay.

Fig. 4 shows how the performance of our collision free model compares with the bound on the delay and maximum achievable throughput in a network with packet collisions. The network parameters for the plots are the following: $\xi = 5 \times 10^4$, $r(n) = 3\sqrt{\frac{\log n}{n}}$, $L = 1$ Kb, $W = 11$ Mbps, and $p(x) = \sqrt{\frac{\log n}{n}}$. Fig. 4a shows that in the presence of collision the service time of a large network may be up to 30% greater than that of our analytical model. However the error is small for smaller network sizes. Also the error in the maximum achievable throughput is almost negligible.

5.3.2. *Effect of deterministic routing*

In the routing model used in this paper, a node forwards a packet to any of its neighbor with equal probability which spreads the traffic evenly throughout the network. On the other hand, a deterministic routing protocol routes each packet belonging to a particular flow (typically identified by source–destination pair) along a deterministic path, determined using some goodness metric. This often leads to a situation where large number of flows pass through a few nodes that have good paths to many flow destinations. This leads to creation of routing bottlenecks leading to large queuing delays intermediate nodes and higher end-to-end delays. The MAC layer delays with deterministic routing are always higher than those with random routing. For high load situations, this translates to end-to-end delays. For low load situations, MAC delays are low for

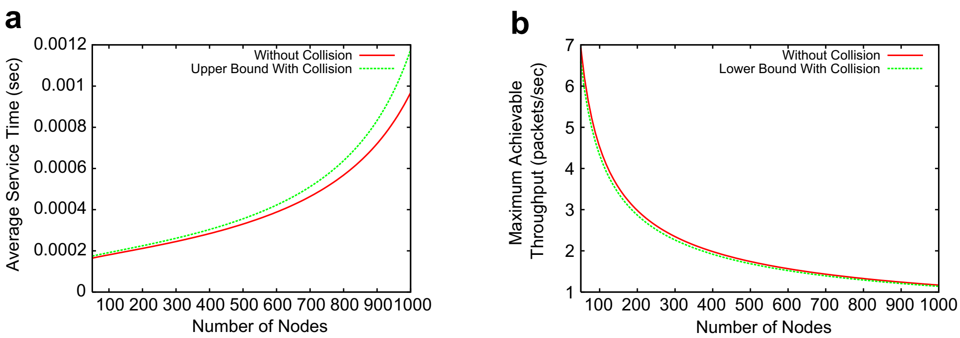

Fig. 4. Comparison of performance of the idealistic scenario with a network with packet collisions. (a) Average time required to successfully transmit a packet to the next hop, with and without collision. (b) Maximum achievable throughput as a function of network size, with and without collision.

both types of routing, and thus the end-to-end delays for deterministic routing could be less than those for random routing (predicted by (21)) because packets travel fewer hops.

6. Simulations

We perform the following simulations:

1. *Simulations of the model*: These simulations verify the validity of the assumptions made in the analysis and the accuracy of diffusion analysis.
2. *Simulations using shortest path routing instead of probabilistic routing*: We compare the analytical results with a practical scenario where the packets are routed along the shortest path rather than undergoing probabilistic routing.
3. *NS simulations*: These simulations provide comparison of the results of our analytical model against results obtained from NS simulations that employ standard MAC (IEEE 802.11) and routing (DSDV) protocols.

The rest of this section presents the results for each of the above simulation sets.

6.1. Simulations for validating the analytical results

We use a simulator written in C in order to simulate the model and compare the analytical results with the simulation results. The simulation setting is the following. The network topology for the simulations consists of n nodes scattered randomly over a torus of unit surface area. Each node can communicate with the nodes within a distance $r(n) = \sqrt{\frac{\log n}{n}}$. The random access MAC protocol used by the nodes is the same as described in Section 4.2. Each node produces packets of size $L = 1$ Kbits at the rate of λ packets/s. The transmission rate of each node is $W = 10^6$ bits/s. The probabilistic routing described in Section 3 is used for the simulations. The simulation time is 500 s. In order to ensure that the network is in a steady state, the first 100 s of the simulations are discarded. The average delay for a particular topology is obtained by averaging the end-to-end delay of all packets produced during the simulation. In order to average out the effect of topology, each simulation is repeated over several topologies. The average end-to-end delay and 95% confidence intervals are obtained by obtained from 35 simulation runs.

Fig. 5 shows how the average end-to-end delay, as obtained from the simulations, varies with the number of nodes for $\lambda = 0.5, \lambda = 0.7$ and $\lambda = 1.0$ with $p(n) = \sqrt{\frac{\log n}{n}}$. Fig. 6 shows how the average end-to-end delay varies with the arrival rate (λ) for $n = 500, 600$ and 800 with $p(n) = \sqrt{\frac{\log n}{n}}$. Fig. 7 shows how the average end-to-end delay varies with the number of nodes for various values of absorption probability with $\lambda = 1$ packets/s. The theoretical values of the average end-to-end delay as obtained from the analytical results are plotted alongside the simulation results in Figs. 5–7. It is observed that the simulation results agree closely with the theoretical values.

6.2. Comparison of results for the shortest path routing with the analytical results

In our model the packets are subjected to probabilistic routing, which is similar to a random walk.

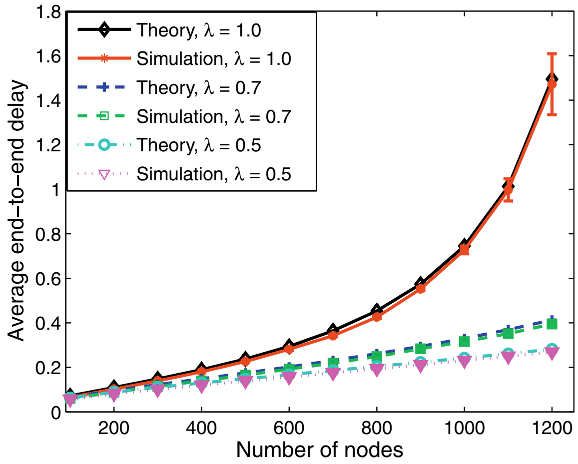

Fig. 5. Average end-to-end delay vs. number of nodes for varying arrival rates.

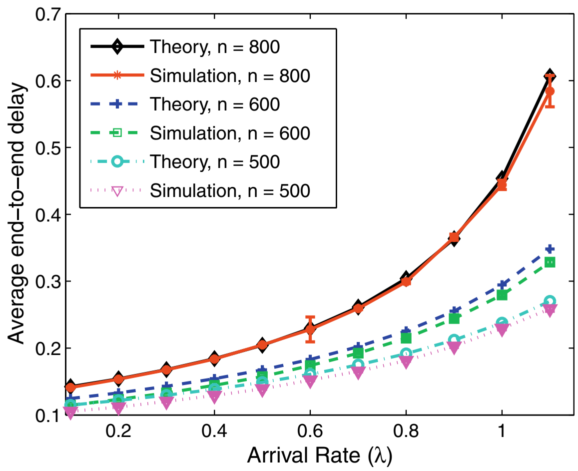

Fig. 6. Average end-to-end delay vs. arrival rate for varying network size.

It would be pertinent to compare the analytical results for our model with the simulations where the packets are routed to the destinations along corresponding shortest paths.

The simulation setting differs from the setting described in the previous subsection in the following. When a new packet is generated at a node, a destination node for the packet is chosen at random. The packet is routed to the destination node along the shortest path. In order to route the packets along shortest paths, each node maintains a routing table. The routing tables are constructed using Bellman Ford algorithm. Fig. 8 shows the simulation

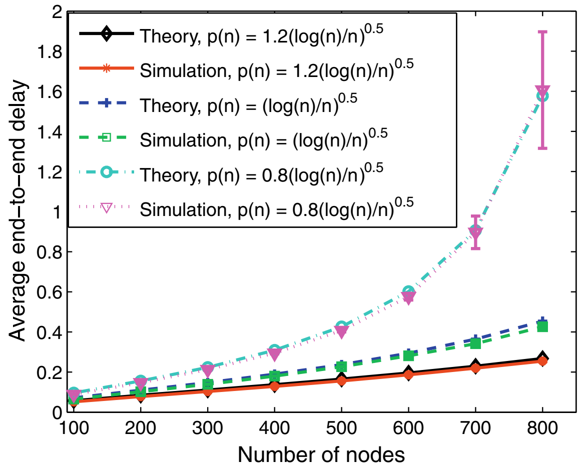

Fig. 7. Average end-to-end delay vs. number of nodes for varying absorption probabilities.

results for the shortest path routing along with the analytical results obtained from our model. The absorption probability of our model is scaled appropriately so that the average number of hops between a source–destination pair for the shortest path routing matches with the average number of hops traversed by a packet in the probabilistic routing model. It is observed that the for low traffic arrival rate or small network size, the simulation results agree closely with the analytical results. However for higher packet arrival rates and large network size the delay obtained from simulation is larger than the analytical result. This deviation may be explained in the following manner. Shortest path

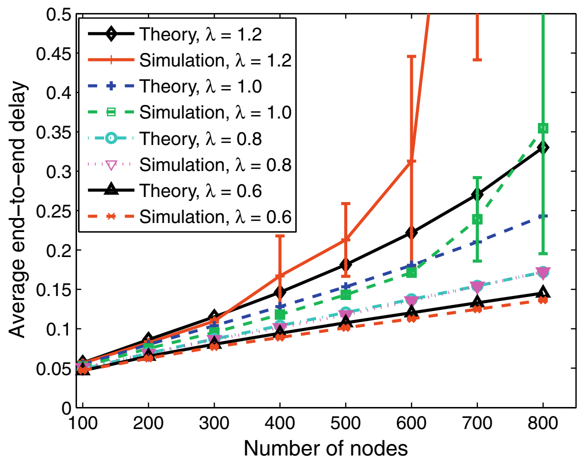

Fig. 8. Comparison of analytical results with simulation results for the shortest path routing.

routing inevitably leads to a situation where some links in the network carry more traffic than the average traffic in the network. These are the links that lie on shortest path of many source–destination pairs. These links are referred to as *routing hotspots*. When the traffic load is high, such routing hotspots become bottleneck and the packet routed through these links experience much larger delays. In contrast, probabilistic routing ensures that the traffic is spread uniformly over the network and thus the average end-to-end for the analytical model increases less rapidly. As the number of nodes in the network increases, it is accompanied by increase in traffic in the network leading to high delays on the bottleneck links. Another factor for the deviation is the following. Since the communication radius is set equal to $\sqrt{\log n/n}$, the diameter of network is approximately $\sqrt{n/\log n}$. For small n, the number of hops traversed by a packet are small therefore routing is more similar to random walk. However as n increases, the diameter of the network increases and packets traverse more hops between a source–destination pair. For this case random walk poorly approximates shortest path routing.

6.3. Comparison against results from NS simulations

As mentioned earlier, the aim of the delay analysis presented in this paper is to capture the effect of random access MAC and queuing delays on the average end-to-end delay and maximum achievable throughput of multihop wireless networks. Our model includes an idealistic random access MAC and probabilistic routing model. The MAC model assumes that the transmission timers of all interfering neighbors of a node freeze as soon as the node starts transmitting. This precludes the possibility of any packet collision at the intended receiver (except for the results in Section 5.3.1). However IEEE 802.11, which is the de facto MAC protocol of wireless networks and simulated in NS, is not free from collisions. The RTS packets transmitted by a node may collide with a transmission at the intended receiver which would then prevent the node from grabbing the channel. (This is more likely in networks with large contention window, e.g. long propagation delays.) Also when a node, in an ad hoc IEEE 802.11 network, starts transmitting, the transmission timers of only a subset of interfering neighbors are frozen. This subset includes only the neighbors of the transmitter and the intended receiver.

In this section we compare the analytical results for our model with the average delay obtained from NS-2 simulations, that use IEEE 802.11 as MAC and DSDV for routing. The purpose of this comparison is to understand how the end-to-end delay in a network based on specific established protocols would differ from the results obtained from our generic model, given that our model includes some simplifying assumptions that does not capture the actual protocols (and their interactions).

The NS simulation set-up is as follows. The network consists of n nodes that are uniformly and independently distributed over a $500\,\text{m} \times 500\,\text{m}$ area. This deployment area is chosen to represent a realistic network deployment. An exponential traffic source is attached to each node, which produces packets of length 1000 bytes at the rate of λ packets per second. Each node chooses a random destination and the traffic is routed to the destination using routes maintained by DSDV.[2] UDP is used as the transport layer protocol in order to avoid delays due to congestion avoidance mechanisms of TCP. The receive threshold[3] of the nodes is set such that each node within a distance of $500 \cdot \sqrt{\log n/n}$ meters from a transmitter is able to listen to the transmission, in absence of any interference. This corresponds to $r(n) = \sqrt{\log n/n}$ for our model where nodes are deployed over unit area. The IEEE 802.11 MAC and free space propagation models are used for the simulations.

In order to compare the simulation results with the analytical results we set the values of the parameters of the analytical model such that they are comparable to that of the simulation. This is done in the following manner. We obtain the values of the average duration of the IEEE 802.11 backoff timer (ξ') and the transmission duration (T'_0)[4] for each simulation setting. We use $\xi = \xi'$ and $\frac{L}{W} = T'_0$ in the analytical results. This ensures that the average backoff duration of MAC protocol and the transmission time of the analytical model is same as that of the simulations. We used the default value for the parameter CSThresh in NS which is 1.5×10^{-11}, while the RXThresh was set such that the transmis-

[2] Since the nodes are stationary, DSDV is the ideal routing protocol.

[3] If the signal strength at a receiver is below receive threshold, the receiver cannot detect the signal. Please see [5] for details.

[4] T'_0 includes the time taken for the exchange of RTS, CTS, data and ACK packets i.e. $T'_0 = T_{\text{RTS}} + T_{\text{SIFS}} + T_{\text{CTS}} + T_{\text{SIFS}} + T_{\text{DATA}} + T_{\text{SIFS}} + T_{\text{ACK}}$.

sion radius is $500 \times \sqrt{\log n/n}$. This means that for the simulations the RXThresh varied from 5.6×10^{-9} to 1.6×10^{-8}.

We perform 35 simulations for each scenario in order to obtain the mean and 95% confidence interval for each point. The simulation time for each simulation run equals 2500 s.

Fig. 9 shows the plots of average end-to-end delay, obtained from NS-2 simulations and analytical model, as a function of number of nodes. We observe that for lightly loaded conditions (small λ and/or n) the simulation results are less than the analytical results. This is because in IEEE 802.11 only the transmission timers of the neighbors of the transmitter and receiver are frozen during a transmission while in our model we assume that the timers of all interfering neighbors are frozen. Thus the number of times the transmission timer of a node is frozen in IEEE 802.11 is smaller than that in our model. So the average time in which the transmission timer of a node expires is less in IEEE 802.11 leading to lesser delays. However the simulation results closely follow the trend indicated by the analytical results. As the traffic size or load in the network increases the average end-to-end delay of the simulation becomes larger than the analytical results. This is due to the following three reasons: (i) The number of collisions of RTS/CTS packets increases. As number of nodes in the network increases, the number of nodes contending for channel increases hence increasing the chances of packet collision. As the packet generation rate increases, the nodes attempt to transmit more often which increases the chances of packet collision. (ii) Due to formation of routing hotspots as a result of shortest path routing. (iii) Due to collision of data packets with routing control packets. The routing update packet which are not preceded by virtual carrier sense and RTS–CTS exchange [4] which makes routing packets vulnerable to collision. As the number of nodes in the network the number of routing update packets that transmitted increases, leading to increased chances of packet collisions and higher delays.

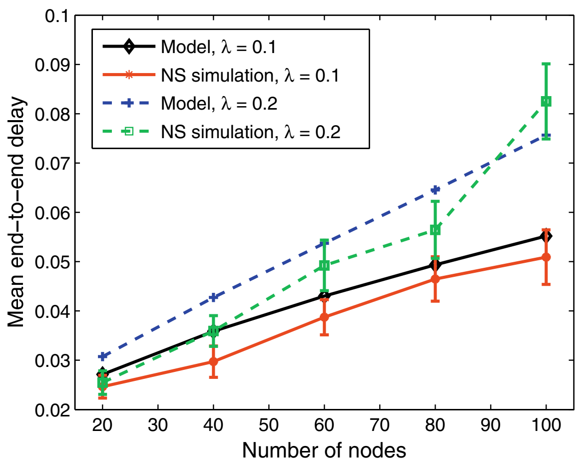

Fig. 9. Average end-to-end delay vs. number of nodes for NS simulations.

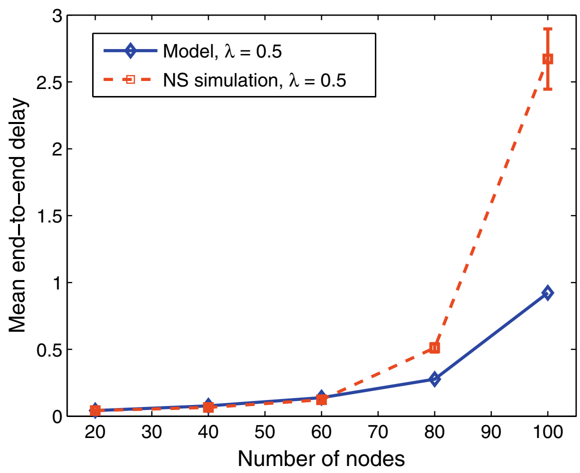

Fig. 10. Average end-to-end delay vs. number of nodes for NS simulations. Due to increased congestion, packet collisions, and control overheads, for higher packet generation rate the NS simulations further deviate from the theoretical results.

Fig. 10 shows how delay obtained from NS simulations varies with the number of nodes for $\lambda = 0.5$ packets/s. It is observed that for higher network sizes and traffic arrival rate the simulation results differ substantially from the theoretical results. The increased deviation with increase in traffic is due to the three reason mentioned previously. These results indicate that the delay and capacity in real networks should be expected to be worse than the results obtained in this paper. This deviation is the impact of issues associated with the existing MAC and routing protocols such as poor load balancing, high collision rates, and protocol overheads. So although our analytical results closely approximate delay in real networks for the light load scenario, there is considerable deviation for the high load scenario. Developing accurate results for the high load scenario will require more detailed modeling of the impact of specific routing and MAC algorithms in ad hoc networks.

7. Conclusion and future work

Characterization of capacity and delay in ad hoc networks has been focus of considerable research. However capacity and delay of networks based on random access MAC, like IEEE 802.11, have not been substantially studied. In this paper we presented delay analysis of random access MAC multihop wireless ad hoc networks. We derived closed form expressions for the average end-to-end delay and maximum achievable throughput. We showed that, for comparable network parameters, the upper bound on maximum achievable throughput is of the same order as the Gupta–Kumar's bound. However for the random access MAC the bound is not achievable. The analytical results are verified using simulations. The NS-2 simulations indicate that under heavy load the performance of the standard wireless protocols is worse than the performance predicted by our model.

The results and framework presented in this paper leads to several venues for future research. Our current directions include the delay analysis and characterization of the maximum achievable throughput for hierarchical networks, many to one communication scenarios, wireless networks with sleeping nodes and wireless networks with other medium access control algorithms.

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Appendix A. Omitted proofs

Proof of Lemma 1. Let $P[i \to j]$ denote the probability that a packet forwarded by node i enters the queue at node j. We define $\beta_{ij}^{j,k} = P[i \to j | j \in N(i), |N(i)| = k]$, $\beta_{ij}^{j} = P[i \to j | j \in N(i)]$ and $\alpha_i^{j,k} = P[|N(i)| = k | j \in N(i)]$. Thus

$$\beta_{ij}^{j,k} = \frac{1}{k}(1 - P[j \text{ absorbs the packet}]) = \frac{1-p(n)}{k}.$$

Since the nodes are uniformly and independently distributed over a unit area, the probability that a node is in neighborhood of the node i equals $A(n)$. Hence $P[j \in N(i)] = A(n)$ and

$$\alpha_i^{j,k} = \binom{n-1}{k-1}(1-A(n))^{n-k}A(n)^{k-1}.$$

Therefore,

$$\overline{\beta_{ij}^{j}} = E\left[\beta_{ij}^{j}\right] = \sum_{k=1}^{n} \beta_{ij}^{j,k}\alpha_i^{j,k} = \frac{1-p(n)}{nA(n)}(1-(1-A(n))^n).$$

Also according to the model node i cannot forward a packet to node j unless $j \in N(i)$. Hence $E[P[i \to j] | j \notin N(i)] = 0$. So the expected forwarding probability is given by

$$\overline{p_{ij}}(n) = \overline{\beta_{ij}^{j}} P[j \in N(i)] = \frac{1-p(n)}{n}(1-(1-A(n))^n).$$

Proof of Lemma 2. The visit ratio of a node in the queuing network is given by (1). Taking expectation of both sides of the equation we have,

$$\overline{e_i} = \frac{1}{n+1} + \sum_{j=1}^{j=n+1} \overline{p_{ji}}(n)\overline{e_j}.$$

Each node of the wireless network is similar, thus from symmetry $\overline{e_i} = \overline{e_j} \forall i; j$. Also $\overline{p_{ij}} = \frac{1-p(n)}{n}(1-(1-A(n))^n)$. Since in our model $A(n)$ is chosen such that the network is connected with high probability, therefore $(1-(1-A(n))^n) \approx 1$ and hence $\overline{p_{ij}}(n) \approx \frac{1-p(n)}{n}$. From symmetry

$$\overline{e_i} = \frac{1}{n+1} + \sum_{j=1, j\neq i}^{j=n+1} \frac{1-p(n)}{n}\overline{e_i}.$$

By rearranging the above equation we get (9).

Proof of Lemma 3. The packet arrival process at each node is an i.i.d. Poisson process with rate λ. So the total external arrival rate, denoted by λ_e, equals $(n+1)\lambda$. According to (2), $\lambda_i = \lambda_e e_i$. Substituting $\overline{e_i}$ from (9) and λ_e we get (10).

Proof of Lemma 4. Let s denote the number of hops between a source and destination, then $P[s = k] = (1-p(n))^{k-1}p(n) k \geqslant 1$. Thus,

$$\bar{s} = E[s] = \sum_{k=1}^{\infty} k \cdot (1-p(n))^{k-1}p(n) = \frac{1}{p(n)}. \tag{53}$$

Proof of Lemma 5. Since the nodes are uniformly distributed over a unit area, the probability that a node is an interfering neighbor of node i equals $\pi(2r(n))^2$. Thus the probability that $H_i = h$ is given by

$$P[H_i = h] = \binom{n}{h}(4\pi r(n)^2)^h \cdot (1-4\pi r(n)^2)^{(n-h)}.$$

Thus H_i has a binomial distribution. (11) and (12) are the first and second moment of the binomial distribution.

Proof of Lemma 6. Let the number of interfering neighbors of node i be H_i. Let $Y_j, 1 \leqslant j \leqslant H_i$, be an indicator random variable associated with node j, indicating whether under steady state node j has a packet to transmit or not. ($Y_j = 1$ if node j has a packet to transmit, $Y_j = 0$ if node j has no packet to transmit). Using (5) $P(Y_j = 1) = \rho_j$, where ρ_j is the utilization factor of node j. By symmetry $\rho_j = \rho \forall j$. M_i is equal to $\sum_{j=1}^{H_i} Y_j$. The expected value of M_i equals

$$E[M_i] = E_{H_i}[E[M_i|H_i = h]] = E_{H_i}\left[\sum_{j=1}^{h} E[Y_j]\right] = \rho E[H_i].$$

Substituting (11), we get (13). Similarly the expected value of M_i^2, given $H_i = h$, is given by

$$E[M_i^2|H_i = h] = E\left[\left(\sum_{j=1}^{h} Y_j\right)\left(\sum_{k=1}^{h} Y_k\right)\right].$$

Since Y_j is independent of Y_k, we get

$$E[M_i^2|H_i = h] = \sum_{j=1}^{h}\sum_{k=1,k\neq j}^{h} E[Y_j]E[Y_k] + \sum_{j=1}^{h} E[Y_j^2]$$
$$\Rightarrow E[M_i^2] = \rho^2 E[H_i^2] + (1-\rho)\rho E[H_i].$$

Substituting (11) and (12), we get (14).

Proof of Lemma 7. Let T_i denote the duration of the back off timer of node i. During a transmission epoch M_i may not remain constant. In order to simplify the analysis we assume that M_i remains constant throughout a transmission epoch of node i. The timer of node i is frozen each time a timer of any of the interfering neighbors of i expires. The timer of each node has an exponential distribution. Thus the probability that $Z_i = z$, given that $M_i = m$ and $T_i = t_i$, is

$$P[Z_i = z|T_i = t_i, M_i = m] = e^{-m\cdot\xi\cdot t_i}\cdot(m\cdot\xi\cdot t_i)^z/z! \Rightarrow E[Z_i|T_i = t_i, M_i = m] = m\xi t_i \Rightarrow E[Z_i|M_i = m] = m\xi E[t_i] = m \Rightarrow E[Z_i] = E[M_i]. \quad (54)$$

Substituting $E[M_i]$ from (13), we get (15).

Proof of Theorem 2. Let $\overline{D_i}$ denote the average delay at a node i. According to Little's Law, $\overline{D_i} = \overline{K_i}/\lambda_i$, where $\overline{K_i}$ is the average number of packets in the queue of node i. Substituting $\overline{K_i}$ from (7) we get $\overline{D_i} = \overline{K_i}/\lambda_i = \rho/(\lambda_i(1-\hat{\rho}))$.

By symmetry the average delay at each node is same. Thus the average end-to-end delay equals the product of the average number of hops traversed by a packet and the average delay at each node. Hence $D(n) = \bar{s}\cdot\overline{D_i}$ which leads to (21).

Proof of Corollary 2. From (16) the utilization factor of a node, ρ_i, is given by

$$\rho_i = \lambda_i \cdot \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i\frac{L}{W}}. \quad (55)$$

For the average delay to be finite ρ_i must be strictly less than 1. Thus the following inequality must be satisfied to ensure finite delay.

$$\lambda_i \cdot \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i\frac{L}{W}} < 1.$$

Substituting $c = \frac{1}{\xi} + \frac{L}{W}$, $\lambda_i = \frac{\lambda}{p(n)}$ and rearranging, we get

$$\lambda < \frac{p(n)}{c + 4nA(n)\frac{L}{W}}. \quad (56)$$

Thus the maximum achievable throughput $\lambda_{\max}$ is $\frac{p(n)}{c+4nA(n)\frac{L}{W}}$. Also $c > 0$, thus $\lambda_{\max} < \frac{p(n)W}{nA(n)L}$. So for a fixed packet size L and transmission rate W, $\lambda_{\max} = o\left(\frac{p(n)}{nA(n)}\right)$. Substituting $p(n) = \frac{1}{\bar{s}}$, $\lambda_{\max} = o\left(\frac{1}{\bar{s}nA(n)}\right)$.

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