Network Layer Scheduling and Relaying in Cooperative Spectrum Sharing Networks

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Abstract

We consider network layer cooperation in spectrum sharing networks whereby some secondary users relay primary users' packets, in return for more favorable spectrum access rules. Under this cooperative scheme, we investigate how primary and secondary networks can be stabilized without explicit knowledge of the packet arrival rates. We consider a primary packet generation process wherein a packet is formed by aggregating constant amount of bits that arrive in every time slot from upper-layers of the primary transmitter. For this primary packet generation model we develop a relaying and scheduling algorithm using Lyapunov drift techniques that does not require knowledge of packet arrival rates. We also construct a *guaranteed stability region* representing packet generation rates for which the algorithm can stabilize the network. The set of secondary packet generation rate vectors for which the network can be stabilized do not decrease under cooperation when the primary packet generation rate is lower than what can be maximally supported without cooperation. For higher primary packet generation rates the algorithm stabilizes the network for a non-empty set of secondary packet generation rate vectors.

Index Terms

Cognitive radio, cooperation, relay, queueing theory, Lyapunov drift, network stability.

I. Introduction

THE increase in the number of wireless devices has resulted in increasing the demand for wireless spectrum. However, at any given time, bands in the licensed spectrum are often under-utilized. This observation has led to the widespread study of dynamic spectrum sharing radio networks. In one realization of spectrum sharing regulations, unlicensed or secondary users can opportunistically access the spectrum when it is not being used by licensed or primary users. Typically primary and secondary networks are thought to be non-cooperative, i.e., the users in the respective networks do not assist in each others transmissions. However if secondary users assist the transmission of primary users, it may reduce the fraction of time the channel is used by primary users. This in turn may increase transmission opportunities for secondary users.

Even in conditions where cooperation benefits some primary users and some secondary users, a question remains on whether this would benefit *all* secondary users. Intuitively, it can be seen that in a general network some secondary users may benefit from cooperation while others may not. For example, a secondary user located near a secondary relay node but far away from a primary transmitter may obtain fewer transmission opportunities when there is cooperation. This occurs due to increased transmission activity by the secondary relays. Therefore, an important network problem is to find a balanced cooperative scheduling policy under which the primary network can improve its performance while maintaining secondary network stability.

Cooperation between primary and secondary networks have been widely studied from a physical layer perspective. Some of those works (e.g. [1]–[3]) study it from an information-theoretic perspective. Other works [4]–[6] consider the primary network leasing the spectrum to secondary nodes in return for

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cooperation. However, interactions between primary and secondary users also affect higher-layer operations such as queuing and prioritized scheduling. The above studies do not address this issue.

Prior works that did study the problem from a network layer perspective include [7]–[12]. In [7] and [8] the authors consider a cooperative network with one secondary node. In [9] and [10] the authors consider a cooperative network with multiple secondary nodes that randomly access the channel. In [11] the authors find optimal cooperative power allocations in a network of multiple secondary users and a single primary user. However the authors only consider primary packet generation rates for which the network is stable even without cooperation. In [12] the authors extend the work in [11] to include cases of higher primary packet arrival rates. In particular, the authors in [12] obtain optimal power allocations by solving a convex optimization problem with the knowledge of primary packet arrival rate. However none of the works in [7]–[12] addressed the case where cooperation is beneficial, with respect to queue stability, to some secondary nodes and *harmful to others* (due to higher interference from relay nodes). Besides, those works have not considered general network models where multiple secondary users can transmit simultaneously. In our work we address those issues.

We study cooperation in a network consisting of a single primary source-destination (s-d) pair and multiple secondary s-d pairs. Secondary users can help primary users without harming primary user traffic (otherwise cooperation would be simply disabled). Our network model is general enough to include cases of multiple secondary s-d pair transmissions and cases where cooperation harms some secondary s-d pairs (due to higher interference). Scheduling and relaying decisions are made by a centralized network controller whose goal is to keep the lengths of all queues in the network bounded. The controller has no knowledge of packet generation rates; however it can observe the instantaneous state of the queues at every node. Moreover, we assume the following about our cooperation and packet transmission model: (a) primary network is *aware of the existence* of the secondary network and can request cooperation from the latter to improve latency of its own packets, and (b) primary packets can be transmitted across multiple time slots. Both assumptions are similar to the *spectrum leasing* model of cognitive radio which has been used in [4]–[6].

Our system model can represent a two tier heterogeneous network where the tier 1 (primary) nodes have higher priority of channel access than the tier 2 (secondary) nodes; nodes can engage in device-to-device communications. Such heterogeneous networks are likely to be part of future 5G networks [13]. One example of a network represented by our system model is the uplink or downlink of a macrocell network co-existing with one or more cognitive femtocells. For the downlink communication, the primary transmitter and receiver in our model represents the macro base-station and macro user respectively[1]; the secondary transmitters and receivers represent femto users and femto base-stations respectively. Femto users relay packets for the macro user using device-to-device communications. The network controller is a femto base-station that determines scheduling for all femto users and indirectly for the macro base-station (by deciding whether the femto users relay packets for the macro user or not). Our system model can also represent the macrocell uplink communication with the primary transmitter and receiver representing the macro user and the macro base-station respectively.

Our main contribution is in applying Lyapunov drift based techniques to efficiently solve the difficult problem of designing scheduling schemes in cognitive networks where primary nodes cooperate with secondary nodes. The work is novel as we consider primary packet generation rates that are *greater* than what is supportable[2] without cooperation and are unknown to the network controller. Prior works have only studied this problem for the case when primary packet generation rate is either less than what is supportable without cooperation [11] or is known to the controller [12]. The difficulty of the problem lies in the coupled nature of the queue evolution processes of primary and secondary nodes. The queue evolution of secondary nodes depends on the queue lengths of primary nodes as secondary packet transmissions have lower priority. On the other hand, the queue evolution of primary nodes, which have higher priority

[1]Note that the analysis in this paper, which is for a network with one primary receiver, can be extended to one with multiple primary receivers as is the case in a cellular setup. We discuss this possible extension in Section IV-F.

[2]By "supportable" we imply packet generation rates for which both the primary and secondary network are stable.

of channel access, depends on the cooperation scheme which in turn depends on the secondary queue lengths.

Coupled interaction between primary and secondary user queues in a cooperative setting was previously studied in [11] for the case of random independent and identically distributed (iid) primary packet arrivals. The authors showed that the problem of designing efficient scheduling schemes in this case is a constrained Markov Decision Problem (MDP), with the system state being the primary queue length. The authors in [11] applied the Lyapunov drift plus penalty method for renewal frames to solve this problem efficiently. Their solution does not suffer from the typical drawbacks associated with solutions to constrained MDPs such as knowledge of state transition probabilities and large convergence times. However, their analysis is valid only when the rate of primary packet arrivals is not higher than what can be maximally supported even without cooperation. This is because the renewal frame technique used requires the mean and second moment of the variable frame lengths to be always finite; this can be guaranteed only when rate of primary packet arrivals is not higher than what can be supported without cooperation. It is not trivial to extend the analysis for higher primary packet arrival rates.

In contrast to the work in [11], we show that when the primary packet generation process is periodic, then one can apply Lyapunov drift based techniques to support even higher primary packet arrival rates. We overcome the limitations of the analysis in [11] by exploiting the periodicity of the primary packet generation process. In particular, we define the system state differently than in [11]. We then construct a relaying and scheduling algorithm such that the length of a renewal frame, under the proposed cooperation scheme, is always constant. The algorithm requires knowledge of only *instantaneous queue lengths* at secondary nodes and *inter-arrival times* of primary packets[3]. Thus our work is in accordance with the wide body of works on max-weight based scheduling policies for communication networks that require knowledge of only instantaneous queue-states. We also show that the algorithm can also support a certain aperiodic primary packet generation process.

In order to analyze the stability performance of the algorithm we construct a region, referred to as guaranteed stability region, consisting of primary and secondary packet generation rates. We show that all packet generation rates in the interior of this region are supportable under our scheduling algorithm. For tractable analysis the definition of guaranteed stability region contains additional constraints, referred to as deadline constraints (discussed in Section III), beyond typical stability constraints. As a result this region is not the capacity region of the network. However, it includes those primary and secondary arrival rates that are supportable even without cooperation. Hence there is no reduction in stability performance of secondary users when the primary packet generation rate is lower than what can be supported without cooperation.

To summarize, our main technical contribution is:

1) the construction of a cooperative scheduling scheme with stability performance guarantees that supports higher primary packet generation rates than possible without cooperation, and
2) development of cooperation scheme for a general cognitive network model where cooperation may harm some secondary users and benefit other secondary users.

The remainder of the paper is organized as follows. Section II describes the system model. In Section III we present our scheduling policy. In Section IV we construct a guaranteed stability region and claim, in Theorem 1, that the network is stable for all packet arrival rates in the interior of this region. In Section V we prove Theorem 1. In Section VI we extend our analysis to a case where the primary packet generation process is not periodic. In Section VII we present simulation results. Section VIII concludes the paper.

II. System model

We consider a single primary s-d pair in the presence of multiple secondary s-d pairs. One or more secondary node(s) can relay primary packets. There is one primary transmitter (PT) and S secondary transmitters ST_1, ST_2, ..., ST_S. We denote the primary receiver as PR and the secondary receiver

[3]In this work we refer to the time difference between the arrival time of a given packet at the primary source node and that of the previous primary packet to be the inter-arrival time of the former packet.

TABLE I
LIST OF NOTATIONS

S	number of secondary transmitters	R	number of relay nodes
PT (PR)	primary transmitter (receiver)	ST_j (SR_j)	j'th secondary transmitter (receiver)
L	set of all links	L_p	links used to transmit primary packets
λ_p	primary packet generation rate (no fluctuation case)	D_p	primary packet size in bits
$d_p(t)$	amount of data-arrival at PT (in bits) at slot t	λ_i	packet generation rate of ST_i
$U_p(t)$	queue length of PT at slot t	$U_i(t)$	queue length at ST_i at slot t
Γ	set of feasible average transmission rate vectors	$\pi_{(n_1,n_2)}$	probability of link (n_1, n_2) being used
π_0	probability of no primary packet transmission	$t_{r,j}$	j'th slot in r'th frame
(n_1, n_2)	link with n_1 as transmitter and n_2 as receiver		
$K_{(n_1,n_2)}$	number of slots required to transmit primary packet via (n_1, n_2)		
$I(n_1, n_2)$	set of secondary transmission rate vectors feasible if (n_1, n_2) is used		
I_0	set of secondary transmission rate vectors feasible if no primary packet being transmitted		
f_i	effective primary packet transmission capacity via ST_i in packets/slot		
f_0	effective packet transmission capacity of link (PT, PR) in packets/slot		

corresponding to ST_i (where $i = 1, 2, \ldots, S$) as SR_i respectively. A transmission link is defined by the ordered nodes-pair (n_1, n_2), where $n_1 \in \{\text{PT}, \text{ST}_1, \ldots, \text{ST}_S\}$, $n_2 \in \{\text{PR}, \text{ST}_1, \ldots, \text{ST}_S, \text{SR}_1, \ldots, \text{SR}_S\}$ and n_2 is located within n_1's transmission range. Links with one of the nodes as PT or PR can be used to transmit primary packets and are referred to as *primary links*. We assume ideal channel sensing process, i.e., the sensing results are always accurate and sensing takes place in an infinitesimal time-duration. Next we present our packet transmission and arrival model, the interference model, the scheduling and queuing model. Important notations are summarized in Table I.

A. Packet Transmission Model

We assume a discrete time model where a new packet transmission begins only at the start of a slot. In this subsection we describe our packet transmission model as follows. First, we discuss the variable transmission power levels used by PT in our model. Second, we define a slot in terms of secondary packet transmissions. Third, we identify which secondary nodes can relay primary packets. Finally, we briefly discuss how PT transmits packets in our model.

We assume that PT either transmits a packet at fixed power level to PR, or at lower power level to some secondary transmitters within its transmission range that can act as intermediate relays. This is motivated by the observation that PT can simultaneously conserve power and achieve better latency for primary packets (than possible without cooperation), by transmitting at lower power level to some secondary node ST_i (where $i \in \{1, \ldots, S\}$) instead of PR. Such a situation can arise when the quality of the (PT, PR) link is poor compared to that of (PT, ST_i) and (ST_i, PR), for example, due to an obstacle in the direct transmission path from PT to PR. For ease of analysis, we assume each secondary transmitter uses a fixed power level to transmit any packet and all secondary transmitter-receiver links have the same capacity (in bits/s).

We define the length of a slot as the duration of a secondary packet transmission. Each secondary packet is shorter than a primary packet [4]. As a result, some primary packet transmission times can span across multiple slots. We denote as $K_{(n_1,n_2)}$ the number of slots required to transmit a primary packet through primary link (n_1, n_2). Clearly, all secondary transmitter-receiver links have the same capacity of 1 secondary packet per slot.

A secondary transmitter ST_j can relay primary packets if the following two conditions hold:

1) ST_j can receive packets from PT and then transmit to PR, i.e., ST_j is within transmission range of PT and PR is within transmission range of ST_j respectively;
2) Transmitting a primary packet via ST_j does not lead to higher latency for the transmitted packet than

[4]Transmitting longer secondary packets requires longer time; this increases the likelihood of such a transmission experiencing loss of spectrum opportunity due to primary user activity [14].

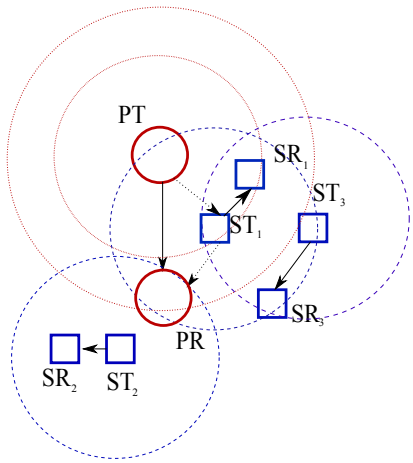


Fig. 1. A network with one primary s-d pair and three secondary s-d pairs. Each of the three blue-dashed circles correspond to transmission range of one of the ST_is (where $i = 1, 2, 3$). The dotted lines from PT and ST_1 represent cooperative transmission with ST_1 as a relay node. Cooperation helps ST_2 and hurts ST_3 by decreasing and increasing interference due to primary packet transmissions respectively.

that obtained by transmitting the packet directly to PR, i.e.,

$$K_{(\mathrm{PT},\mathrm{ST}_j)} + K_{(\mathrm{ST}_j,\mathrm{PR})} \leq K_{(\mathrm{PT},\mathrm{PR})}. \tag{1}$$

Let R (where $1 \leq R \leq S$) denote the number of such relay nodes. Without loss of generality (w.l.o.g.) we assume ST_1,..., ST_R are those nodes. Let L denote the set of all links that can be used for packet transmissions, i.e., $L = \{(\mathrm{PT}, \mathrm{PR}), (\mathrm{PT}, \mathrm{ST}_i), (\mathrm{ST}_i, \mathrm{PR}), (\mathrm{ST}_j, \mathrm{SR}_j) : 1 \leq i \leq R, 1 \leq j \leq S\}$. We denote the set of links that are used exclusively for primary packet transmission by L_p, i.e., L_p=$\{(\mathrm{PT}, \mathrm{PR}), (\mathrm{PT}, \mathrm{ST}_i), (\mathrm{ST}_i, \mathrm{PR}) : 1 \leq i \leq R\}$. Fig. 1 shows example of a network with $R = 1$ and S=3.

PT transmits packets whenever its buffer is non-empty because primary packets have higher transmission priority in the channel. If PT begins transmitting a packet to PR directly at slot t, then clearly in slots [t, $t + K_{(\mathrm{PT},\mathrm{PR})} - 1$] it is busy transmitting the packet. Instead if at slot t the packet is scheduled to be relayed via ST_j (where $1 \leq j \leq R$), then PT transmits the packet to ST_j during slots [t, $t + K_{(\mathrm{PT},\mathrm{ST}_j)} - 1$]; during slots [$t + K_{(\mathrm{PT},\mathrm{ST}_j)}$, $t + K_{(\mathrm{PT},\mathrm{ST}_j)} + K_{(\mathrm{ST}_j,\mathrm{PR})} - 1$], node ST_j relays the packet to PR.

B. Primary Packet Arrival Model

The packet generation process at PT results from aggregation of constant stream of data that arrives from higher layer to the transmission layer of PT. This packet generation process models a real-time Constant Bit Rate (CBR) voice or video source. A CBR voice source is one where packets generated during the silence periods of a voice call are not suppressed and packets are transmitted to the network periodically [15]. A CBR video data source can, for example, be a codec with appropriate rate control mechanisms to generate constant bit rate traffic from compressed video data [16]. Analysis of networks with video data source is important since video traffic is likely to account for more than half of global mobile data traffic by 2019 [17].

We assume the system begins at slot 0 with no data being initially present at transmission layer of PT. At slot t, $d_p(t)$ bits arrive from the upper layers of PT to its transmission layer. We assume $d_p(t)$ is constant and equal to b_p for every t. Let D_p denote the size of a primary packet (in bits), where $D_p > b_p$. Whenever the accumulated data is greater than D_p bits, those D_p bits are aggregated as a primary packet and moved to the transmission queue of PT. We denote the number of primary packet arrivals in slot t as $A_p(t) \in \{0, 1\}$. The primary packet generation rate, denoted as λ_p, is $\lambda_p = \frac{b_p}{D_p}$. Since b_p, D_p are integers, λ_p is a rational number, i.e., $\lambda_p \in Q$. Further, the primary packet generation process is periodic. The period is the denominator of λ_p after it has been expressed as the ratio of two co-prime integers.

For example, consider the case when $\lambda_p = \frac{5}{13}$. The first primary packet is generated at slot 2 when the accumulated data is $\frac{15D_p}{13}$ bits. Then the sequence of $A_p(t)$ starting at $t = 0$ is: 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, ... The process is periodic with each period consisting of 13 slots. The inter-arrival time of the first and second packet is 3 slots and that of the third one is 2 slots.

C. Secondary Packet Arrival Model

For secondary nodes we consider a more general case (compared to primary packets) of random secondary packet arrivals. Every slot a secondary packet arrives at the transmission layer of ST_i with probability λ_i (where $i = 1, 2, \ldots, S$). The packet generation process is iid across slots. We denote as $A_i(t)$ (where $A_i(t) \in \{0, 1\}$) the number of packet arrivals at ST_i at slot t. We denote the vector $(\lambda_1, \ldots, \lambda_S)^T$ as $\boldsymbol{\lambda}_s$.

D. Interference Model

The interference model is based on the protocol model of interference whereby a node can transmit to another node within its transmission range; this transmission is allowed only if the receiving node is not within the transmission range of another node that is transmitting in the same slot [18].

We represent a set of simultaneously occurring transmissions by an *activation vector*. Each component of the vector corresponds to a unique link in L; its length is equal to the cardinality of L. The cardinality of L is $S+2R+1$ corresponding to S links between secondary transmitter and receivers, R links between PT and secondary relay nodes, R links between secondary relay nodes and PR, and 1 link between PT and PR. Since every component in an activation-vector $\mathbf{E}$ corresponds to a unique link, with slight abuse of notation, we denote as $E_{(n_1,n_2)}$ the component of $\mathbf{E}$ corresponding to link (n_1, n_2). An activation vector is binary; for every $(n_1, n_2) \in L$ the component $E_{(n_1,n_2)}$ in $\mathbf{E}$ is set to 1 if n_1 is transmitting to n_2, otherwise it is set to 0.

We construct an activation vector $\mathbf{E}$ that is feasible under the protocol model of interference as follows: any component $E_{(n_1,n_2)}$ is set to 1 only if $E_{(n_3,n_4)} = 0$ for every $(n_3, n_4) \in L$ s.t. n_2 is within the transmission range of n_3. This is because, a node n_1 can transmit to a node n_2 within its transmission range only if n_2 is not within transmission range of another node n_3 that is transmitting simultaneously. The set of all feasible activation vectors is denoted by Φ.

E. Scheduling and Relaying Decisions

Whenever PT is about to transmit a new packet, the network controller makes a relaying decision, i.e., whether to transmit the packet directly to PR or via a relay node. At every slot t the controller also makes scheduling decision, i.e., it offers secondary transmission rate vector $\boldsymbol{\mu}_s(t)$ to secondary nodes for their own transmissions where $\boldsymbol{\mu}_s(t) = (\mu_1(t), \ldots, \mu_S(t))^T$. The variable $\mu_i(t) \in \{0, 1\}$ denotes the transmission rate offered (in secondary packets/slot) at slot t for transmission from ST_i to SR_i (where $i \in \{1, 2, \ldots, S\}$). Recall, all secondary transmitter-receiver links have the same capacity of 1 secondary packet per slot. Note that the relaying decision for a primary packet at any slot affects the scheduling decision in successive slots when that packet is being transmitted.

We now introduce some new notations to represent the feasible set of secondary transmission rate vectors at any slot. This notations will be used extensively throughout the rest of the paper. First, consider the mapping $\Pi\colon \Phi \mapsto R^S$; $\Pi(\mathbf{E})$ denotes the offered secondary transmission rate vector corresponding to a feasible activation vector $\mathbf{E}$. For every feasible activation vector $\mathbf{E}$, the offered secondary transmission rate vector $\Pi(\mathbf{E})$ is obtained by eliminating from $\mathbf{E}$ the components corresponding to primary links[5]. We denote as $I(n_1, n_2)$ the set of offered secondary transmission rate vectors in any slot in which n_1 is transmitting a primary packet to n_2 (where $(n_1, n_2) \in L_p$). This set can be written as, $I(n_1, n_2)$=$\{\Pi(\mathbf{E}) : \mathbf{E} \in \Phi,\ E_{(n_1,n_2)} = 1\}$. We denote as I_0 the set of offered secondary transmission rate vectors in any slot in which no node is transmitting any primary packet. This set can be written as, $I_0 = \{\Pi(\mathbf{E}) : \mathbf{E} \in \Phi, E_{(n_1,n_2)} = 0 \quad \forall (n_1, n_2) \in L_p\}$.

F. Queuing Model

Let $U_p(t)$ and $U_i(t)$ denote the queue lengths of PT and ST_i (where $i = 1, 2, \ldots, S$) at slot t respectively. Queue length at PT evolves as

$$U_p(t+1) = U_p(t) - C(t) + A_p(t), \tag{2}$$

[5]For example, for the network in Fig. 1, the offered secondary transmission rate vector $\Pi(\mathbf{E})$ corresponding to the activation vector $\mathbf{E} = (E_{(\text{PT},\text{PR})}, E_{(\text{PT},\text{ST}_1)}, E_{(\text{ST}_1,\text{PR})}, E_{(\text{ST}_1,\text{SR}_1)}, E_{(\text{ST}_2,\text{SR}_2)}, E_{(\text{ST}_3,\text{SR}_3)})^T$ is the vector $(E_{(\text{ST}_1,\text{SR}_1)}, E_{(\text{ST}_2,\text{SR}_2)}, E_{(\text{ST}_3,\text{SR}_3)})^T$.

where $C(t)$ is an indicator variable which is 1 if a primary packet transmission is completed at t and is 0 otherwise. The queue lengths for secondary packets evolve as:

$$U_i(t+1) = \max[U_i(t) - \mu_i(t), 0] + A_i(t). \tag{3}$$

Let $\boldsymbol{U}_s(t)$ denote the queue length vector $(U_1(t), \ldots, U_S(t))^T$ at slot t.

Throughout the paper we use the notion of strong stability of queues, defined as follows.

Definition 1: A discrete time queue $U(t)$ is strongly stable if $\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[|U(\tau)|] < \infty$. The network is strongly stable if every queue in the network is strongly stable.

III. Dynamic relaying and scheduling policy

In this section we present a dynamic Scheduling and Cooperative Relay Policy (SCRP). The policy is based on Lyapunov drift techniques introduced by Tassiulas and Ephremides in [19], which has been widely used to develop throughput-optimal algorithms in computer networks. This section is organized as follows. First, in Section III-A, we introduce some notations that are frequently used in the SCRP algorithm. Then, in Section III-B, we discuss the construction methodology of the SCRP algorithm. In particular, in Section III-B we discuss how the SCRP algorithm determines the primary packet transmission scheme by solving a certain optimization problem; we also discuss how the SCRP algorithm determines the secondary packet transmission scheme. Then, in Section III-C, we formally describe the SCRP algorithm. Finally, in Section III-D, we comment on the difficulty of implementing the SCRP algorithm in practice.

A. Additional Notations

We first introduce notations to represent the maximum primary packet arrival rate that can be supported by using a given relay node. We then introduce notations to classify slots based on ongoing primary packet transmission activity. Finally, we introduce notations to represent secondary transmission rate vectors obtained as solution to certain maximization problems.

Let f_i (where $1 \le i \le R$) denote the maximum primary packet arrival rate that can be supported if every primary packet is transmitted via relay ST_i. Clearly, this is just reciprocal of the overall latency of a primary packet transmitted via ST_i, i.e., $f_i = \frac{1}{K_{(\text{PT},\text{ST}_i)} + K_{(\text{ST}_i,\text{PR})}}$. We assume w.l.o.g. that $\text{ST}_1, \ldots, \text{ST}_R$ are indexed in ascending order of their capacity to relay primary packets, i.e., $f_j \le f_{j+1}$ for every $j \in \{1, \ldots, R-1\}$. Let f_0 denote the maximum primary packet arrival rate that can be supported if every primary packet is directly transmitted. Clearly, $f_0 = \frac{1}{K_{(\text{PT},\text{PR})}}$. Due to (1), this rate is lower than that supported by the relays, i.e., $f_0 \le f_1$.

A slot is considered *idle* if there is no primary packet transmission anywhere in the network or equivalently, there is no packet at PT; otherwise it is considered *busy*. A *busy period* consists of a contiguous collection of busy slots.

For any slot t and for each link $(n_1, n_2) \in L_p$, we denote as $\mathbf{v}^*_{(n_1,n_2)}(t)$ the solution to the following maximization problem:

$$\boldsymbol{v}^*_{(n_1,n_2)}(t) \in \operatorname*{argmax}_{\boldsymbol{v} \in I(n_1,n_2)} \boldsymbol{U}_s{}^T(t)\boldsymbol{v}. \tag{4}$$

For any slot t we denote as $\boldsymbol{v}^*_0(t)$ the solution to the following maximization problem:

$$\boldsymbol{v}^*_0(t) \in \operatorname*{argmax}_{\boldsymbol{v} \in I_0} \boldsymbol{U}_s{}^T(t)\boldsymbol{v}. \tag{5}$$

B. Construction Methodology of the SCRP Algorithm

We first discuss construction of the primary packet transmission scheme under SCRP. We then discuss construction of the secondary packet transmission scheme under SCRP.

1) Primary Packet Transmission Scheme Under SCRP

Under the SCRP algorithm we schedule transmission of the HOL packet at PT at any slot t, if it is not being served currently, by solving an optimization problem. In particular, we determine the transmission scheme for this packet, either via the direct link (PT, PR) or via a relay, by minimizing the conditional drift of a Lyapunov function of secondary user queue lengths. We first discuss this minimization problem. We then present two simpler equivalent maximization problems: P1 and P2. The primary packet transmission scheme obtained by solving P1 and P2 minimizes the aforementioned conditional drift when the inter-arrival time of the HOL packet at PT is greater than or equal to $K_{(\text{PT},\text{PR})}$ slots, and less than $K_{(\text{PT},\text{PR})}$ slots

respectively.

We now discuss the conditional drift minimization problem whose solution determines the transmission scheme of the HOL packet at PT at slot t. Suppose, the inter-arrival time of this packet is k slots, where k is any positive integer. Then we transmit this packet by selecting the primary packet transmission scheme that minimizes the k-slot conditional drift[6] of the Lyapunov function $\sum_{n=1}^{S} U_n^2(t)$ and also transmits the packet to PR before slot $t+k$. This minimization is performed considering only one primary packet transmission[7] in slots $[t, t+k-1]$. Note that, in the minimization problem we considered only those relay nodes (and the direct transmission path) as feasible options to transmit the primary packet, for which the overall primary packet transmission time is not greater than the packet's inter-arrival time. This restriction, referred to as *deadline constraint*, is required to analyze the stability performance of the SCRP algorithm (to be discussed in detail in the next section).

We now present equivalent maximization problem P1 as follows. Consider the primary packet transmission scheme that minimizes the k-slot conditional drift when k is greater than or equal to $K_{(\text{PT,PR})}$. This scheme also maximizes the dot-product of $\boldsymbol{U}_s(t)$ and the vector sum of offered secondary transmission rate vectors in slots $[t, t+K_{(\text{PT,PR})}-1]$, considering only one primary packet transmission in those slots. Therefore, we next find the transmission scheme that maximizes this dot-product as follows. We first obtain the maximum value of this dot-product, conditioned on the primary packet transmission scheme:

1) If the primary packet is directly transmitted to PR, then the maximum value of the dot-product term is $\boldsymbol{U}_s{}^T(t) K_{(\text{PT,PR})} \boldsymbol{v}^*_{(\text{PT,PR})}(t)$. This is obtained by selecting $\boldsymbol{v}^*_{(\text{PT,PR})}(t)$ as the offered secondary transmission rate vector in slots $[t, t+K_{(\text{PT,PR})}-1]$.
2) If the primary packet is relayed via ST_n (where $1 \le n \le R$), then the maximum value of the dot-product term is $\boldsymbol{U}_s{}^T(t)\{K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t) + K_{(\text{ST}_n,\text{PR})}\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t) + (K_{(\text{PT,PR})} - K_{(\text{PT,ST}_n)} - K_{(\text{ST}_n,\text{PR})})\boldsymbol{v}^*_0(t)\}$. This is obtained by selecting $\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t)$, $\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t)$ and $\boldsymbol{v}^*_0(t)$ as the offered secondary transmission rate vectors in slots $[t,\ t+K_{(\text{PT,ST}_n)}-1]$, $[t+K_{(\text{PT,ST}_n)},\ t+K_{(\text{PT,ST}_n)}+K_{(\text{ST}_n,\text{PR})}-1]$ and $[t+K_{(\text{PT,ST}_n)}+K_{(\text{ST}_n,\text{PR})},\ t+K_{(\text{PT,PR})}-1]$ respectively.

Using the maximum values of the dot-product term for each primary packet transmission scheme, we formulate the dot-product maximization problem as:

$$\begin{aligned}\text{P1:}\ \underset{n\in\{1,\ldots,R\}}{\operatorname{argmax}}\ & \boldsymbol{U}_s{}^T(t)\{K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t) \\ & + K_{(\text{ST}_n,\text{PR})}\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t) + (K_{(\text{PT,PR})} \\ & - K_{(\text{PT,ST}_n)} - K_{(\text{ST}_n,\text{PR})})\boldsymbol{v}^*_0(t)\}\end{aligned} \tag{6}$$

s.t.

$$\begin{aligned}\boldsymbol{U}_s{}^T(t) K_{(\text{PT,PR})}\boldsymbol{v}^*_{(\text{PT,PR})}(t) \ \le\ & \boldsymbol{U}_s{}^T(t)\{K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t) \\ & + K_{(\text{ST}_n,\text{PR})}\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t) \\ & + (K_{(\text{PT,PR})} - K_{(\text{PT,ST}_n)} \\ & - K_{(\text{ST}_n,\text{PR})})\boldsymbol{v}^*_0(t)\}\end{aligned} \tag{7}$$

If problem P1 has no feasible solution it means the dot-product is maximized by transmitting the primary packet via direct link (PT, PR). Otherwise, the dot-product is maximized by transmitting the primary packet via the relay node whose index is in the solution of problem P1.

We now present equivalent maximization problem P2 as follows. Consider the primary packet transmission scheme that minimizes the k-slot conditional drift when k is less than $K_{(\text{PT,PR})}$. This scheme also maximizes the dot-product of $\boldsymbol{U}_s(t)$ and the vector sum of offered secondary transmission rate vectors in slots $[t, t+K_{(\text{PT,ST}_{k'})}+K_{(\text{ST}_{k'},\text{PR})}-1]$, considering only one primary packet transmission during those slots. Here $\text{ST}_{k'}$, where $k' \in \{1,\ldots,R\}$, is the relay node with the smallest index that can still satisfy the deadline constraint for this primary packet, i.e., $\frac{1}{f_{k'-1}} > k \ge \frac{1}{f_{k'}}$. We formulate the dot-product

[6]The k-slot conditional drift of a Lyapunov function of queue lengths, $V(\mathbf{U_s}(t))$ is $E[V(t+k)-V(t)|\mathbf{U_s}(t)]$ [20].

[7]This is not an assumption about the system model but is simply a way to interpret the objective of the minimization problem.

maximization problem, similar to the way we obtained problem P1, as:

$$\text{P2: } \underset{n\in\{k',\ldots,R\}}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(t)\{K_{(\text{PT},\text{ST}_n)}\boldsymbol{\nu}^*_{(\text{PT},\text{ST}_n)}(t) + K_{(\text{ST}_n,\text{PR})}\boldsymbol{\nu}^*_{(\text{ST}_n,\text{PR})}(t) + (K_{(\text{PT},\text{ST}_{k'})} + K_{(\text{ST}_{k'},\text{PR})} - K_{(\text{PT},\text{ST}_n)} - K_{(\text{ST}_n,\text{PR})})\boldsymbol{\nu}^*_0(t)\}. \tag{8}$$

The dot-product is maximized by transmitting the primary packet via the relay node whose index is in the solution of problem P2.

2) Secondary Packet Transmission Scheme Under SCRP

At any busy slot, secondary packets are transmitted according to backpressure policy where links used for or interfered by primary packet transmission are excluded. At any idle slot, secondary packets are transmitted according to the backpressure policy.

C. Description of the SCRP Algorithm

The SCRP algorithm makes the following scheduling and relaying decisions:

1) *Cooperative Relaying Decision for Primary Packets:* If the transmission queue of PT is non-empty and the HOL packet in its queue is not served at slot t, then its service begins at t in the following manner. If the inter-arrival time of the primary packet is greater than or equal to $K_{(\text{PT},\text{PR})}$ slots, find solution to problem P1; otherwise, find solution to problem P2. Denote the solution as i^*. In case of multiple solutions pick an i^* arbitrarily. Then use ST_{i^*} as relay and transmit the HOL primary packet from PT to ST_{i^*} in slots $[t,\, t + K_{(\text{PT},\text{ST}_{i^*})} - 1]$ and from ST_{i^*} to PR in slots $[t + K_{(\text{PT},\text{ST}_{i^*})},\, t + K_{(\text{PT},\text{ST}_{i^*})} + K_{(\text{ST}_{i^*},\text{PR})} - 1]$. If inter-arrival time of the primary packet is greater than or equal to $K_{(\text{PT},\text{PR})}$ slots and there is no solution to problem P1, then directly transmit this packet to PR in slots $[t,\, t + K_{(\text{PT},\text{PR})} - 1]$.
2) *Secondary Packet Scheduling Decision in Busy Slots:* If the decision about transmitting the primary packet in the previous step was to relay the same via ST_{i^*}, then the offered secondary transmission rate vector $\boldsymbol{\mu}^*_s(\tau)$ in slots $\tau \in [t,\, t + K_{(\text{PT},\text{ST}_{i^*})} + K_{(\text{ST}_{i^*},\text{PR})} - 1]$ are obtained as follows:
(i) For slots $\tau \in [t,\, t + K_{(\text{PT},\text{ST}_{i^*})} - 1]$, i.e., during transmission from PT to ST_{i^*}, $\boldsymbol{\mu}^*_s(\tau)$ is obtained as:
$$\boldsymbol{\mu}^*_s(\tau) \in \underset{\boldsymbol{\nu}\in I(\text{PT},\text{ST}_{i^*})}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{\nu}. \tag{9}$$
(ii) For slots $\tau \in [t + K_{(\text{PT},\text{ST}_{i^*})},\, t + K_{(\text{PT},\text{ST}_{i^*})} + K_{(\text{ST}_{i^*},\text{PR})} - 1]$, i.e., during transmission from ST_{i^*} to PR, $\boldsymbol{\mu}^*_s(\tau)$ is obtained as:
$$\boldsymbol{\mu}^*_s(\tau) \in \underset{\boldsymbol{\nu}\in I(\text{ST}_{i^*},\text{PR})}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{\nu}. \tag{10}$$
If the decision about transmitting the primary packet in the previous step was to directly transmit the same, then for slots $\tau \in [t,\, t + K_{(\text{PT},\text{PR})} - 1]$, i.e., during transmission from PT to PR, $\boldsymbol{\mu}^*_s(\tau)$ is obtained as:
$$\boldsymbol{\mu}^*_s(\tau) \in \underset{\boldsymbol{\nu}\in I(\text{PT},\text{PR})}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{\nu}. \tag{11}$$
3) *Secondary Packet Scheduling Decision in Idle Slots:* At any idle slot τ, the offered secondary transmission rate vector $\boldsymbol{\mu}^*_s(\tau)$ is obtained as:
$$\boldsymbol{\mu}^*_s(\tau) \in \underset{\boldsymbol{\nu}\in I_0}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{\nu}. \tag{12}$$
4) *Transmission and Queue-update:* Transmit $\min(\mu^*_i(t), U_i(t))$ secondary packets from ST_i in slot t for every $i = 1, \ldots, S$. If t is the last slot in any busy period, remove the HOL primary packet from PT's transmission queue at the end of t.

Note that the SCRP algorithm makes decision based only on instantaneous queue-states and requires no knowledge of the primary packet generation rate.

D. Difficulties With Practical Implementation of the SCRP Algorithm

Implementing the SCRP algorithm might be difficult in practice as it involves a centralized controller collecting queue length information from all secondary nodes and then selecting a transmission rate vector by searching from a combinatorial set of transmission rate vectors. The worst case computational

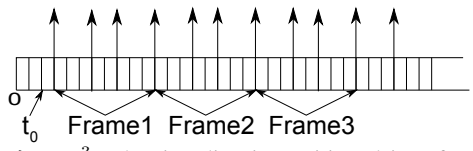


Fig. 2. Primary packet arrival process when $\lambda_p = \frac{3}{8}$. The time-line is partitioned into frames starting at slot 3 with $t_0 = 2$. Each small rectangle represents a slot. The arrival of a primary packet at the transmission queue of PT during any slot is indicated by a vertical arrow at the boundary between that slot and the one immediately after it.

complexity of SCRP is at least as high as that of the backpressure algorithm since, in idle slots, it is identical to the backpressure algorithm for secondary nodes. In literature, greedy maximal scheduling algorithms have been suggested as approximation to such max-weight based algorithms [21]. In addition to low complexity, such algorithms are also suited for distributed implementation [22]. These algorithms typically work by selecting the link with maximum weight at any time, followed by the link with next highest weight among the set of remaining links and so on. We do not analyze any such greedy algorithm for our network model. However in Section VII, we compare the stability performance, via simulations for a particular network, of the SCRP algorithm with one such greedy algorithm.

IV. Stability Analysis

In this section we present a guaranteed stability region for the network under the SCRP algorithm. The region is constructed by imposing deadline constraints for transmitted primary packets in addition to regular flow-conservation constraints at the primary and secondary nodes. This formulation is similar to the capacity region description in [23]. We first discuss some properties of the primary packet generation process (Section IV-A), the motivation behind having deadline constraints (Section IV-B) and the effective transmission rate for secondary users (Section IV-C). Then we use those concepts to define the guaranteed stability region in Section IV-D. We then state Theorem 1 in Section IV-D where we formally claim that the network is indeed stable under the SCRP algorithm for all arrival rate vectors within the guaranteed stability region. Then in Section IV-E we discuss some observations that immediately follow from Theorem 1 and illustrate them with a numerical example. Finally, in in Section IV-F, we discuss possible extensions to the system model.

A. Properties of the Primary Packet Generation Process

Since λ_p is rational it can be expressed as the ratio of two co-prime integers, denoted as $M(\lambda_p)$ and $N(\lambda_p)$ respectively, i.e., $\lambda_p = \frac{M(\lambda_p)}{N(\lambda_p)}$. $N(\lambda_p)$ and $M(\lambda_p)$ signify the length of period of $A_p(.)$ and the number of primary packet arrivals in that period respectively. Note that there exists a unique positive integer k_1 s.t. $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1}$. Then,

Property 1: the inter-arrival time of any primary packet is either $k_1 + 1$ slots or k_1 slots.

For any λ_p we denote as $\kappa^{(1)}(\lambda_p)$ and $\kappa^{(2)}(\lambda_p)$ the number of primary packet arrivals, during any interval of length $N(\lambda_p)$ slots, with inter-arrival times of $k_1 + 1$ and k_1 slots respectively. Then,

Property 2: the total number of primary packet arrivals within any period is $M(\lambda_p)$, i.e.,

$$\kappa^{(1)}(\lambda_p) + \kappa^{(2)}(\lambda_p) = M(\lambda_p), \text{ and} \tag{13}$$

Property 3: the sum of inter-arrival times of all primary packet arrivals within any period is equal to the length of the period, i.e.,

$$(k_1 + 1)\kappa^{(1)}(\lambda_p) + k_1\kappa^{(2)}(\lambda_p) = N(\lambda_p). \tag{14}$$

For example, Fig. 2 shows the primary packet arrival process when λ_p is $\frac{3}{8}$. We observe that the inter-arrival time of any primary packet is either 3 or 2 slots which is consistent with $\frac{3}{8}$ being less than $\frac{1}{2}$ but greater than $\frac{1}{3}$. The total number of packet arrivals in any 8 slots is 3. In any 8 slots, there are 2 and 1 primary packet arrivals with inter-arrival time of 3 and 2 slots respectively and the sum of inter-arrival times is therefore 8.

In the rest of the section we assume $\lambda_p \in Q$ and refer to $M(\lambda_p)$ and $N(\lambda_p)$ simply as M and N whenever there is no confusion.

B. Intuition Behind Deadline Constraints

Deadline constraints allow us to apply Lyapunov-drift techniques and analyze the stability performance of SCRP. The explanation is provided below.

Lyapunov-drift techniques are helpful in constructing efficient backpressure-type scheduling policies without knowledge of packet arrival rates. However, it is difficult to develop such policies in networks where there is cooperation between primary and secondary users. This is because such policies assign higher priority of transmission to queues with high backlogs. But in our model, primary users always have highest priority of transmission regardless of queue length. Moreover, the evolution of the primary user queue depends on the actions of secondary users. We solve this problem by using renewal frame-based techniques. We first define the *system state* to be the queue length of PT at the slot following arrival of $(nM+1)$'th primary packet for every $n = 0, 1, 2, \ldots$. The system state is *refreshed* every time its value becomes 1. A renewal frame is defined as the collection of time slots between successive system state refresh events. Existence of renewal frames whose frame-length is constant facilitates construction of a Lyapunov drift based algorithm [20]. The use of primary queue lengths as system state and then applying renewal frame based techniques is motivated by [11]. We observe that a renewal frame whose length is N slots can be obtained if M primary packets are transmitted every N slots. A sufficient condition to ensure this is the deadline constraint, i.e., to require that the overall transmission time of every primary packet is not higher than its inter-arrival time.

Assume that the system starts at slot 0 and denote the slot in which the first primary packet arrives as t_0. The time-line can then be partitioned into a finite interval $[0, t_0]$ and successive non-overlapping frames of length N slots each as: $[t_0+1, t_0+N]$, $[t_0+N+1, t_0+2N]$,.... Fig. 2 shows the partition of the time-line into frames for the case where $\lambda_p = \frac{3}{8}$.

C. Effective Transmission Rate for Secondary Users

Since for every slot t, $\boldsymbol{\mu}_s(t)$ depends on which primary link is being used in that slot, the effective transmission rate offered to secondary users is defined in a time average sense as follows. Let $\pi_{(n_1,n_2)}$ denote, under some policy, the time-averaged probability of the event[8]: "n_1 is transmitting to n_2" (where $(n_1, n_2) \in L_p$). Let π_0 denote the time-averaged probability of the event: "no node is transmitting a primary packet". Note that all such events are mutually exclusive. Assume $\lambda_p \in [\frac{1}{k_1+1}, \frac{1}{k_1})$, where k_1 is a positive integer. Let $\pi^{(1)}_{(n_1,n_2)}$ and $\pi^{(2)}_{(n_1,n_2)}$ denote the long-term average probability of the events "node n_1 is transmitting a packet with inter-arrival time of k_1+1 slots to node n_2" and "node n_1 is transmitting a packet with inter-arrival time of k_1 slots to node n_2" respectively. The time-averaged offered secondary transmission rate vector during the event: "n_1 is transmitting to n_2" belongs to the convex hull[9] of $I(n_1, n_2)$ since any transmission rate vector in $I(n_1, n_2)$ can be used during this event. Similarly the time-averaged secondary transmission rate vector offered in idle slots belongs to the convex hull of I_0. Averaging over all such events, we observe that any effective transmission rate vector offered to secondary users belongs to the set $\Gamma(\boldsymbol{\pi})$ defined as,

$$\begin{aligned}\Gamma(\boldsymbol{\pi}) &= \pi_{(\mathrm{PT},\mathrm{PR})}\mathbf{conv}I(\mathrm{PT},\mathrm{PR}) \\ &+ \sum_{j=1}^{R}\Big\{\pi_{(\mathrm{PT},\mathrm{ST}_j)}\mathbf{conv}I(\mathrm{PT},\mathrm{ST}_j) \\ &+ \pi_{(\mathrm{ST}_j,\mathrm{PR})}\mathbf{conv}I(\mathrm{ST}_j,\mathrm{PR})\Big\} + \pi_0\mathbf{conv}I_0. \qquad (15)\end{aligned}$$

where $\boldsymbol{\pi}$ denotes the vector $(\pi_{(\mathrm{PT},\mathrm{PR})}, \pi_{(\mathrm{PT},\mathrm{ST}_1)}, \ldots, \pi_{(\mathrm{PT},\mathrm{ST}_R)}, \pi_{(\mathrm{ST}_1,\mathrm{PR})}, \ldots, \pi_{(\mathrm{ST}_R,\mathrm{PR})}, \pi_0)^T$. The "+" operator in (15) indicates Minkowski addition of sets[10].

[8]For the time being assume that such an average exists. In the next section we show the existence of a stationary policy which achieves such averages.

[9]The convex hull of a set W is the set of all convex combinations of elements $w \in W$.

[10]The Minkowski addition of two sets of vectors W_1 and W_2 is the set formed by adding every vector in W_1 to every vector in W_2 i.e. the set $\{w_1 + w_2 | w_1 \in W_1, w_2 \in W_2\}$ [24].

D. Guaranteed Stability Region

The guaranteed stability region is defined as the set $\{(\lambda_p, \boldsymbol{\lambda}_s) : \lambda_p \leq f_R, \boldsymbol{\lambda}_s \in \text{Interior}(\Lambda(\lambda_p))\}$ where $\Lambda(\lambda_p)$ is defined as follows. Suppose k_1 is the unique positive integer s.t. λ_p is less than $\frac{1}{k_1}$ but is greater than or equal to $\frac{1}{k_1+1}$. Then $\Lambda(\lambda_p)$ is the set of secondary packet arrival rate vectors $\boldsymbol{\lambda}_s$ for which there exists non-negative variables $\pi_0, \pi^{(i)}_{(n_1,n_2)}$ (where $(n_1, n_2) \in L_p$ and $i = 1, 2$) s.t.:

$$\frac{\kappa^{(i)}(\lambda_p)}{N} = \frac{\pi^{(i)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}} + \sum_{1\leq j\leq R} \frac{\pi^{(i)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}} \tag{16}$$

$$\pi_0, \pi^{(i)}_{(n_1,n_2)} \geq 0 \tag{17}$$

$$\frac{\pi^{(i)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}} = \frac{\pi^{(i)}_{(\text{ST}_j,\text{PR})}}{K_{(\text{ST}_j,\text{PR})}} \quad 1 \leq j \leq R \tag{18}$$

$$\pi^{(1)}_{(\text{PT,PR})} = 0, \quad \text{if } K_{(\text{PT,PR})} > k_1 + 1 \tag{19}$$

$$\pi^{(2)}_{(\text{PT,PR})} = 0, \quad \text{if } K_{(\text{PT,PR})} > k_1 \tag{20}$$

$$\pi^{(1)}_{(\text{PT,ST}_j)} = 0 \quad \forall 1 \leq j \leq R, \quad \text{if } K_{(\text{PT,ST}_j)} + K_{(\text{ST}_j,\text{PR})} > k_1 + 1 \tag{21}$$

$$\pi^{(2)}_{(\text{PT,ST}_j)} = 0 \quad \forall 1 \leq j \leq R, \quad \text{if } K_{(\text{PT,ST}_j)} + K_{(\text{ST}_j,\text{PR})} > k_1 \tag{22}$$

$$\pi_{(n_1,n_2)} = \pi^{(1)}_{(n_1,n_2)} + \pi^{(2)}_{(n_1,n_2)} \tag{23}$$

$$\pi_0 = 1 - \sum_{(n_1,n_2)\in L_p} \pi_{(n_1,n_2)} \tag{24}$$

$$\lambda_n \leq X_n \quad \forall n = 1, 2, \ldots, S \tag{25}$$

for some $\boldsymbol{X}_s = (X_1, \ldots, X_S)^T \in \Gamma(\boldsymbol{\pi})$. The equality constraint (16) is a conservation constraint at PT: arrival rates of primary packets of inter-arrival time $k_1 + 1$ and k_1 slots are equal to their respective departure rates. Terms $\frac{\pi^{(1)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}}$ and $\frac{\pi^{(2)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}}$ in (16) represent the average number of primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots transmitted per slot via ST_j respectively. Similarly terms $\frac{\pi^{(1)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}}$ and $\frac{\pi^{(2)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}}$ in (16) represent the average number of primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots transmitted per slot directly to PR respectively. Constraint (18) represents that the average number of primary packets of either type that enter any relay node is equal to that transmitted by the relay node. The constraints (19)-(22) are deadline constraints. The relations (17), (23) and (24) represent that probability of all events are non-negative and add up to 1. The inequality constraint (25) represents stability condition for secondary transmitters.

Theorem 1: For all $\lambda_p < f_R$, under the SCRP algorithm, $U_p(t)$ and $U_i(t)$ (where $i \in \{1, \ldots, S\}$) are strongly stable for all $\boldsymbol{\lambda}_s$ in the interior of $\Lambda(\lambda_p)$.

When λ_p is 0, the algorithm reduces to traditional back-pressure theorem whose stability performance was analyzed in [19]. In the next section we prove the theorem for $\lambda_p \neq 0$.

E. Observations From Theorem 1

Before proving Theorem 1 we discuss some observations that follow immediately from it.

1) Due to the deadline constraints (19)-(22) the guaranteed stability region is not the capacity region of the network under cooperation. However, when λ_p is not greater than what can be supported without cooperation, i.e., f_0, the set of $\boldsymbol{\lambda}_s$ for which the secondary network is stable under SCRP is the same as that under any stationary algorithm. This is because, the deadline constraints are redundant for those values of λ_p.
2) Corresponding to any $\lambda_p \leq f_0$, the set of $\boldsymbol{\lambda}_s$ for which the secondary network is stable without

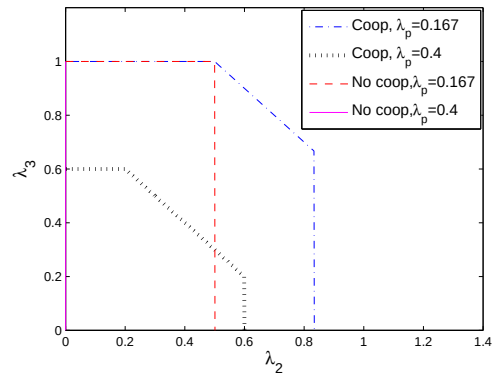


Fig. 3. Stability region without cooperation and guaranteed stability region under SCRP for the network in Fig. 1 when λ_1 is 0 and λ_p is 0.4 and 0.167 respectively.

cooperation, denoted as $\Lambda_0(\lambda_p)$, does not decrease with cooperation. This is because $\Lambda_0(\lambda_p)$ can be obtained by setting $\pi^{(i)}_{(\text{PT},\text{ST}_j)}$ to 0 in (16)-(25) (i.e. the relations used to obtain the guaranteed stability region) for every $j \in \{1, \ldots, \text{R}\}$ and $i = 1, 2$. Ignoring the set of $\boldsymbol{\lambda}_s$ forming the boundary of $\Lambda(\lambda_p)$ we observe that cooperation does not adversely affect the stability performance of secondary nodes when $\lambda_p \leq f_0$.

3) The set of $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized, for a given λ_p not greater than f_0, is expanded with cooperation. For higher λ_p, the set of $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized with cooperation may not include all $\boldsymbol{\lambda}_s$ for which the network can be stabilized without cooperation. However, in this case cooperation may result in win-win scenarios for both PT and some secondary transmitter.

We illustrate the last 2 observations by the following example. Consider the network in Fig. 1 with $K_{(\text{PT},\text{ST}_1)} = K_{(\text{ST}_1,\text{PR})} = 1$ and $K_{(\text{PT},\text{PR})} = 3$. The interference model is described as follows.
When PT transmits to PR, only ST_3 can transmit. When PT transmits to ST_1 only ST_2 can transmit. When ST_1 transmits to PR, no secondary node can transmit. When there is no primary packet transmission, ST_2, and either ST_1 or ST_3 can transmit.

The maximum λ_p that can be supported are 0.5 and 0.33 with and without cooperation respectively. In Fig. 3 we plot the set of $\boldsymbol{\lambda}_s$, in terms of λ_2 and λ_3, that belongs to $\Lambda(\lambda_p)$ when λ_p are 0.4 and 0.167 respectively and λ_1 is 0. We also plot, in terms of λ_2 and λ_3, all $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized without cooperation when λ_p are 0.4 and 0.167 respectively and λ_1 is 0. When λ_p is 0.4 this region is just the vertical line from the point (λ_2, λ_3)=(0,0) to (0,1). The values of 0.4 and 0.167 are arbitrarily chosen as examples of λ_p for which the primary network is stabilizable only with and even without cooperation respectively.

All points, with λ_2 greater than 0.5, in the region below the line marked "Coop, $\lambda_p = 0.167$" represent an increase in the set of $\boldsymbol{\lambda}_s$ supportable due to cooperation when λ_1 is 0 and λ_p is 0.167. All points, with positive λ_2, in the region below the line marked "Coop, $\lambda_p = 0.4$" represent $\boldsymbol{\lambda}_s$ that constitute win-win scenarios for PT and ST_2. This is because, without cooperation ST_2 can not transmit any packet if λ_p is greater than or equal to 0.33. Note that this region does not include the entire set of $\boldsymbol{\lambda}_s$ supportable without cooperation when λ_p is 0.4. Simply disabling cooperation will allow the secondary network to be stabilized for all $\boldsymbol{\lambda}_s$ in the latter set.

F. Possible Extensions to the System Model

Our system model can be extended to accommodate multiple primary receivers and power control for the whole network, for example, by considering a network in which each secondary node can select its transmission power from a finite set of discrete power levels. For both those extensions, the changes that need to be made in the system model will arise due to change in the set of links (L). Accommodating multiple primary receivers will increase the set of primary links (L_p). Accommodating power control for the whole network will change the set of links as each link will be defined as the 3-tuple (transmitter,

receiver, transmission power). Moreover, accommodating power control for the whole network will also require new definition of a slot as all secondary transmitter-receiver links may not have same capacity. In this case, we can define a slot duration to be the longest secondary packet transmission time, considering all power levels. Under this new definition, some secondary transmitter-receiver links can transmit multiple secondary packets per slot. All the above changes will subsequently lead to changes in the definition of activation vectors ($\mathbf{E}$), the set of feasible activation vectors (Φ) and the set of feasible average transmission rate vectors (Γ). The rest of the analysis, i.e., the construction of an efficient scheduling algorithm and proving its stability performance can be performed in a similar manner as in the construction of the SCRP algorithm and Theorem 1 respectively. However, the analysis for both extensions will be substantial enough to require a separate manuscript.

Our system model can also be extended to account for fairness among the users by using the Lyapunov drift plus penalty technique instead of just the Lyapunov drift technique.

V. Proof of Theorem 1

To prove Theorem 1 we first construct a Stationary Scheduling Policy (SSP) that stabilizes the network for all arrival rates in the guaranteed stability region without any knowledge of secondary queue lengths (Lemma 1). Then we define an Alternate Scheduling Policy (ASP) that maximizes a utility function corresponding to the conditional drift in each frame among all policies satisfying the deadline constraint. We compare the value of this utility function under ASP with that under SSP (Lemma 2). We then compare the value of this utility function under ASP with that under SCRP (Lemma 3). Finally, we use those comparisons to prove Theorem 1.

A. Stationary Scheduling Policy

Consider the variables $X_1, \ldots, X_S$ and $\pi_0, \pi^{(i)}_{(n_1,n_2)}$ (where $i = 1, 2$ and $(n_1, n_2) \in L_p$) that satisfy (16)-(25) for any $\boldsymbol{\lambda}_s \in \Lambda(\lambda_p)$. Suppose we index vectors in the set $I(n_1, n_2)$ (where $(n_1, n_2) \in L_p$) and I_0 as $\boldsymbol{\nu}_{(n_1,n_2),1}, \ldots, \boldsymbol{\nu}_{(n_1,n_2),|I(n_1,n_2)|}$ and $\boldsymbol{\nu}_{0,1}, \ldots, \boldsymbol{\nu}_{0,|I_0|}$ respectively. Then from (15) it follows that there exists non-negative variables $p^{\text{SSP}}_{(n_1,n_2),m}$, $p^{\text{SSP}}_{0,l}$ (where $(n_1, n_2) \in L_p$, $1 \le m \le |I(n_1, n_2)|$ and $1 \le l \le |I_0|$) s.t.

$$\begin{aligned}\boldsymbol{X}_s = \pi_{(\text{PT},\text{PR})} &\sum_{m=1}^{|I(\text{PT},\text{PR})|} p^{\text{SSP}}_{(\text{PT},\text{PR}),m} \boldsymbol{\nu}_{(\text{PT},\text{PR}),m} \\ + \sum_{j=1}^{R} \{ (\pi_{(\text{PT},\text{ST}_j)} &\sum_{m=1}^{|(I(\text{PT},\text{ST}_j)|} p^{\text{SSP}}_{(\text{PT},\text{ST}_j),m} \boldsymbol{\nu}_{(\text{PT},\text{ST}_j),m}) \\ + (\pi_{(\text{ST}_j,\text{PR})} &\sum_{m=1}^{|(I(\text{ST}_j,\text{PR})|} p^{\text{SSP}}_{(\text{ST}_j,\text{PR}),m} \boldsymbol{\nu}_{(\text{ST}_j,\text{PR}),m}) \} + \pi_0 \sum_{l=1}^{|I_0|} p^{\text{SSP}}_{0,l} \boldsymbol{\nu}_{0,l} \end{aligned} \tag{26}$$

$$\sum_{m=1}^{|I(\text{PT},\text{PR})|} p^{\text{SSP}}_{(\text{PT},\text{PR}),m} = 1, \quad \sum_{l=1}^{|I_0|} p^{\text{SSP}}_{0,l} = 1 \tag{27}$$

$$\sum_{m=1}^{|I(\text{PT},\text{ST}_j)|} p^{\text{SSP}}_{(\text{PT},\text{ST}_j),m} = 1, \quad \sum_{m=1}^{|I(\text{ST}_j,\text{PR})|} p^{\text{SSP}}_{(\text{ST}_j,\text{PR}),m} = 1, \quad \forall 1 \le j \le R. \tag{28}$$

The policy SSP performs the following steps in each frame:

1) *Cooperative Relay Decision for Primary Packets:* Transmit all primary packets with inter-arrival time of $k_1 + 1$ and k_1 slots directly with probability $\pi^{(1)}_{(\text{PT},\text{PR})}$ and $\pi^{(2)}_{(\text{PT},\text{PR})}$ respectively, or transmit them via ST_j with probability $\pi^{(1)}_{(\text{PT},\text{ST}_j)}$ and $\pi^{(2)}_{(\text{PT},\text{ST}_j)}$ respectively.
2) *Secondary Packet Scheduling in Busy Slots:* At any slot t in which a primary packet is being transmitted from n_1 to n_2 use $\boldsymbol{\nu}_{(n_1,n_2),m}$ as the vector $\boldsymbol{\mu}_s(t)$ with probability $p^{\text{SSP}}_{(n_1,n_2),m}$.
3) *Secondary Packet Scheduling in Idle Slots:* At any idle slot t use $\boldsymbol{\nu}_{0,l}$ as the vector $\boldsymbol{\mu}_s(t)$ with probability $p^{\text{SSP}}_{0,l}$.
4) *Queue Update:* Update the queues as in SCRP.

We show that for any $\lambda_p \le f_R$, SSP stabilizes the network for all $\boldsymbol{\lambda}_s$ in the region $\Lambda(\lambda_p)$.

Lemma 1: Under the SSP policy for any $\lambda_p \leq f_R$ and $\boldsymbol{\lambda}_s \in \Lambda(\lambda_p)$ the queue length at PT is bounded and for all $r = 1, 2, \ldots$, we have

$$E[\sum_{\tau=t_0+1+(r-1)N}^{t_0+rN} \mu_i^{\text{SSP}}(\tau)] \geq \lambda_i N \quad \forall i = 1, \ldots, S, \tag{29}$$

where $\mu_i^{\phi}(t)$ denotes the transmission rate offered to ST_i at slot t under policy ϕ.

Proof: Since M primary packets are transmitted in each frame, the queue length at PT is bounded. Since relays and $\boldsymbol{\mu}_s(.)$ vectors are selected based on a solution of (16)-(25), from (25) it follows that $E[\sum_{\tau=t_0+1+(r-1)N}^{t_0+rN} \mu_i^{\text{SSP}}(\tau)] = X_i N \geq \lambda_i N$ for every $i \in \{1, 2, \ldots, S\}$. ■

B. Alternate Scheduling Policy

Next we consider the ASP algorithm. ASP maximizes, for every strictly positive integer r, the following function in the r'th frame over the set of all scheduling policies ϕ in which the overall transmission time of every primary packet is less than or equal to its inter-arrival time:

$$\psi^{\phi}(t_{r,1}) \triangleq \sum_{i=1}^{S} U_i(t_{r,1}) E[\sum_{\tau=t_{r,1}}^{t_{r,1}+N-1} \mu_i^{\phi}(\tau) | \boldsymbol{U}_s(t_{r,1})], \tag{30}$$

where $t_{r,j}$ denotes the j'th slot (where j=1,..., N) in r'th frame.

Lemma 2: For any $\lambda_p \leq f_R$ and $r = 1, 2, \ldots$ we have

$$\psi^{\text{ASP}}(t_{r,1}) \geq \psi^{\text{SSP}}(t_{r,1}). \tag{31}$$

Proof: ASP maximizes $\psi^{\phi}(t_{r,1})$ among all policies ϕ in which transmission time of every primary packet is no greater than its inter-arrival time. Since SSP is one such policy therefore $\psi^{\text{ASP}}(t_{r,1}) \geq \psi^{\text{SSP}}(t_{r,1})$. ■

By comparing $\psi^{\text{ASP}}(.)$ with $\psi^{\text{SCRP}}(.)$ for each frame we observe the following.

Lemma 3: For any $\lambda_p \leq f_R$ and $r = 1, 2, \ldots$

$$\psi^{\text{SCRP}}(t_{r,1}) \geq \psi^{\text{ASP}}(t_{r,1}) - B, \tag{32}$$

where $B \geq 0$ is a finite constant.

Proof: Proof is shown in Appendix A. ■

Proof of Theorem 1: Consider $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ where $\lambda_p < f_R$. Note that under SCRP and for any slot t, if $\lambda_p < f_R$, then M packets are transmitted in each frame. Therefore $U_p(t) \leq M$ and the queue length process U_p is strongly stable.

We define $Z_n(r) \triangleq U_n(t_{r,1})$ for each $n \in \{1, \ldots, S\}$ and $r = 1, 2, \ldots$. We denote the vector $(Z_1(r), \ldots, Z_S(r))^T$ as $\boldsymbol{Z}_s(r)$. We define a Lyapunov function $V(\boldsymbol{Z}_s(r)) \triangleq \sum_{n=1}^{S} Z_n^2(r)$ and its conditional drift $\triangle(r)$ as

$$\triangle(r) \triangleq E[V(\boldsymbol{Z}_s(r+1)) - V(\boldsymbol{Z}_s(r)) | \boldsymbol{Z}_s(r)]. \tag{33}$$

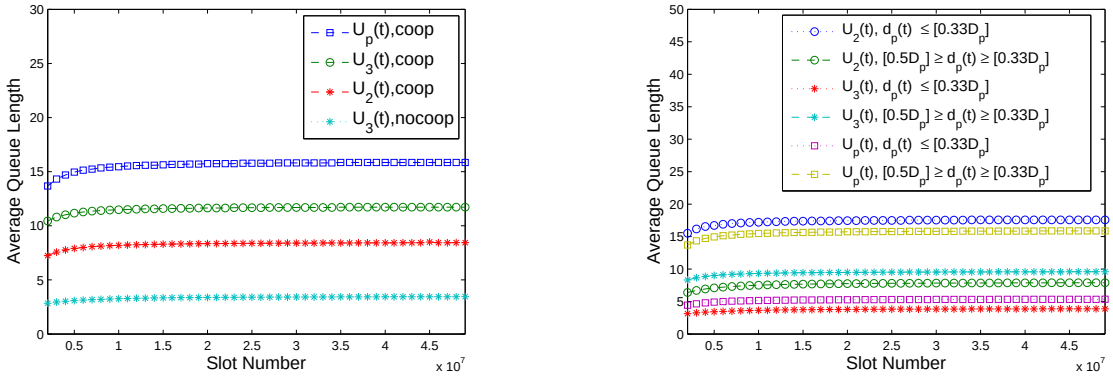
Now for every $n \in \{1, 2, \ldots, S\}$,

$$\begin{aligned} U_n(t_{r,1} + N) \leq \max[U_n(t_{r,1}) - \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SCRP}}(\tau), 0] \\ + \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau). \end{aligned} \tag{34}$$

Since in every slot the queue length for secondary packets at each secondary transmitter increases or decreases by at most 1,

$$\begin{aligned} \triangle(r) &\leq SN^2(1+1) \\ &-2\sum_{n=1}^{S} Z_n(r) E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SCRP}}(\tau) - A_n(\tau) | \boldsymbol{Z}_s(r)] \end{aligned} \tag{35}$$

$$\begin{aligned} &\leq 2SN^2 + 2B - 2E[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{ASP}}(\tau) | \boldsymbol{Z}_s(r)] \\ &+ \sum_{n=1}^{S} Z_n(r) 2E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau)] \end{aligned} \tag{36}$$



(a) Constant primary data arrival process

(b) Fluctuating primary data arrival process

Fig. 4. Plot of time-averaged queue lengths of PT, ST_2 and ST_3 versus number of time slots under SCRP with constant and fluctuating primary data arrival process for a network with one relay. Fig. 4a plots time-averaged queue lengths under SCRP and a non-cooperative policy when λ_p is 0.4167 and $\boldsymbol{\lambda}_s$ is $(0,0.2,0.2)^T$. Fig. 4b plots time-averaged queue lengths under SCRP when $d_p(t)$ is between 0 and $[\frac{D_p}{3}]$,$\boldsymbol{\lambda}_s$ =$(0,0.6,0.2)^T$; $d_p(t)$ is between $[\frac{D_p}{3}]$ and $[\frac{D_p}{2}]$, $\boldsymbol{\lambda}_s$ =$(0,0.2,0.2)^T$.

$$\leq 2SN^2 + 2B - 2E[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SSP}}(\tau)|\boldsymbol{Z}_s(r)]$$
$$+ \sum_{n=1}^{S} Z_n(r) 2E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau)] \tag{37}$$
$$\leq 2SN^2 + 2B - \sum_{n=1}^{S} 2N\epsilon Z_n(r) \tag{38}$$

where $\epsilon > 0$ is a constant s.t. $(\lambda_1 + \epsilon, \ldots, \lambda_S + \epsilon)^T \in \Lambda(\lambda_p)$. Inequalities (36), (37) and (38) follow from Lemma 3, 2 and 1 respectively. Hence by Theorem 4.1 of [20] the process Z_n is strongly stable for every $n = 1, \ldots, S$. Since N is finite, each process U_n is strongly stable. ■

VI. Primary Data Arrival Process with Fluctuations

In this section we extend the stability analysis to a case where the amount of primary data that arrives in each slot is not constant but has bounded fluctuations around a fixed number. In particular, we consider the primary data arrival process d_p that satisfies the following:

1) $d_p(t)$ either lies in the interval $(\frac{D_p}{k+1}, \frac{D_p}{k})$ for each slot t, where k is an integer such that $f_0 \leq \frac{1}{k+1} < \frac{1}{k} \leq f_R$, or lies in the interval $(0, f_0 D_p)$ for each slot t.
2) Time-averaged value of $d_p(t)$ converges to a rational number $\lambda_p D_p$ i.e., for any t and strictly positive ϵ there exists a finite integer N_0 s.t. for all $n > N_0$ we have
$$\left| \frac{\sum_{\tau=t}^{t+n} d_p(\tau)}{D_p n} - \lambda_p \right| < \epsilon \tag{39}$$
This property is satisfied almost surely, by strong law of large numbers, when $d_p(t)$ variables are iid with expected mean $D_p \lambda_p$.

We claim that, for a certain set of λ_p, the set of supportable secondary arrival rate vectors under SCRP, is same as that of a system with constant primary data arrival rate of $D_p \lambda_p$ bits per slot:

Lemma 4: SCRP stabilizes the network for any $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ where λ_p is a rational number in the interval $(0, f_0)$ or $(\frac{1}{k+1}, \frac{1}{k})$, with k being any integer such that $f_0 \leq \frac{1}{k+1} < \frac{1}{k} \leq f_R$.

Proof: Due to lack of space, we provide a sketch of the proof in Appendix B and the detailed proof in our technical report [25]. ■

VII. Simulations

We validate the performance of the SCRP algorithm via simulations in C-programming language.

A. Performance of SCRP With Constant and Fluctuating Primary Data Arrival Process

In Fig. 4 we observe the performance of the SCRP algorithm for two cases: one in which the primary data-arrival process d_p has fluctuations and one in which it does not. We consider the network depicted in Fig. 1 with $K_{(\text{PT,PR})}$ as 3 and $K_{(\text{PT,ST}_1)} = K_{(\text{ST}_1,\text{PR})}$ as 1. In Fig. 4 we set λ_1 as 0 and plot time-averaged queue lengths of PT, ST_2 and ST_3, starting at slot number 2000000, for a single run; simulation runtime is 50000000 slots.

In Fig. 4a we compare the performance of the SCRP algorithm with that of a non-cooperative policy when the process d_p does not have any fluctuations. The non-cooperative policy is based on the work in [26] wherein a backpressure-type scheduling is performed at secondary transmitters when PT is not transmitting. This policy is throughput-optimal for the secondary network among the set of all non-cooperative policies. In Fig. 4a we plot the time-averaged queue lengths under SCRP and the non-cooperative policy with λ_p and $\boldsymbol{\lambda}_s$ as 0.4167 and $(0, 0.2, 0.2)^T$ respectively. Those values of λ_p and $\boldsymbol{\lambda}_s$ correspond to a case where the queues at PT and ST_2 are both unstable without cooperation, but $\boldsymbol{\lambda}_s \in \Lambda(\lambda_p)$. In Fig. 4a we do not plot for PT and ST_2 under the non-cooperative policy because their queue length were observed to be very high (greater than 100000) at slot number 2000000 and increasing monotonically with time. All queue lengths are bounded in Fig.4a under SCRP. This demonstrates that the network is unstable under the non-cooperative policy but is stable under SCRP for the selected values of λ_p and $\boldsymbol{\lambda}_s$.

Next in Fig. 4b we observe the performance of the SCRP algorithm when the primary data-arrival process d_p has fluctuations. In Fig. 4b we plot the time-averaged queue lengths versus slot under SCRP for two cases. In the first case, $d_p(t)$ is any integer between 0 and $[\frac{D_p}{3}]$ for every slot t and $\boldsymbol{\lambda}_s$ is $(0,0.6,0.2)^T$; in the second case, $\text{d}_p(t)$ is any integer between $[\frac{D_p}{3}]$ and $[\frac{D_p}{2}]$ for every t and $\boldsymbol{\lambda}_s$ is $(0,0.2,0.2)^T$. For any real x, the function $[x]$ denotes the largest integer not greater than x. In both cases, $d_p(t)$ is an iid uniformly distributed random variable for each t. We select D_p as 8192 representing a packet of size 1KB. We observe that for those network parameters, the network is stable under SCRP even for a variable primary data arrival process.

B. Comparison of the Performance of SCRP With That of a Greedy Scheduling Policy

In Fig. 5 we compare the performance of the SCRP algorithm with that of a greedy scheduling algorithm and the non-cooperative scheduling algorithm mentioned in Section VII-A. The greedy algorithm differs from the SCRP algorithm in the calculation of $\boldsymbol{\mu}_s(.)$ vectors. At any slot, the greedy algorithm obtains an offered secondary transmission rate vector by first selecting the secondary node with highest secondary queue length that can transmit, followed by the secondary node with next highest queue length that can simultaneously transmit and so on.

For the plot in Fig. 5 we consider a network with $S = 12$ and $R = 2$. We select $K_{(\text{PT, PR})} = 4$ and $K_{(\text{PT,ST}_1)} = K_{(\text{PT,ST}_2)} = K_{(\text{ST}_1,\text{PR})} = K_{(\text{ST}_2,\text{PR})} = 1$. The interference model is as follows. None of the secondary nodes can transmit when PT is transmitting. When PT does not transmit, any secondary node ST_i (where $i \in \{1, \ldots, 12\}$) can transmit a packet if and only if neither $\text{ST}_{i'}$ nor $\text{ST}_{i''}$ simultaneously transmits a packet, where $i' = (i-1) \mod 12$ and $i'' = (i+1) \mod 12$. We select this network as for this secondary network, when λ_p is zero, greedy scheduling policies achieve lower stability region than the back-pressure policy [27].

In Fig. 5 we plot the average length of any queue in the network, averaged over all queues as well as simulation runtime, under SCRP, greedy and the non-cooperative algorithm when $\boldsymbol{\lambda}_s$=$(0.09, 0.09, 0.09, 0.49, 0.09, 0.49, 0.09, 0.49, 0.09, 0.49, 0.09, 0.49)^T$ and λ_p is varied from 0 to 0.4. Each simulation in Fig. 5 is run for 10000000 slots. We observe that the greedy algorithm performs as well as the SCRP algorithm when $\lambda_p \leq 0.3$. The network is unstable under the greedy algorithm for $\lambda_p = 0.4$ and the non-cooperative algorithm for $\lambda_p \geq 0.2$.

VIII. Conclusion

In this work we studied opportunistic cooperation in a spectrum sharing network where some nodes may benefit from cooperative relaying while others may suffer loss of transmission of opportunities. For a deterministic periodic primary packet arrival process we develop a scheduling algorithm using Lyapunov drift techniques that balances the trade off between cooperation and network stability. We exploit the

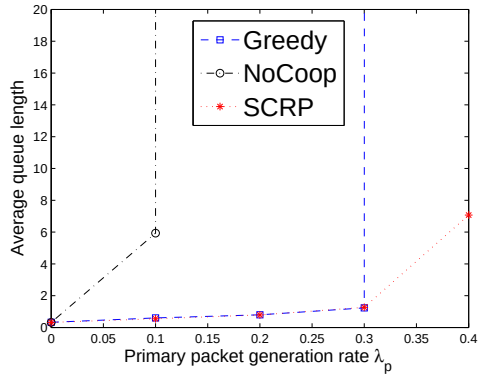


Fig. 5. Plot of average queue lengths in a network versus primary packet generation rate under SCRP, a non-cooperative algorithm and a greedy algorithm. The network consists of 1 primary s-d pair, 12 secondary s-d pairs and 2 relay nodes.

periodicity of the primary packet arrival process to perform stability analysis even for primary packet generation rates greater than what is supportable without relays. Interesting research possibilities would be to extend this analysis to networks where the service-time of packets are stochastic and cases involving multiple primary s-d pairs.

APPENDIX A
PROOF OF LEMMA 3

Proof: For any two integers h_1 and h_2 we define δ_{h_1,h_2} as $\delta_{h_1,h_2} \triangleq |h_1 - h_2|$. Since arrival rate and maximum transmission rate of secondary packets per slot per node are both no greater than 1, for any two slots t_1 and t_2 (where t_2 is greater than t_1 and $n = 1, 2, \ldots, S$),

$$|U_n(t_2) - U_n(t_1)| \quad \leq \quad \delta_{t_1,t_2} \tag{40}$$

Consider any λ_p s.t. $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_R$ (where k_1 is a positive integer). We index primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots generated in r'th frame as $x_{r,1}, x_{r,2}, \ldots, x_{r,\kappa^{(1)}(\lambda_p)}$ and $\grave{x}_{r,1}, \grave{x}_{r,2}, \ldots, \grave{x}_{r,\kappa^{(2)}(\lambda_p)}$ respectively. Let $w^{\phi}_{r,i}$ denote the slot when primary packet $x_{r,i}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_1} (where $1 \leq n_1 \leq R$) for which $\frac{1}{f_{n_1}} \leq k_1 + 1$, then it creates $(k_1 + 1) - \frac{1}{f_{n_1}}$ idle slots in the same frame denoted as $y^{n_1,\phi}_{r,i,1}, y^{n_1,\phi}_{r,i,2}, \ldots, y^{n_1,\phi}_{r,i,(k_1+1-\frac{1}{f_{n_1}})}$ respectively. Similarly for all $\grave{i}$ s.t. $1 \leq \grave{i} \leq \kappa^{(2)}(\lambda_p)$ let $\grave{w}^{\phi}_{r,\grave{i}}$ denote the slot when primary packet $\grave{x}_{r,\grave{i}}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_2} (where $1 \leq n_2 \leq R$) for which $\frac{1}{f_{n_2}} \leq k_1$ then it creates $k_1 - \frac{1}{f_{n_2}}$ idle slots in r'th frame denoted as $\grave{y}^{n_2,\phi}_{r,\grave{i},1}, \grave{y}^{n_2,\phi}_{r,\grave{i},2}, \ldots, \grave{y}^{n_2,\phi}_{r,\grave{i},(k_1-\frac{1}{f_{n_2}})}$ respectively. We denote idle slots created when $x_{r,i}$ is transmitted directly from PT to PR under policy ϕ as $y^{0,\phi}_{r,i,1}$, $y^{0,\phi}_{r,i,2}, \ldots, y^{0,\phi}_{r,i,(k_1+1-\frac{1}{f_0})}$ respectively.

Let $U^{\text{SCRP}}_n(\tau)$ and $U^{\text{ASP}}_n(\tau)$ (where $\tau \in [t_{r,1}, t_{r,1} + (N-1)]$) denote the transmission queue-length of ST_n at time-slot τ under SCRP and ASP respectively. Similarly we denote the vector $\boldsymbol{v}^*_0(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2)}(\tau)$ (where $(n_1, n_2) \in L_p$) obtained under policies SCRP and ASP as $\boldsymbol{v}^*_{0,\text{SCRP}}(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2),\text{SCRP}}(\tau)$ and $\boldsymbol{v}^*_{0,\text{ASP}}(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2),\text{ASP}}(\tau)$ respectively.

Next assume $x_{r,i}$ is relayed via ST_{θ_2} under ASP and by ST_{θ_1} under SCRP (where $1 \leq \theta_1, \theta_2 \leq R$ and $\frac{1}{f_{\theta_1}}, \frac{1}{f_{\theta_2}} \leq (k_1 + 1)$). For convenience, we abbreviate (PT, PR), $(\text{PT}, \text{ST}_{\theta_2})$, $(\text{ST}_{\theta_2}, \text{PR})$, $(\text{PT}, \text{ST}_{\theta_1})$, $(\text{ST}_{\theta_1}, \text{PR})$, SCRP, ASP as z_0, z_1, z_2, z_3, z_4, ϕ_1 and ϕ_2 respectively. Then we have

$$E[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu^{\phi_1}_n(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_3}-1} \sum_{n=1}^{S}(U_n^{\phi_1}(\tau)-\delta_{\tau,t_{r,1}})\mu_n^{\phi_1}(\tau)$$

$$+ \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_3}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S}(U_n^{\phi_1}(\tau)-\delta_{\tau,t_{r,1}})\mu_n^{\phi_1}(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S}(U_n^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1})$$

$$-\delta_{y_{r,i,\hat{n}}^{\theta_1,\phi_1},t_{r,1}})\mu_n^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1})|\boldsymbol{U}_s(t_{r,1})] \tag{41}$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_3}-1} \sum_{n=1}^{S} U_n^{\phi_1}(\tau)\mu_n^{\phi_1}(\tau)$$

$$+ \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_3}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S} U_n^{\phi_1}(\tau)\mu_n^{\phi_1}(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S} U_n^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1})\mu_n^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1})|\boldsymbol{U}_s(t_{r,1})] - 3N^2S \tag{42}$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_3}-1} (\boldsymbol{U}_s^{\phi_1}(\tau))^T \boldsymbol{v}_{z_3,\phi_1}^*(w_{r,i}^{\phi_1})$$

$$+ \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_3}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} (\boldsymbol{U}_s^{\phi_1}(\tau))^T \boldsymbol{v}_{z_4,\phi_1}^*(w_{r,i}^{\phi_1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} (\boldsymbol{U}_s^{\phi_1}(y_{r,i,\hat{n}}^{\theta_1,\phi_1}))^T \boldsymbol{v}_{0,\phi_1}^*(w_{r,i}^{\phi_1})|\boldsymbol{U}_s(t_{r,1})] - 3N^2S \tag{43}$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_3}-1} (\boldsymbol{U}_s^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_3,\phi_1}^*(w_{r,i}^{\phi_1})$$

$$+ \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_3}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} (\boldsymbol{U}_s{}^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_4,\phi_1}^*(w_{r,i}^{\phi_1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} (\boldsymbol{U}_s{}^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{0,\phi_1}^*(w_{r,i}^{\phi_1})|\boldsymbol{U}_s(t_{r,1})] - 6N^2S \tag{44}$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_1}-1} (\boldsymbol{U}_s^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_1,\phi_2}^*(t_{r,1})$$

$$+\sum_{\tau=w_{r,i}^{\phi_1}+K_{z_1}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_2}}-1}(\boldsymbol{U}_s^{\phi_1}((w_{r,i}^{\phi_1}))^T\boldsymbol{v}_{z_2,\phi_2}^*(t_{r,1})$$

$$+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}}(\boldsymbol{U}_s^{\phi_1}((w_{r,i}^{\phi_1}))^T\boldsymbol{v}_{0,\phi_2}^*(t_{r,1})|\boldsymbol{U}_s(t_{r,1})]-6N^2S \tag{45}$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_1}-1}(\boldsymbol{U}_s(t_{r,1}))^T\boldsymbol{v}_{z_1,\phi_2}^*(t_{r,1})$$

$$+\sum_{\tau=w_{r,i}^{\phi_1}+K_{z_1}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_2}}-1}(\boldsymbol{U}_s(t_{r,1}))^T\boldsymbol{v}_{z_2,\phi_2}^*(t_{r,1})$$

$$+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}}(\boldsymbol{U}_s(t_{r,1}))^T\boldsymbol{v}_{0,\phi_2}^*(t_{r,1})|\boldsymbol{U}_s(t_{r,1})]-9N^2S \tag{46}$$

$$=E[\sum_{\tau=w_{r,i}^{\phi_2}}^{w_{r,i}^{\phi_2}+\frac{1}{f_{\theta_2}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_2}(\tau)$$

$$+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_2}(y_{r,i,\hat{n}}^{\theta_2,\phi_2})|\boldsymbol{U}_s(t_{r,1})]-9N^2S. \tag{47}$$

Inequality (41) follows from (40). Inequality (42) follows because each of k_1+1, K_{z_3}, K_{z_4} and $\delta_{\tau,t_{r,1}}$ is less than N. Inequalities (43), (45) follows from the definition of SCRP: at any slot SCRP selects transmission-rate vectors based on instantenous queue-lengths. Inequalities (44) and (46) follows from (40) and because $k_1+1, K_{z_3}, K_{z_4}, K_{z_1}, K_{z_2}, \delta_{\tau,t_{r,1}}$ is less than N. Repeating the above analysis for cases where either SCRP or ASP directly transmit $x_{r,i}$ to PR we obtain

$$E[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{i'}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_1}(\tau)$$

$$+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{i'}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_1}(y_{r,i,\hat{n}}^{i',\phi_1})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq E[\sum_{\tau=w_{r,i}^{\phi_2}}^{w_{r,i}^{\phi_2}+\frac{1}{f_{i''}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_2}(\tau)$$

$$+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{i''}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_2}(y_{r,i,\hat{n}}^{i'',\phi_2})|\boldsymbol{U}_s(t_{r,1})]-B_b, \tag{48}$$

where $B_b>0$ is a finite constant and i', i'' are some non-negative integer less than R s.t. $\frac{1}{f_{i'}}$ and $\frac{1}{f_{i''}}$ are the transmission time of $x_{r,i}$ under SCRP and ASP respectively. Repeating the above procedure for $\grave{x}_{r,\grave{i}}$s and assuming packet $\grave{x}_{r,\grave{i}}$ is transmitted with transmission time of $\frac{1}{f_{\grave{\theta}_2}}$ and $\frac{1}{f_{\grave{\theta}_1}}$ slots under ASP and SCRP respectively (where $0\leq\grave{\theta}_2,\grave{\theta}_1\leq R$ and $\frac{1}{f_{\grave{\theta}_2}},\frac{1}{f_{\grave{\theta}_1}}\leq k_1$) we obtain

$$E[\sum_{\tau=\grave{w}_{r,\grave{i}}^{\phi_1}}^{\grave{w}_{r,\grave{i}}^{\phi_1}+\frac{1}{f_{\grave{\theta}_1}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu_n^{\phi_1}(\tau)$$

$$+ \sum_{\hat{n}=1}^{k_1-\frac{1}{f_{\grave{\theta}_1}}} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\phi_1}(\hat{y}_{r,\grave{i},\hat{n}}^{\grave{\theta}_1,\phi_1})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq E[\sum_{\tau=\grave{w}_{r,\grave{i}}^{\phi_2}}^{\grave{w}_{r,\grave{i}}^{\phi_2}+\frac{1}{f_{\grave{\theta}_2}}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\phi_2}(\tau)$$

$$+ \sum_{\hat{n}=1}^{k_1-\frac{1}{f_{\grave{\theta}_2}}} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\phi_2}(\hat{y}_{r,\grave{i},\hat{n}}^{\grave{\theta}_2,\phi_2})|\boldsymbol{U}_s(t_{r,1})] - B_c, \tag{49}$$

where $B_c > 0$ is a finite constant. Adding the results from (47), (48) and (49) for all $x_{r,i}$ and $\grave{x}_{r,\grave{i}}$ we obtain

$$\psi^{\text{SCRP}}(t_{r,1}) \geq \psi^{\text{ASP}}(t_{r,1}) - B \tag{50}$$

where $B = M\max(B_b, B_c, 9N^2S) > 0$ is a finite constant. ■

APPENDIX B
SKETCH OF THE PROOF OF LEMMA 4

We prove Lemma 4 only for the case when λ_p is a rational number in the interval $(\frac{1}{k+1}, \frac{1}{k})$, with k being any integer such that $f_0 \leq \frac{1}{k+1} < \frac{1}{k} \leq f_R$; the proof for $\lambda_p \in (0, f_0)$ can be obtained similarly. The proof requires results stated in Lemma 5, 6, 7 and 8 respectively whose proofs are provided in [25]. In [25] we also provide an outline of the proof for $\lambda_p \in (0, f_0)$.

Lemma 5: If $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ for some rational $\lambda_p \in (\frac{1}{k+1}, \frac{1}{k})$, where k is any integer such that $f_0 \leq \frac{1}{k+1} < \frac{1}{k} \leq f_R$, then there exists a rational number, $\hat{\lambda}_p > \lambda_p$ s.t. $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\hat{\lambda}_p))$ and $\hat{\lambda}_p \in (\frac{1}{k+1}, \frac{1}{k})$.

We refer to the system with fluctuating primary data-arrival process $d_p(t)$ as System 1 and that with constant primary data arrival rate of $\hat{\lambda}_p D_p$ bits per slot as System 2.

Lemma 6: For any finite positive number n_3 there exists finite positive constants Y_1, Y_2 s.t. number of primary packet arrivals with inter-arrival time of k slots during slots [0, $Y_2Y_1N(\hat{\lambda}_p)$] in System 2 is greater than that in System 1 by at least n_3.

We define ϵ_2 as $\epsilon_2 = \frac{\hat{\lambda}_p - \lambda_p}{2}$. We partition the time-line into a collection of frames of length $Y_2Y_1N(\hat{\lambda}_p)$ each with the first frame beginning at slot $[\frac{1}{\hat{\lambda}_p}]$. Such frames are renewal frames for System 2 but not for System 1. Assume SCRP and ASP are employed as scheduling policies for System 1 and 2 respectively. Using Lemma 6 we show the following.

Lemma 7: Consider System 1. For any Y_1, Y_2 s.t. $Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than $(l_1+5) + \left[\frac{l_1-1}{k}\right]$, where l_1 is a finite positive integer, and Y_2 is a multiple of k, there exists at least l_1 primary packets, in every frame, with inter-arrival time of $k+1$ slots s.t. :

1) transmission of all packets begin and end in that frame and,
2) if any of those packets is relayed via ST$_i$, for any $i \in \{1, \ldots, R\}$ s.t. $k+1 \geq \frac{1}{f_i}$, it creates $k+1-\frac{1}{f_i}$ idle slots within that frame.

In the rest of the paper we assume Y_1, Y_2 is s.t. $Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than $2k+5+[\frac{2k-1}{k}]$ and Y_2 is a multiple of k. Hence there exists at least $2k$ primary packets with inter-arrival time of $k+1$ slots in System 1 in each frame that satisfies the conditions stated in Lemma 7. Let $\mu_n^{\text{SYS1}}(\tau)$ and $\mu_n^{\text{SYS2}}(\tau)$ denote the transmission rate offered to ST$_n$ in System 1 under policy SCRP and in System 2 under policy ASP respectively.

Lemma 8: For every $r = 1, \ldots,$ the value of the utility function $\psi^{\text{SCRP}}(t_{r,1})$ in System 1 is greater than $\psi^{\text{ASP}}(t_{r,1})$ in System 2 minus a finite positive constant, i.e.,

$$E[\sum_{\tau=t_{r,1}}^{t_{r+1,1}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq E[\sum_{\tau=t_{r,1}}^{t_{r+1,1}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS2}}(\tau)|\boldsymbol{U}_s(t_{r,1})] - \hat{B} \tag{51}$$

where $\hat{B} > 0$ is a finite constant.

Proof of Lemma 4: Since every primary packet is transmitted in at most as many slots as its inter-arrival time, queue at PT is strongly stable. For $n = 1, 2, ..., S$ define $Z_n(r) \triangleq U_n(t_{r,1})$ (where $r = 1, 2, \ldots$). We denote the vector $(Z_1(r), Z_2(r)\ , \ldots, Z_S(r))^T$ as $\boldsymbol{Z}_s(r)$. We define a Lyapunov function $V(\boldsymbol{Z}_s(r)) \triangleq \sum_{n=1}^{S} Z_n^2(r)$ with its conditional drift $\triangle(r)$ as in (33). Then we can show, using Lemma 8, 5, 1 and 2 and following similar steps as in (35)- (38), that

$$\triangle(r) \leq 2S(Y_2 Y_1 N(\hat{\lambda}_p))^2 + 2\hat{B} - 2\epsilon_3 \sum_{n=1}^{S} Z_n(r) \tag{52}$$

where $\epsilon_3 > 0$ is a finite constant. Therefore by Theorem 4.1 of [20] the secondary queue length processes are strongly stable. ■

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REFERENCES

[1] I. Maric, R. Yates, and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3536–3548, 2007.

[2] I. Marić, A. Goldsmith, and G. Kramer, "On the capacity of interference channels with one cooperating transmitter," *European Transactions on Telecommunications*, vol. 19, no. 4, pp. 405–420, 2008.

[3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.

[4] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 203–213, 2008.

[5] I. Stanojev and A. Yener, "Facilitating flexible multihop communication via spectrum leasing," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2012 IEEE 23rd International Symposium on*, 2012, pp. 2160–2165.

[6] J. Zhang and Q. Zhang, "Stackelberg game for utility-based cooperative cognitiveradio networks," in *Proceedings of the tenth ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2009, pp. 23–32.

[7] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," *IEEE Trans. Commun.*, vol. 55, no. 12, pp. 2351–2360, 2007.

[8] I. Krikidis, N. Devroye, and J. Thompson, "Stability analysis for cognitive radio with multi-access primary transmission," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 72–77, 2010.

[9] A. Fanous and A. Ephremides, "Stable throughput in a cognitive wireless network," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 3, pp. 523–533, 2013.

[10] A. El-Sherif, A. Sadek, and K. Liu, "Opportunistic multiple access for cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 704–715, 2011.

[11] R. Urgaonkar and M. Neely, "Opportunistic cooperation in cognitive femtocell networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 607–616, 2012.

[12] E. Matskani, N. Chatzidiamantis, L. Georgiadis, I. Koutsopoulos, and L. Tassiulas, "Optimal primary-secondary user cooperation policies in cognitive radio networks," *arXiv preprint arXiv:1307.5613*, 2013.

[13] E. Hossain, M. Rasti, H. Tabassum, and A. Abdelnasser, "Evolution towards 5g multi-tier cellular wireless networks: An interference management perspective," *arXiv preprint arXiv:1401.5530*, 2014.

[14] X. Y. Wang, A. Wong, and P.-H. Ho, "Stochastic medium access for cognitive radio ad hoc networks," *IEEE J. on Sel. Areas Commun.*, vol. 29, no. 4, pp. 770–783, 2011.

[15] A. Kumar, D. Manjunath, and J. Kuri, "Communication networking: an analytical approach," 2004.

[16] A. Ukhanova, E. Belyaev, L. Wang, and S. Forchhammer, "Power consumption analysis of constant bit rate video transmission over 3g networks," *Computer Communications*, vol. 35, no. 14, pp. 1695–1706, 2012.

[17] Ericsson, "Ericsson mobility report, on the pulse of the networked society," *Computer Communications*, vol. 35, no. 14, pp. 1695–1706, 2012.

[18] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, Mar 2000.

[19] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936–1948, 1992.

[20] M. J. Neely, "Stochastic network optimization with application to communication and queueing systems," *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, pp. 1–211, 2010.

[21] C. Joo, X. Lin, and N. B. Shroff, "Understanding the capacity region of the greedy maximal scheduling algorithm in multihop wireless networks," *IEEE/ACM Transactions on Networking (TON)*, vol. 17, no. 4, pp. 1132–1145, 2009.

[22] C. Joo and N. B. Shroff, "Local greedy approximation for scheduling in multihop wireless networks," *IEEE Transactions on Mobile Computing*, vol. 11, no. 3, pp. 414–426, 2012.

[23] M. J. Neely, E. Modiano, and C. E. Rohrs, "Dynamic power allocation and routing for time-varying wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 1, pp. 89–103, 2005.
[24] E. Oks and M. Sharir, "Minkowski sums of monotone and general simple polygons," *Discrete & Computational Geometry*, vol. 35, no. 2, pp. 223–240, 2006.
[25] D. Das, A. Abouzeid, and M. Codreanu, "Network-layer scheduling and relaying in cooperative spectrum sharing networks," 2014. [Online]. Available: http://www.ecse.rpi.edu/homepages/abouzeid/preprints/2014stabrel.pdf
[26] R. Urgaonkar and M. J. Neely, "Opportunistic scheduling with reliability guarantees in cognitive radio networks," *IEEE Transactions on Mobile Computing*, vol. 8, no. 6, pp. 766–777, 2009.
[27] B. Birand, M. Chudnovsky, B. Ries, P. Seymour, G. Zussman, and Y. Zwols, "Analyzing the performance of greedy maximal scheduling via local pooling and graph theory," *IEEE/ACM Trans. Networking*, vol. 20, no. 1, pp. 163–176, 2012.

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