ADAPTIVE: A Dynamic Index Auction for Spectrum Sharing with Time-Evolving Values

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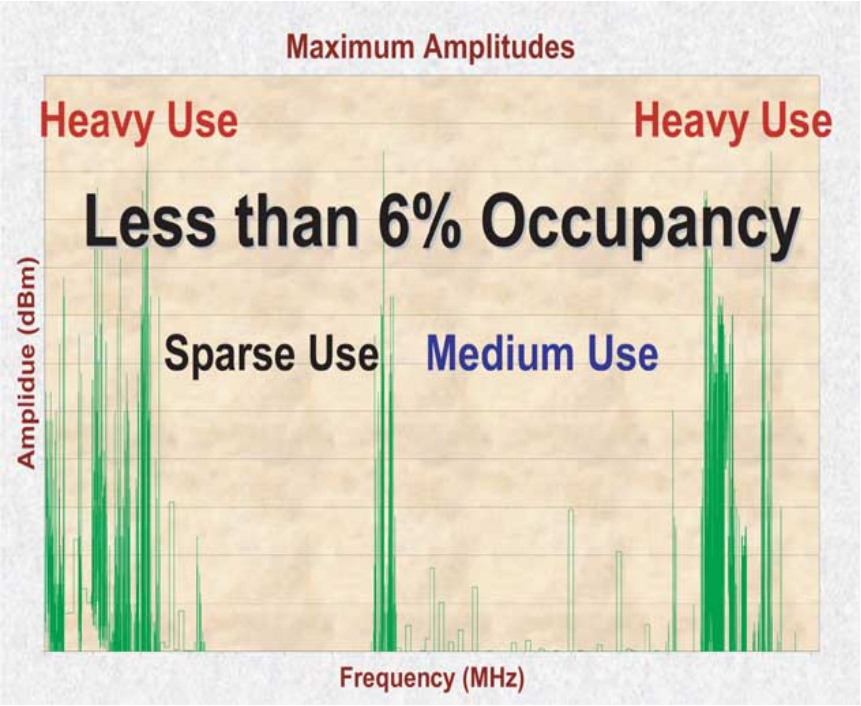
Wireless Spectrum

- Refers to the part of the electromagnetic spectrum corresponding to radio frequencies $<$ around 300 GHz
- Spectrum is a finite and valuable resource
  - only 50 MHz remain un-assigned
  - The Federal Communications Commission (FCC) auctions for about 4% of US spectrum raised $78 billion since 1994.
  - Indirect value of radio spectrum: 5-10% of US economy ($\sim$ 1.4 trillion/year)

Spectrum Utilization

- FCC reports that many of the allocated spectrum bands are idle most of the times or not used in some areas.



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Dynamic Spectrum Sharing

- A promising approach to improve spectrum utilization
- Realized by cognitive radio technology
  - A radio that can change its transmitter parameters according to the interactions with the environment in which it operates.
- Unlicensed secondary users (SU) are allowed to utilize the radio spectrum owned by a primary owner (PO).

Auction-Based Spectrum Sharing

- Why Auctions?
  - The seller is not assumed to know any prior information about the valuation of items to the buyers
  - Auctions can be designed to maximize buyers' valuations.
  - Requires minimum interaction between seller and buyers.

Spectrum Auctions

- In the simplest form of a spectrum auction
  - There is a set of SUs (buyers) who bid to obtain channel access
  - A PO (auctioneer) who collects these bids and determines the winner (or winners) and payments
- Two components of every auction:
  - the allocation rule
  - the payment rule
- Main objectives:
  - revenue maximization (optimality) for the auctioneer
  - social welfare maximization (efficiency)

ADAPTIVE, a Dynamic Spectrum Auction

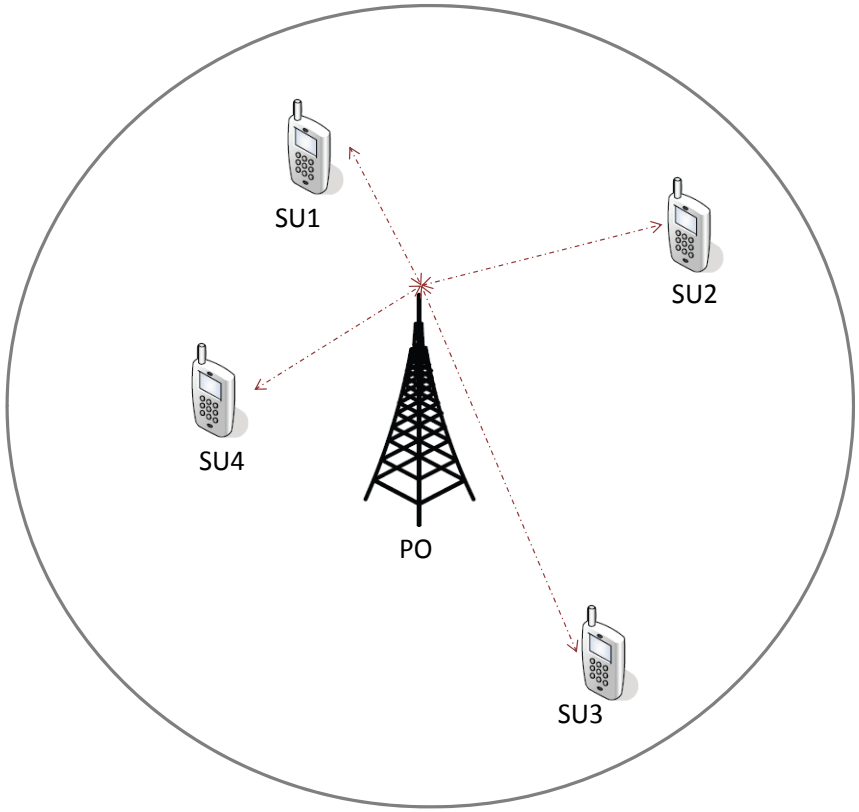
- Existing spectrum auctions assume that SUs have static and known values for the channels.
  - In reality, however, SUs do not know the exact value of channel access a priori, but they learn it over time.
- Here, we study spectrum auctions in a dynamic setting where SUs can change their valuations based on their experiences with the channel.

ADAPTIVE, a Dynamic Spectrum Auction

- We propose ADAPTIVE, a dynAmic inDex Auction for sPectrum sharing with TIme-evolving ValuEs, that
  - maximizes the social welfare
  - has desired economic properties

Network Model

- The PO (a base station or an access point) is willing to auction its idle channel to the SUs.



System Model

- In the ADAPTIVE mechanism:
  - PO is the auctioneer
  - SUs are the bidders
  - Channel is the auctioned item
  - θ_i denotes the type of SU i
    - A real number reflecting monetary value of channel access for SU i
    - Captures the urgency for channel access
  - $e_{i,t}$ denotes SU i's experience at time t
    - we consider SU's experience as SNR of the channel
    - SU's experience evolves only when he gets the channel, otherwise its experience does not change
    - An SU's experience at the instants that it gets the channel evolves in a Markovian model

System Model (Cont'd)

- SU's valuation for the channel is a stationary function of its type and experience:

$$v(\theta_i, e_{i,t}) = \theta_i \; B \log(1 + e_{i,t})$$

  Where B is the channel bandwidth.

- The function v takes into account both the channel quality experienced by SUs and SU's monetary value that reflects urgency for channel access.

The ADAPTIVE Mechanism

- At each time step, SUs report $(\theta_i, e_{i,t})$ to the PO who determines two outputs:
  - The channel allocation denoted by Q that contains $q_{i,t} \in \{0, 1\}$ determining the winner at time t
  - The payment of SU i at time t denoted by $p_{i,t}$

- The expected future social welfare at time t can be defined as:

$$S(\theta, e_t) = \max_{Q \in \mathbb{Q}} \mathbb{E}\left[\sum_{t'=t}^{\infty} \sum_{i} \delta^{t'-t} \; q_{i,t'} \; v(\theta_i, e_{i,t'}) \middle| \theta, e_t \right]$$

Where $0 < \delta < 1$ is the common discount factor.

Efficient Allocation Policy of ADAPTIVE

- We cast the channel allocation problem into a *multi-armed bandit* problem
- In a multi-armed bandit problem, there is an operator that chooses to operate exactly one of the machines at each time step. The chosen machine generates a reward and updates its state. The operator's objective is to maximize the sum of rewards.

Efficient Allocation Policy of ADAPTIVE (Cont'd)

- The channel allocation problem in ADAPTIVE can be transformed into a multi-armed bandit problem.
  - an SU $\rightarrow$ an arm in the bandit model
  - SUs' valuations $\rightarrow$ rewards generated by pulling arms
  - Allocating the channel to an SU $\rightarrow$ pulling an arm
  - experience update of the winning SU $\rightarrow$ State change in the bandit model
- Now, we can use the Gittins index policy to solve the efficient allocation problem

Efficient Allocation Policy of ADAPTIVE (Cont'd)

- The PO gives the channel to the SU with the highest Gittins index:

$$G_i(\theta_i, e_{i,t}) = \max_{\tau_i} \mathbb{E}\left[\frac{\sum_{t'=t}^{\tau_i} \delta^{t'-t}\, v(\theta_i, e_{i,t'})}{\sum_{t'=t}^{\tau_i} \delta^{t'-t}} \middle| \theta_i, e_{i,t}\right]$$

- Gittins index of each SU can be computed independently in polynomial time.

The Payment Rule

- We specify payments such that each SU's utility coincides with its marginal contribution to the social welfare
- The winning SU i at time t pays:

$$p_{i,t} = (1 - \delta)\, S_{-i}(\theta, e_t)$$

Where $S_{-i}(\theta, e_t)$ is the expected future social welfare without SU i

Economic Properties

- The ADAPTIVE mechanism has the following economic properties:
  - *Periodic Ex Post Incentive Compatibility*; for every bidder and at any time, truth-telling is the best response to the truthfulness of the other bidders.
  - *Periodic Ex Post Individual Rationality*; bidders do not suffer as a result of participating in the auction.

Numerical Results

- We compare the performance of ADAPTIVE which is a dynamic valuation auction with the well-known Vickrey auction (also called second price auction) as the representative of static auctions.
- We set the common discount factor, δ, to 0.7 and change the number of SUs from 3 to 21. Each setting is run 500 times in MATLAB.

Numerical Results

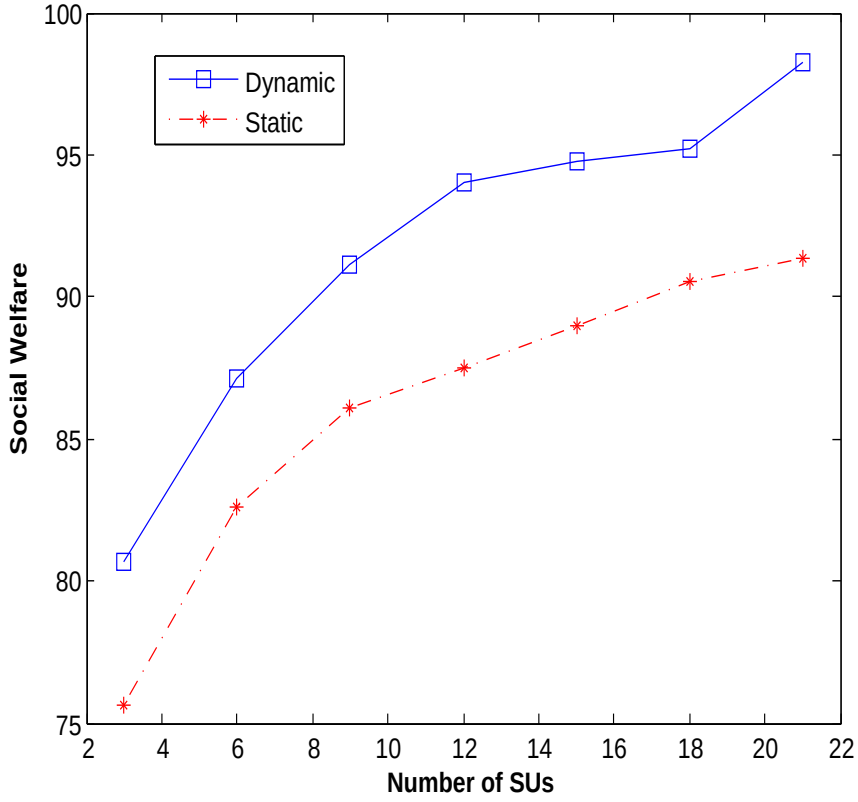


Figure : Social welfare Vs the number of SUs.

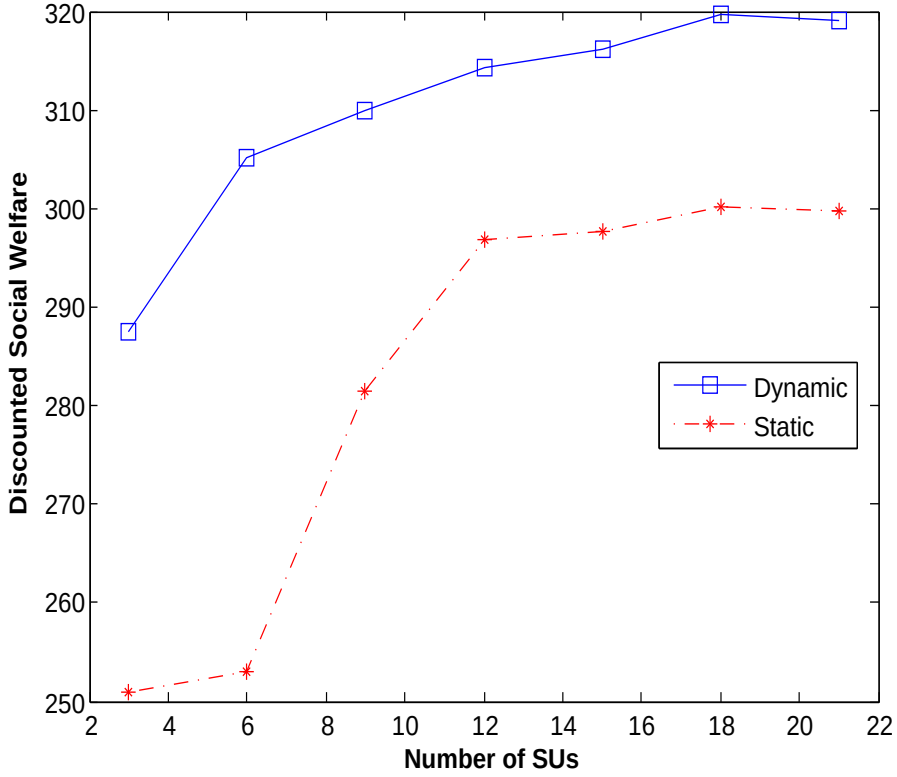


Figure : Discounted social welfare Vs the number of SUs.

Numerical Results

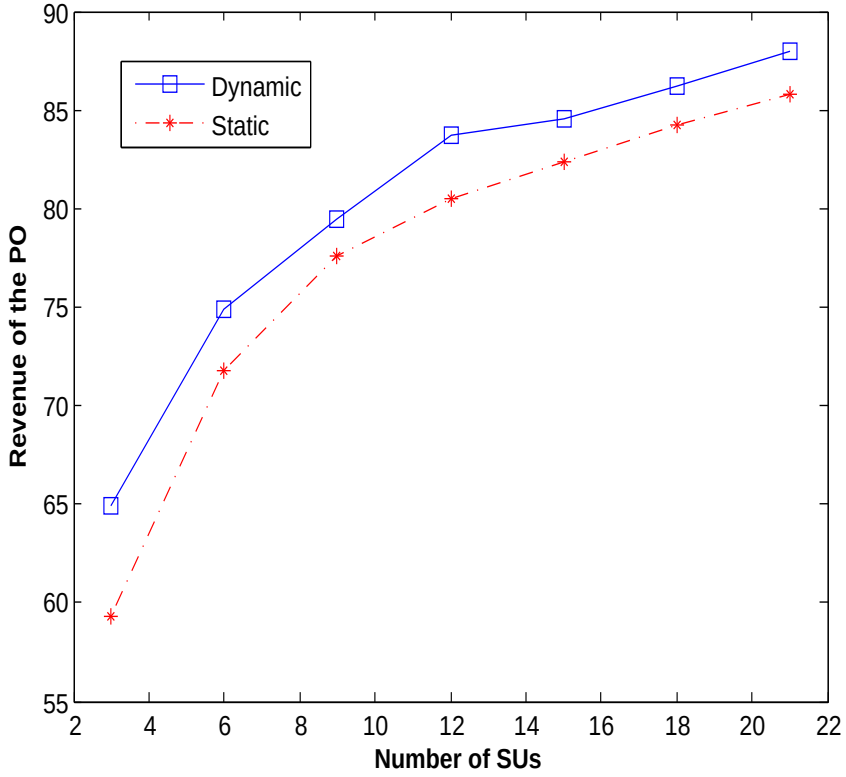


Figure : Revenue of the PO Vs the number of SUs.

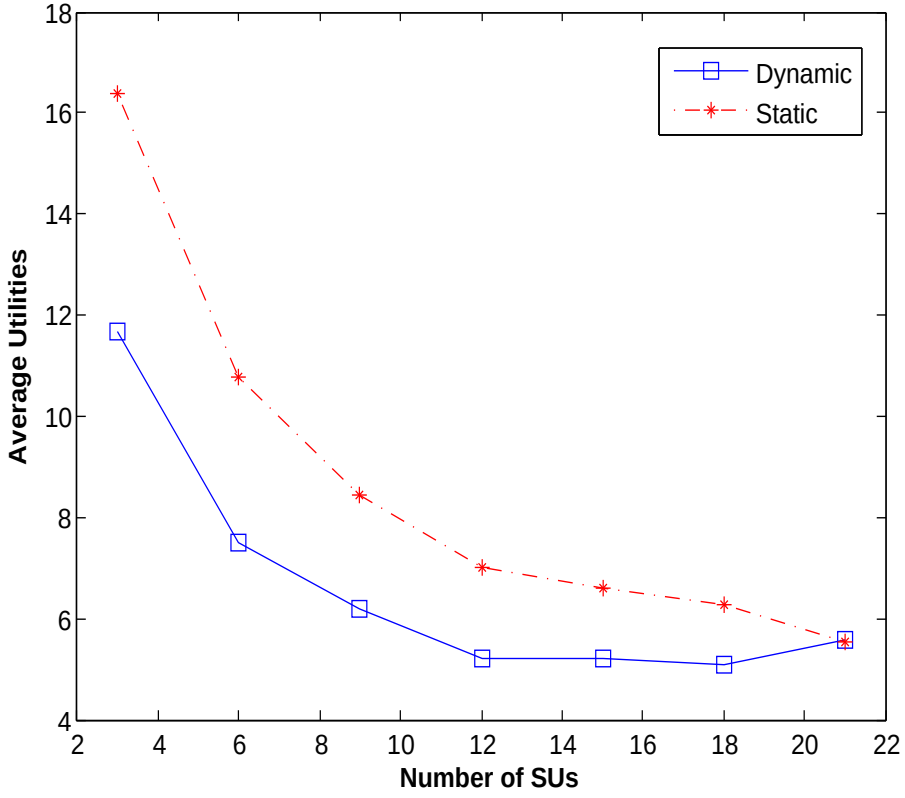


Figure : Average utilities Vs the number of SUs.

Numerical Results

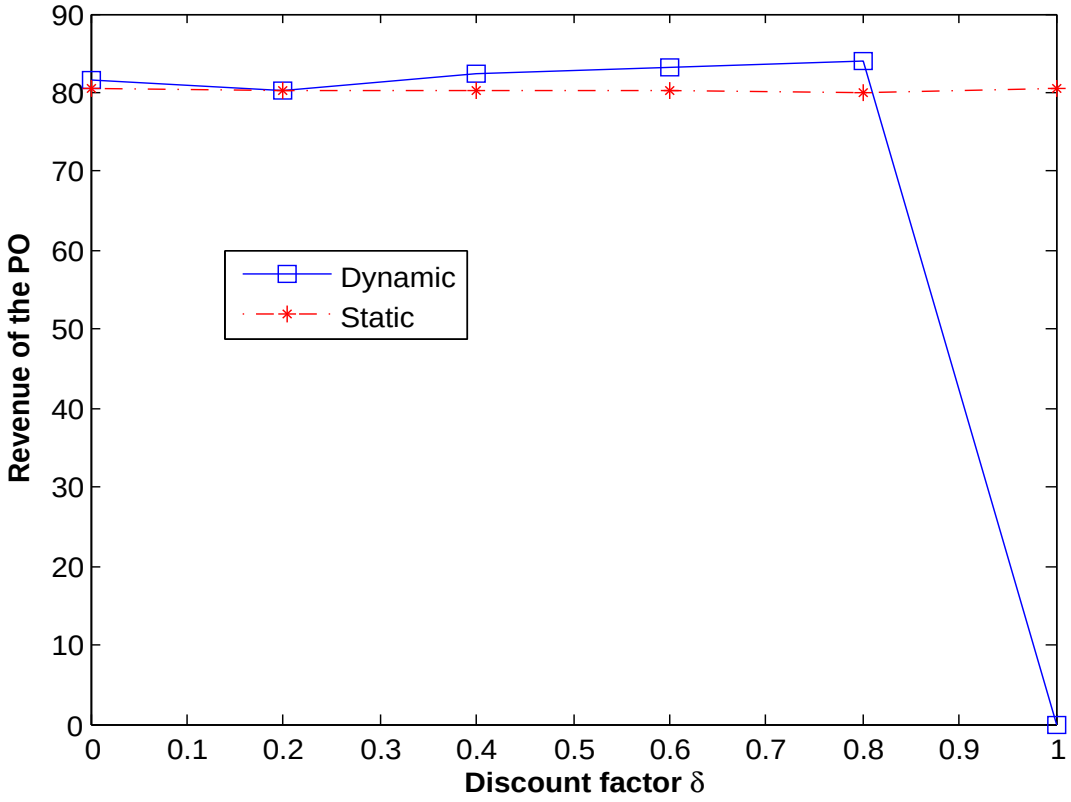


Figure : Revenue of the PO Vs δ, with 12 SUs.

Conclusion

- ADAPTIVE is the first spectrum auction that considers dynamically evolving values
- ADAPTIVE runs in polynomial time and results in efficient allocation with desired economic properties
- A possible direction for future work
  - Extend ADAPTIVE to a dynamic population model that will be a dynamic population and dynamic valuation mechanism.

Acknowledgement

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Thank You!