

ECSE 2010
Electric Circuits
Fall 2006 - **Millard**
Exam 2

Name SOLUTIONS *Al*

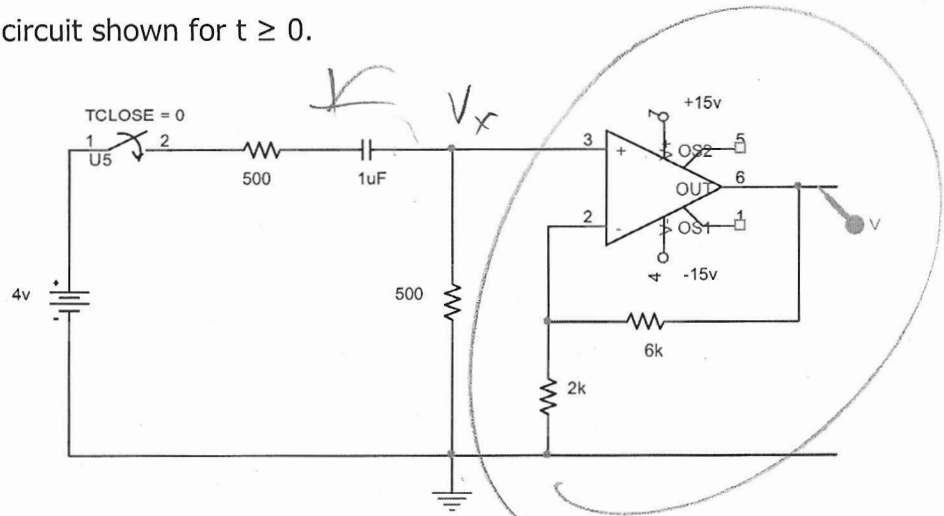
Problem No.	Pts.	Score
1	10pts	<i>x/10</i>
2	25pts	<i>x/25</i>
3	25pts	<i>x/25</i>
4	20pts	<i>x/20</i>
5	20pts	<i>x/20</i>
Total	100pts	<i>x/100</i>

Please Note:

- * Please place your answers in the spaces provided.
- * You must show your work to receive credit.

Problem 1 (10pts)

Find $V(t)$ in the circuit shown for $t \geq 0$.



$$V_x = V_R$$

$$A_v = 1 + \frac{6k}{2k} = 4$$

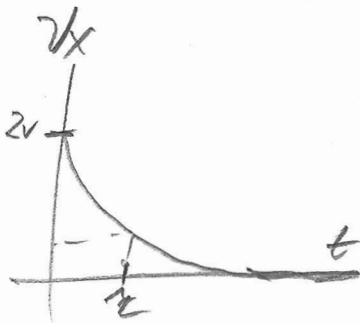
(2pts) $V_x(t=0^+) = 4V \frac{500}{500+500} = 2V$

$$V_{out} = 4 V_x$$

(2pts) $V_x(t \rightarrow \infty) = 0V$ (I → 0)

(2pts)

$$\tau = RC = (1k \times 1\mu F) = 1ms$$

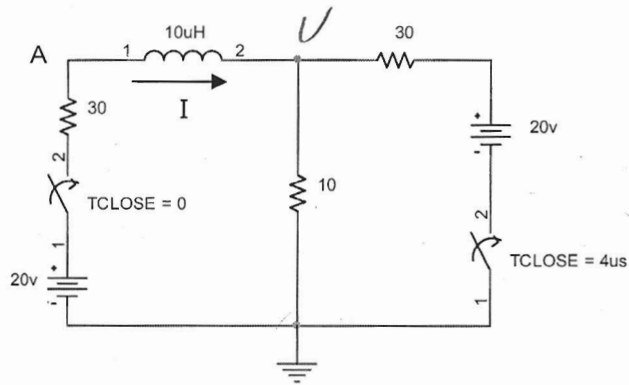


$V(t)$	$8 e^{-t/1ms} = 8 e^{-1000t}$
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(4pts)

Problem 2 (25pts)

a.) Find I (through the 10uH inductor) at each of the times indicated for the circuit shown. (15pts)



$I(t < 4\mu s):$

$i(t) = I_{SS} + (I_{INIT} - I_{SS})e^{-t/\tau}$

$I_{SS} = \frac{20V}{30+10} = .5A; I_{INIT} = 0.$

$\tau = \frac{L}{R} = \frac{10\mu H}{40} = .25\mu s$
 (t = 1us = 4τ)
 24τ: reaches SS

$i(4\tau) = .5(1 - e^{-4}) = .49$

$\partial t = 4\mu s$

$i'(4\mu s^-) = I_{SS} = .5A$

KCL at V_{SS}:

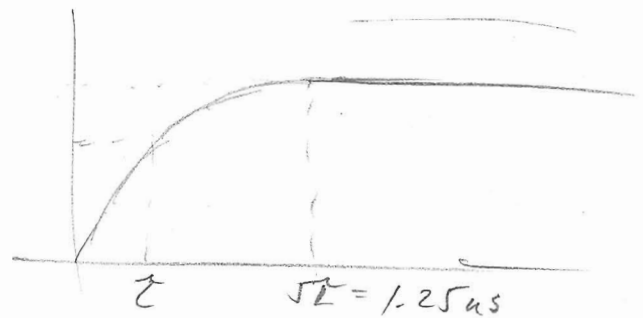
$\frac{V}{10} + \frac{2(V-20)}{30} = 0$

$3V + 2V - 40 = 0$

$5V = 40$

$V = 8$

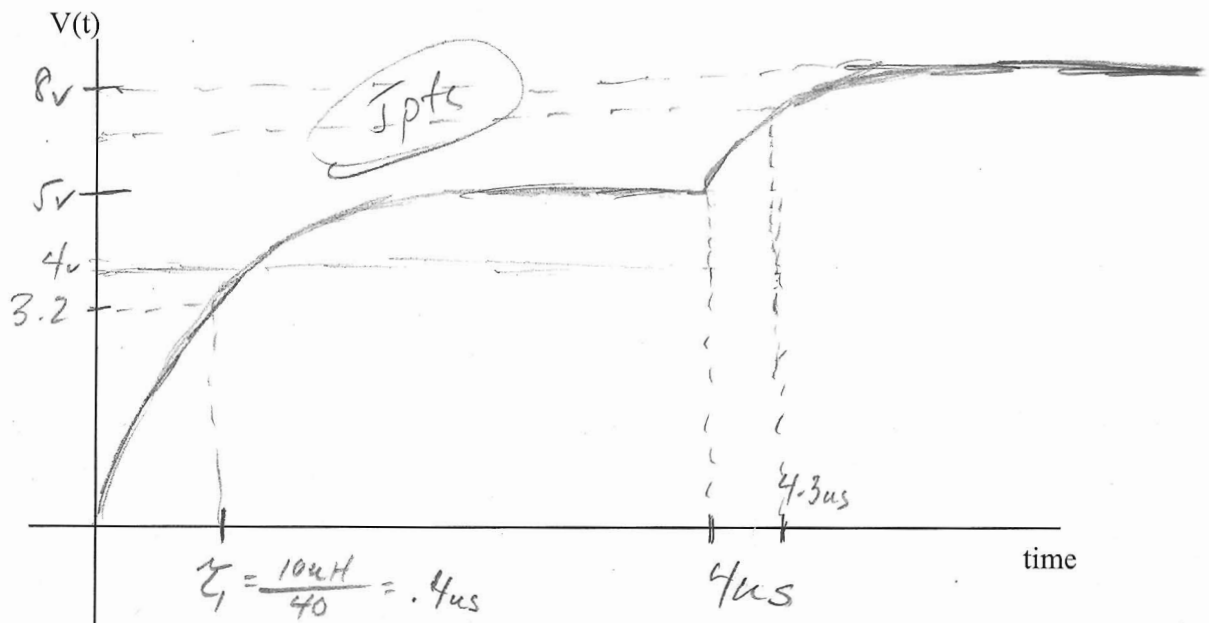
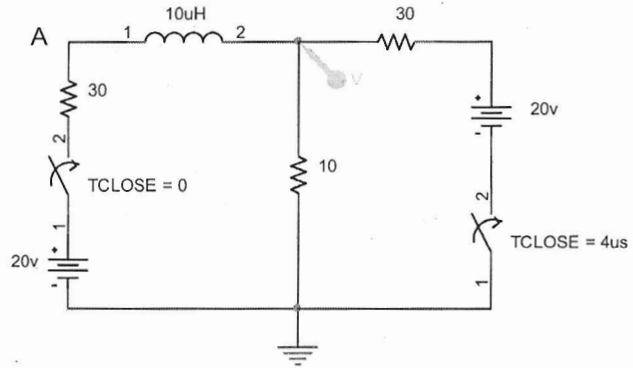
$I = \frac{20-8}{30} = \frac{12}{30} = .4A$



$I(1\mu s)$	$.49A$	5pts
$I(4\mu s)$	$.5A$	5pts
$I(8\mu s)$	$.4A$	5pts

Problem 2 (cont)

b.) Sketch $V(t)$ (across the 10ohm resistor) for $0 \leq t < 10\mu s$; showing ALL pertinent values at the critical points in time. (10pts)



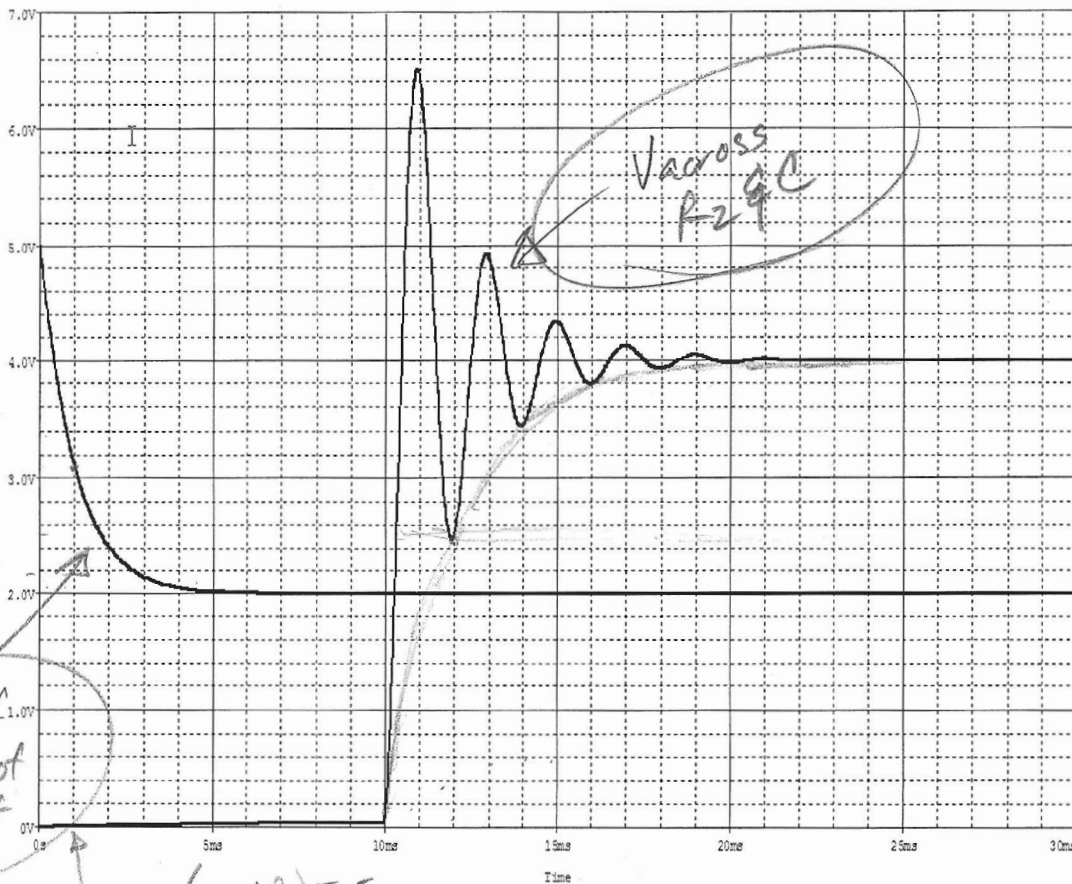
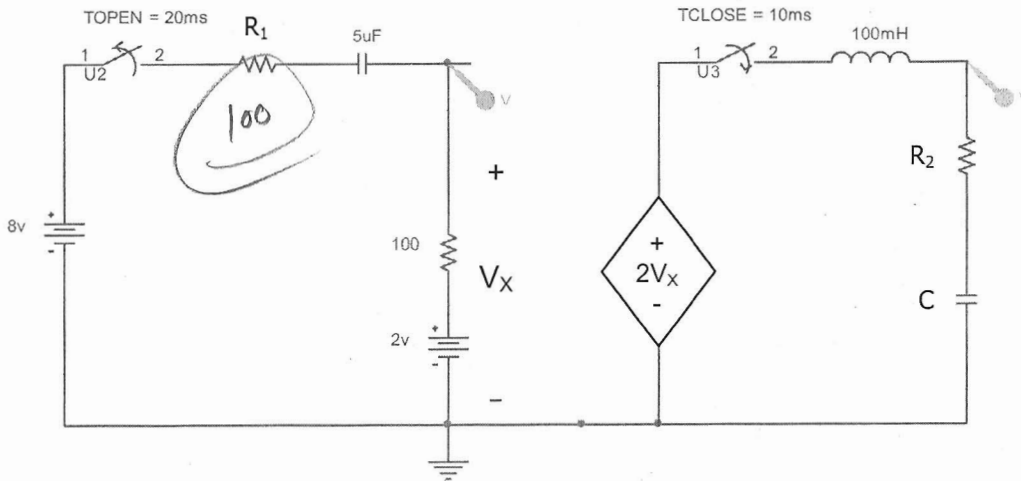
$$\tau_2 = \frac{L}{R_{eq}}$$

$$\alpha = \frac{R_{eq}}{L} = \frac{(10//30) + 30}{10\mu H}$$

$$\xi = \frac{1}{\alpha} = .276\mu s$$

Problem 3 (25pts)

Given the corresponding plots for the circuit shown; find the values of the circuit's components (R_1 , R_2 , and C).



$$f = \frac{1}{T} = 500$$

$$T = 2ms$$

$$\beta = \frac{2\pi f}{T}$$

$$\beta = 3.14 Kbps$$

$$\tau_1 = 1ms = (100 + R_1) 5\mu F$$

$$R_1 = 100$$

$$\beta = \sqrt{\omega_0^2 - \alpha^2} = 3.14 Kbps$$

$$\tau \approx 2ms = \frac{1}{\alpha}; \alpha = 500; \alpha^2 = 250 \times 10^3$$

5pts

Problem 3 (cont)

Right side is a RLC ckt

$$H(s)_{RLC} = \frac{K}{s^2 + \frac{R_2}{L}s + \frac{1}{LC}}$$

5pts

$$\alpha = \frac{R_2}{L} = 1000 ;$$

$$R_2 = (1000)(.1) = 100$$

$$\omega_0^2 = \frac{1}{LC} ; C = \frac{1}{\omega_0^2 L}$$

$$\beta^2 = \omega_0^2 - \alpha^2 ; \omega_0^2 = \beta^2 + \alpha^2 = 9.86 \times 10^6 + 250 \times 10^3$$

$$\omega_0^2 \approx 10 \times 10^6 = 10^7$$

$$C = \frac{1}{(\underbrace{10 \times 10^6}_{\omega_0^2}) \times (\underbrace{.1}_{L})} \approx 1 \times 10^{-6} = 1 \mu F$$

R ₁ (5pts)	100
R ₂ (10pts)	100
C (10pts)	1 μF

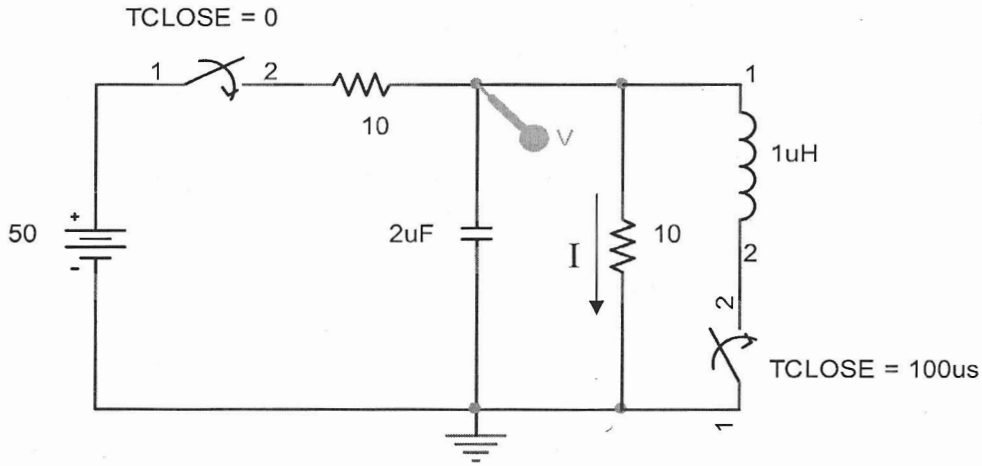
5pts

5pts

5pts

Problem 4 (20pts)

Given the following circuit find $I(100_{\mu s}^-)$, $V(100_{\mu s}^+)$, $I(\infty)$ and $V(\infty)$. Please provide justification for your responses.



$I(100_{\mu s})$: $\tau (< 100_{\mu s}) = RC = (10 || 10)(2_{\mu F}) = 10_{\mu s}$

CAP \rightarrow open

$I = \left(\frac{50}{20}\right) = 2.5A$

$V(0^-) = \frac{10}{10+10} \times 50 = 25V$

$I(\infty)$:

C \rightarrow open

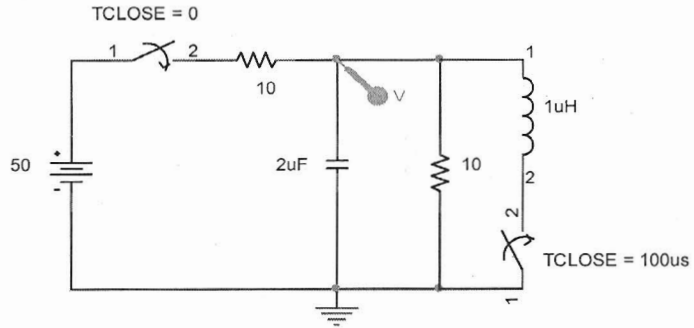
L \rightarrow short; $V \rightarrow 0$

if $V \rightarrow 0$; $I \rightarrow 0$

$I(0^-)$ $I(100^-)$	2.5A	5pts
$V(0^-)$ $V(100^-)$	25V	5pts
$I(\infty)$	0	5pts
$V(\infty)$	0	5pts

Problem 5 (20pts)

a.) Find $V(t)$ (across the capacitor) in the circuit shown for $t < 100\mu s$. (5pts)



$t < 100\mu s$

$V_{INIT} = 0$

$V_{SS} = 25$

Therefore:

$$V(t < 100\mu s) = 25 + (0 - 25)e^{-t/\tau} ; \tau = 5 \cdot 2\mu F$$

$\tau = 10\mu s$

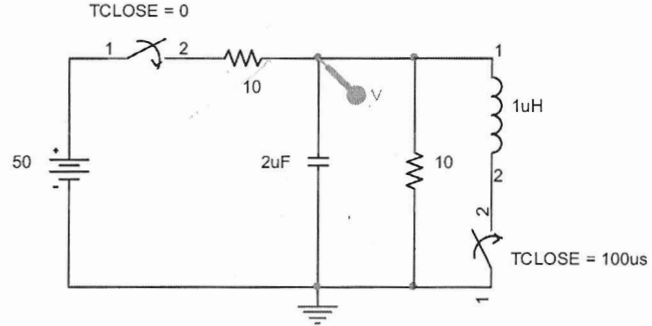
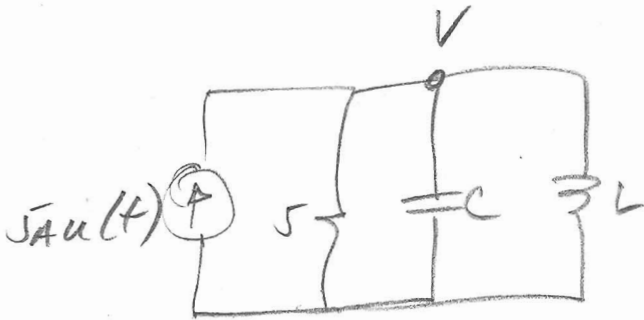
$$V(t < 100\mu s) = 25(1 - e^{-t/10\mu s})$$

$V(t)$	$25(1 - e^{-t/10\mu s})$
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5pts

Problem 5 (cont)

b.) Find the response type (over, critically, or under) and $V(t)$ (across the capacitor) in the circuit shown for $t > 100\mu s$. (15pts)



$$H(s)_{\text{parallel RLC}} = \frac{\frac{s}{C}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$\omega_0^2 = \frac{1}{LC} = 500 \times 10^9$$

$$\alpha^2 = \left(\frac{1}{2RC}\right)^2 = 2.5 \times 10^9 ; \alpha = \underline{50 \times 10^3 \text{ s}^{-1}}$$

$$\omega_0^2 \gg \alpha^2 \text{ (under)}$$

$$\beta = \sqrt{\omega_0^2 - \alpha^2} \approx \underline{705 \times 10^3 \text{ rps}}$$

$K = 3.54$ (+2)
 $\phi = -90^\circ$

Type	underdamped	5pts
V(t)	$K e^{-\alpha t} \cos(\beta t + \phi)$	10pts

α/β