ECSE 2010
Electric Circuits
Exam 2
Spring 2006

Name Solutions - OK

Section (please circle one)

MR	MR	MR
10-12	2-4	4-6
Millard	Salama	Kraft

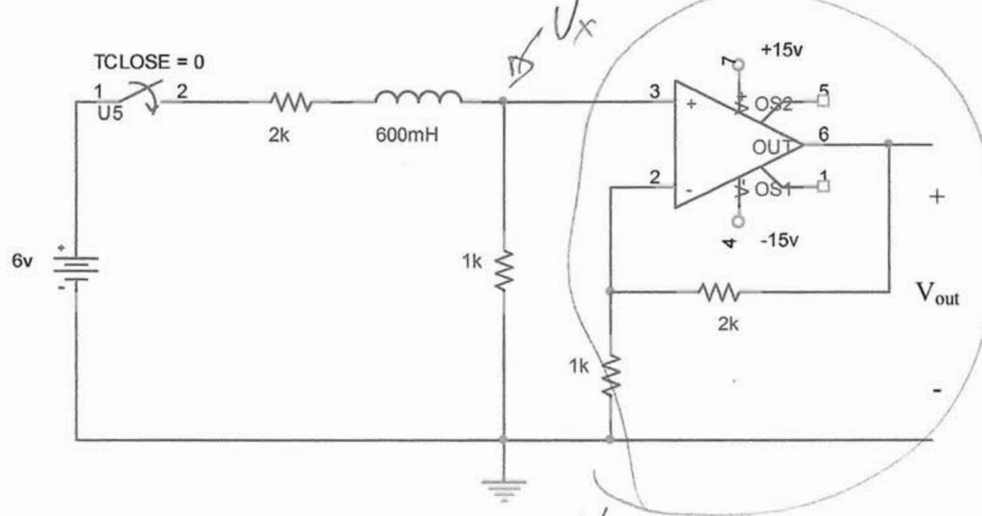
Please Note:

- Place all your answers in the spaces provided.
- You MUST show your work to receive any credit.
- Assume all resistances are in ohms, if not otherwise indicated.

Problem No.	Pts.	Score
1	20pts	
2	20pts	
3	20pts	
4	20pts	
5	20pts	
Total	100pts	

Problem 1 (20pts) Name Solutions

a) Find the differential equation for $V_{out}(t)$ for the circuit shown below. (assume that the op-amp has the correct supplies).



$$\frac{V_{OUT}}{V_X} = A_V = 1 + \frac{2K}{1K} = 3$$

$$6u(t) - 2Ki - L\frac{di}{dt} - 1Ki = 0$$

$$L\frac{di}{dt} + 3Ki = 6u(t)$$

5pts

$$V_x = (1K)\cdot 2mA(1-e^{-t/\tau})$$

$$\frac{di}{dt} + 5000i = 10u(t)$$

$$V_{OUT} = 3V_x = 6(1-e^{-t/\tau})$$

$$SI(s) + \underset{(R/L)}{5000}I(s) = \frac{10}{s}$$

$V_{out}(t)$	$6v(1-e^{-5000t})u(t)$

15pts

$$I(s) = \frac{10}{s(s+5000)}$$

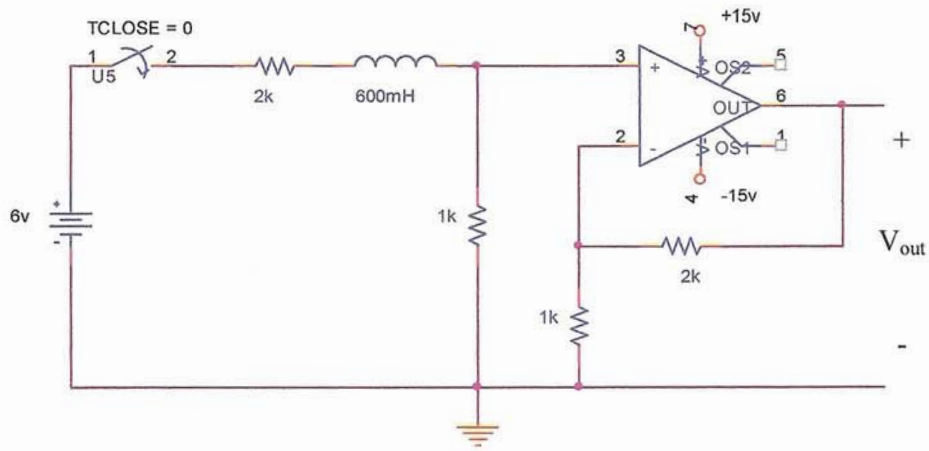
10pts

$$I_{SS} = \frac{6}{3K} = 2mA \quad I_{INIT} = 0 \quad i(t) = 2mA(1-e^{-t/\tau})$$

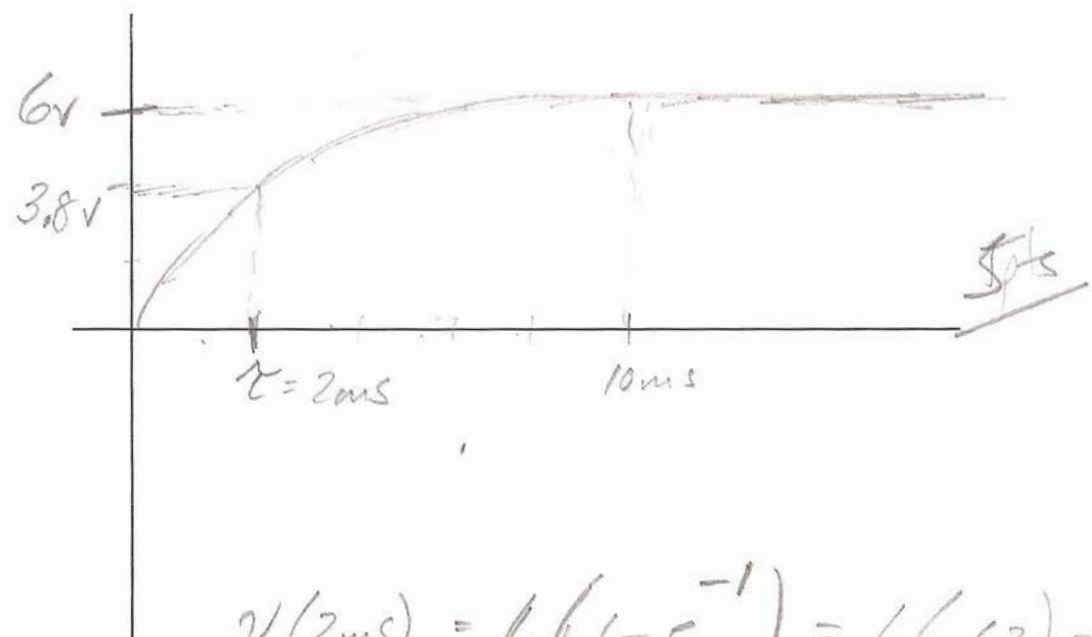
$$\tau = \frac{L}{R} = \frac{.6}{3K} = .2ms$$

Problem 1 (cont) Name Solutions

b) Sketch the $V_{out}(t)$ for the circuit shown below including all pertinent values (e.g time constants, etc.).



$V_{out}(t)$



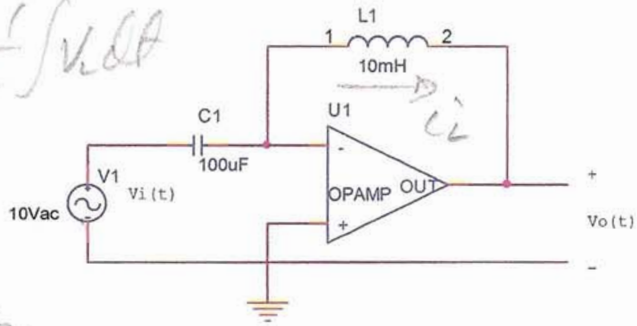
$$v(2ms) = 6(1-e^{-1}) = 6(.63) = 3.78$$

$$v(10ms) \cong 6v$$

Problem 2 (20pts) Name Solutions

a) For the circuit, find the differential equation relating $i_c(t)$ (the current in the capacitor) to $v_i(t)$ and the differential equation relating $i_L(t)$ (the current in the inductor) to $v_o(t)$. (assume that the op-amp has the correct supplies).

$v_L(t) = L\frac{di}{dt}$, $i_L = \frac{1}{L}\int v_L\,dt$



$$i_C = C\frac{dv_c}{dt}$$

$$i_C = C\frac{d(v_i - 0)}{dt} = C\frac{dv_i}{dt}$$

$$i_L = \frac{1}{L}\int v_L\,dt = \frac{1}{L}\int (0 - v_{OUT})\,dt$$

$$i_L = -\frac{1}{L}\int v_{OUT}\,dt$$

$i_C(t)$	$C\frac{dv_i}{dt}$	5pts
$i_L(t)$	$-\frac{1}{L}\int v_{OUT}\,dt$	5pts

Problem 2 (cont) Name Solutions

b) Find an expression for $v_o(t)$ in terms of $v_i(t)$ for the circuit shown in part a.

$$i_C = i_L$$

$$C\frac{dv_{IN}}{dt} = \frac{-1}{L}\int v_{OUT}\,dt$$

$$v_{OUT} = -LC\frac{d^2 v_i}{dt^2}$$

$v_o(t)$	$-LC\dfrac{d^2 v_i}{dt^2}$

7 pts

c) What is $v_{out}(t)$ if the input is: $v_i(t) = 10\cos(1000t)$?

$$\frac{dv_i}{dt} = -10(1000)\sin(1000t)$$

$$LC = .01 \cdot 100u$$

$$LC = 10^{-6}$$

$$\frac{d^2 v_i}{dt^2} = -10(1000)^2 \cos(1000t)$$

$v_o(t)$	$10\cos(1000t)$

5 pts

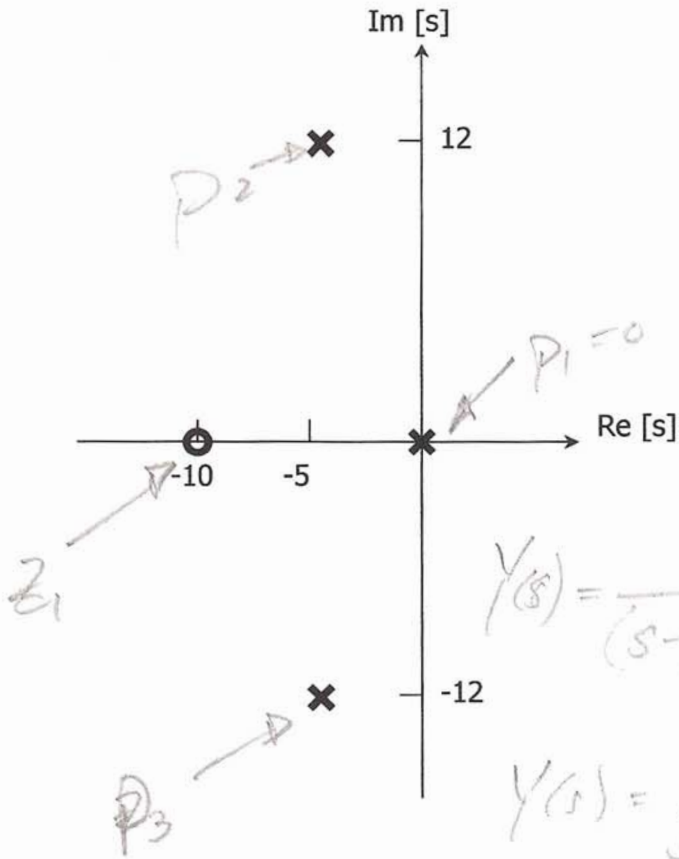
$$V_o(t) = -(-10\,\cancel{(1000)^2}\,\cancel{LC}\cos(1000t))$$

LC

$$= +10\cos(1000t)$$

Problem 3 (20pts) Name ______________________

a) The output Y(s) of a circuit is the product of the input X(s) and the network function H(s). The Y(s) for a circuit has the following pole/zero diagram. Find Y(s) as a ratio of polynomials in s. (Ignore any scaling or gain values.)



$$Y(s) = \frac{s - z_1}{(s-p_1)(s-p_2)(s-p_3)}$$

$$Y(s) = \frac{(s+10)}{s(s+5+j12)(s+5-j12)}$$

$p_2 = -5 + j12$
$p_3 = -5 - j12$

$\beta = 12 = \sqrt{\omega_0^2 - \alpha^2}$

$\omega_0^2 = \beta^2 + \alpha^2 = 144 + 25$
$= 169$

Y(s)	$\dfrac{s+10}{s(s^2+10+169)}$

(10pts)

Problem 3 (cont) Name Solutions

b) If Y(s) is the output when X(s) results from a unit step function 1u(t), classify the circuit's damping (please support your response):

$\alpha^2 < \omega_0^2$

(5pts)

OVERDAMPED	CRITICALLY DAMPED	UNDERDAMPED	OSCILLATOR

c) Find the damping ratio ζ, for the circuit.

$$\zeta \omega_0 = \alpha$$

$$\zeta = \frac{\alpha}{\omega_0} = \frac{5}{13} \cong .4$$

(3pts)

d) Find y(t), the circuit's response and inverse Laplace transform of Y(s), if x(t) is still a unit step function 1u(t).

$$X(s) = \frac{1}{s}$$

(2pts)

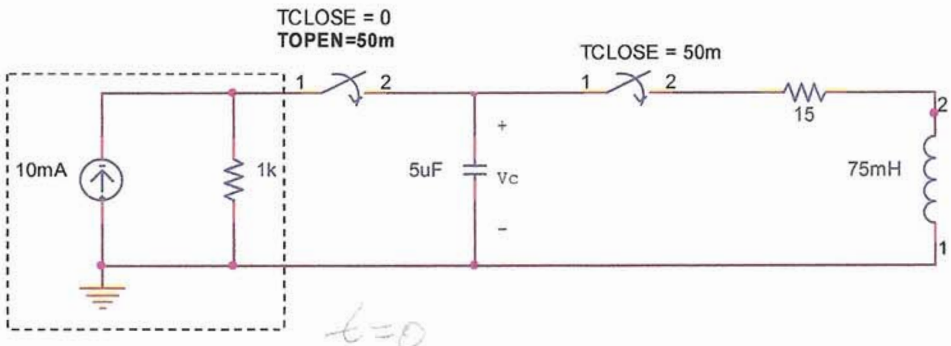
$$Y(s) = \frac{A_0}{s} + \frac{A_1}{(s+5+j12)} + \frac{A_1^*}{(s+5-j12)}$$

$$y(t) = A_0 + 2A_1 e^{-\alpha t} \cos(\beta t + \phi)$$

$$y(t) = .06 + .08 e^{-5t} \cos(12t - 135°)$$

Problem 4 (20pts) Name Solutions

a) For the circuit find an expression for $v_C(t)$ for $0 \le t < 50ms$.
Assume that $v_C(0-) = 0V$ and the switches change state at t=50ms.



$$10 - RC\frac{dv_c}{dt} - v_c = 0$$

$$\tau = (1K)(5uF)$$
$$\tau = 5ms$$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{10}{RC}u(t)$$

$$v_c(t) = V_{ss} + (V_{INIT} - V_{ss})e^{-t/\tau}$$

$$v_{css} = 10V\ ;\ v_{c_{INIT}} = 0 \qquad v_c(t) = 10 + (0-10)e^{-t/\tau}$$
$$\tau = 5ms$$

$v_C(t)$	$10\left(1 - e^{-t/5ms}\right) \cdot u(t)$	$0 \le t < 50ms$

8pts

Problem 4 (cont) Name Solutions

b) At time t = 50ms the first switch opens and the second switch closes. Assuming the capacitor has charged to 10V at t = 50ms, find an expression for $v_C(t)$ for t ≥ 50ms.

Series RLC: $2\alpha = \frac{R}{L}$ $\omega_0^2 = \frac{1}{LC} = 2.7 \times 10^6$

$$\frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$\alpha = \frac{R}{2L} = \frac{15}{2(.075)} = 100$

$\alpha^2 = (100)^2 = 10^4$ $\alpha^2 \ll \omega_0^2$!

$v_C(t)$	$10e^{-100t}\cos(\beta t + \phi)$	t ≥ 50ms

5pts

$\beta = \sqrt{\omega_0^2 - \alpha^2} \approx \omega_0 = 1640$

c) Find the differential equation relating $i_L(t)$ (the current in the inductor) for t ≥ 50ms.

$i_L(t) = -i_c = -C\frac{dv_c}{dt}$ 1pt

$i_L(t) = -C\frac{dv_c(t)}{dt} = -C\Big[(-1000e^{-100t}\cos(1640t+\phi)) - (16400e^{-100t}\sin(1640t+\phi))\Big]$ 1pt

$i_L(t)$		t ≥ 50ms

2pts

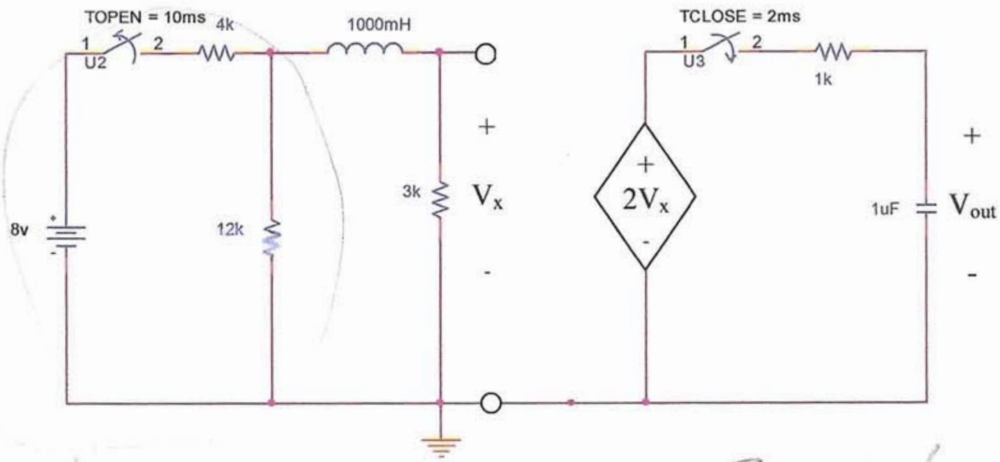
d) Classify the solution for $v_C(t)$ for t ≥ 50ms. (circle one)

OVERDAMPED	CRITICALLY DAMPED	UNDERDAMPED	OSCILLATOR

5pts

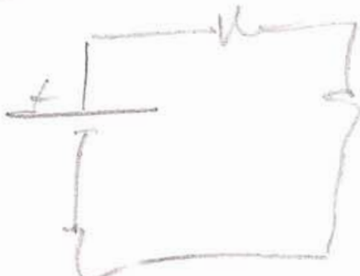
Problem 5 (20pts) Name Solutions

a) Find the value of V_x at t=3ms, given the circuit shown.



Thevenin to 3k

$5\tau = 5 \cdot \frac{1}{6K} = \frac{5}{6000} < 2mS$



$\frac{12}{16} \times 8 = 6v$

$3K \quad V_x = \frac{1}{2} 6v = 3v$

$\tau = \frac{1}{6K} = .00017$

V_x (3ms)	3v

5pts

b) Find V_x at t = 10ms.

$i_L(10^-) = \frac{6v}{6K} = 1mA$

$i_L(0^+) = i_L(0^-) = 1mA$

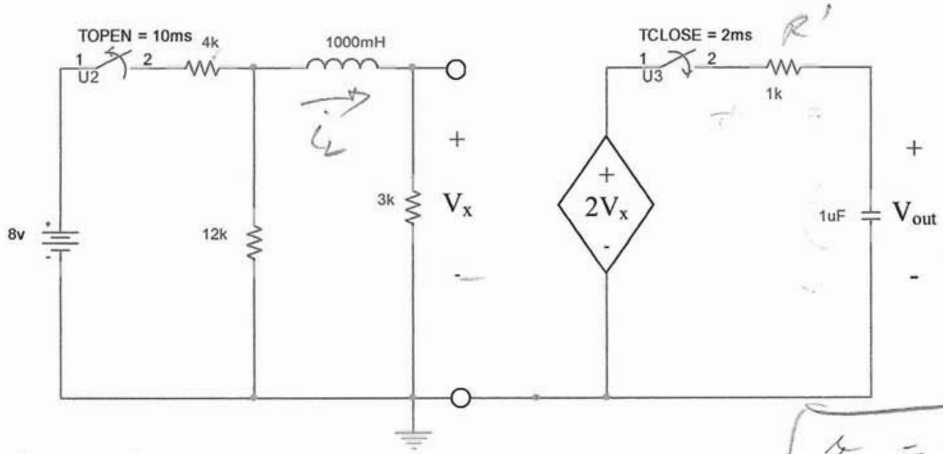
$V_x = 3K \cdot 1mA = 3v$

V_x (10ms)	3v

5pts

Problem 5 (cont) Name Solutions

c) Find the differential equation for $V_{out}(t)$ (for 20ms > t ≥ 2ms) given the circuit shown.



for $2ms < t \leq 10ms$

$i_L = 1mA$, $V_x = 3v$; $2V_x = 6v$

$6v - iR' - V_{out} = 0$

$6v - R'C\frac{dV_{out}}{dt} - V_{out} = 0$

$\frac{dV_{out}}{dt} + \frac{1}{R'C}V_{out} = \frac{6}{R'C}$

$\tau_{RC} = 1ms$
$\tau_{LR} = \frac{L}{R} = \frac{1}{15k} \ll \tau_{RC}$

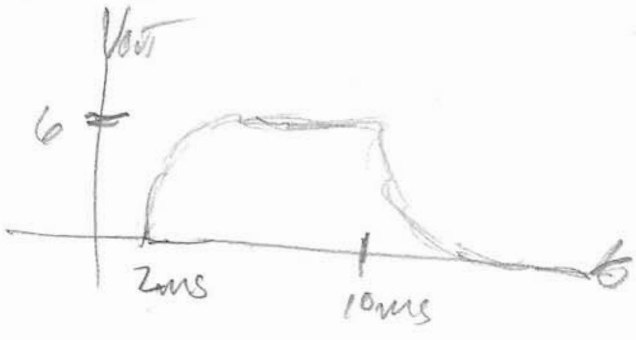
for $10ms < t < 20ms$

$R'C\frac{dV_{out}}{dt} + V_{out} = 0$

$\frac{dV_{out}}{dt} + \frac{1}{R'C}V_{out} = 0$

(5pts)

d) Find V_{out} at time t = 10ms.



V_{out} (10ms)	6v

5pts