1. (20 points) Representations of Logic Functions

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a. (5 points) Fill up the above truth table representing \( G(a, b, c) = (\overline{a} + b + c) \cdot (a + c) \cdot b \)

b. (5 points) Write the shorthand minterm and Maxterm representations for \( G(a, b, c) \):

\[
\bar{Z}_m(3, 6, 7) \quad \bar{P}_M(0, 1, 2, 4, 5)
\]

c. (5 points) Write the canonical minterm and Maxterm representations for \( G(a, b, c) \):

- Canonical minterm: \( \overline{a}bc + abc + \overline{a}b\overline{c} \)
- Canonical Maxterm: \( (a+b+c) \cdot (a+b+c) \cdot (a+b+c) \cdot (\overline{a}+b+c) \cdot (\overline{a}+b+c) \)

d. (5 points) Prepare an espresso input file representing \( G(a, b, c) \):

```
Solution #1:
0 3
1 3
0 1 1 1
1 0 1
1 1 1
1 1 1 1
```

```
Solution #2:
0 1
1 8
0 1 1 0 0 1 1 1
```

e. (5 points) Sketch a logic circuit that implements \( G(a, b, c) \) using NOR gates only.

\[
G(a, b, c) = \overline{a}bc + \overline{a}b + bc + ab + bc
\]

![Logic Circuit Diagram]
2. (20 points) Number & Symbol Representation, Arithmetic

a. (5 points) Convert the number $123_{10}$ to two’s complement binary representation using 8 bits. Show your work.

\[
\begin{align*}
1/2 & = 61 \\
6/2 & = 30 \\
3/2 & = 15 \\
15/2 & = 7 \\
7/2 & = 3 \\
3/2 & = 1 \\
123_{10} & = 01111011_2
\end{align*}
\]

b. (5 points) Give the two’s complement representation of $-123_{10}$. Show your work.

\[
123_{10} = 01111011
\]

One's complement: \[ 10000100 \]
Two's complement: \[ 10000101 \]

Therefore, \[ 123_{10} = -123_{10} \]

\[
\begin{align*}
\text{One's complement:} & \quad 10000100 \\
\text{Two's complement:} & \quad 10000101
\end{align*}
\]

\[
123_{10} = -123_{10}
\]

c. (10 points) Consider the following two 8-bit 2s-complement binary integers expressed in hexadecimal form:

\[
A = \text{FF}_{16} \\
B = \text{7F}_{16}
\]

Calculate the difference $A - B$, and $B - A$. Show your work. Does overflow occur for each case? Why?

\[
\begin{align*}
A - B & = \text{FF}_{16} - \text{7F}_{16} \\
& = \text{FF} + \text{10} \times \text{00000001} \\
& = \text{FFFF} + \text{00000001} \\
& = \text{00000000} \\
& \text{No overflow.}
\end{align*}
\]

\[
\begin{align*}
B - A & = \text{7F}_{16} - \text{FF}_{16} \\
& = \text{7F} + \text{10} \times \text{00000001} \\
& = \text{7FFF} + \text{00000001} \\
& = \text{00000000} \\
& \text{different sign, overflow.}
\end{align*}
\]
3. (15 points) Boolean Algebra

a. (5 points) Simplify the following Boolean function to sum-of-products form using the laws of
Boolean algebra. Show the process.

\[ F = (a + a'b)' + c' \]
\[ = [a'((a+b') + c)]' \]
\[ = [a'b' + c]' = (a + b) c' \]
\[ = ac' + bc' \]

b. (5 points) Simplify the following logic function using Boolean algebra theorems.
Show the process.

\[ F = T'UVWX + T'UV'XZ + T'UWXY'Z \]
\[ = T'UX(VW + U'Z + WY'Z) \]
\[ = T'UX( VW + UIZ + WY'Z ) \]
\[ = T'UX( VW + UIZ + WZ ) \]
\[ = T'UX( VW + UIZ ) \]
\[ = T'UX VU + T'UX U'Z . \]

c. (5 points) Given two variables \( x \) and \( y \), we can define their exclusive-OR as follows:

\[ x \oplus y = x \overline{y} + \overline{x}y \]

where the symbol \( \oplus \) denotes the exclusive-OR of \( x \) and \( y \). Using this definition, and the laws of
Boolean algebra, prove the following equivalence:

\[ x(y \oplus z) = xy \oplus xz \]

\[ \text{Right side: } (xy)(x+z)' + (xy)'(xz) \]
\[ = (xy)(x'+z') + (y+xy')xz \]
\[ = xy z' + y z y' \]
\[ = x(y z' + z y') \]
\[ = x(y \oplus z) \]
\[ = \text{left side. done!} \]
4. (25 points) Systematic Minimization with K-Maps
Consider the following K-map for a Boolean function \( F(A,B,C,D) \).
The X's denote don't care terms.
a. (5 points) Derive the minimum sum of products (SOP) expression for \( F(A,B,C,D) \).

\[
F = AC' + A'D' 
\]
b. (5 points) Derive the minimum product of sums (POS) expression for \( F(A,B,C,D) \). The K-map has been repeated for your convenience.

\[
F' = AC + A'D \\
F = (F')' = (A' + C') (A + D')
\]
c. Given the following logic diagram

1) (5 points) Determine its output $F$ as a function of its input

$$F = x'y'z + xy'2 + x'y2' + xy2$$

2) (5 points) Determine the minimal SOP and POS of $F$ using K-map $X Y$

$$F_{SOP} = x'y'z' + xz + y'z$$

$$F' = xz' + y'z' + y'$$

3) (5 points) Implement the minimum SOP of $F$ using the PLA template below. Label all wires clearly and using crosses to represent connections.
5. (20 points) Combinational Building Blocks (MUX/DMUX)

Implement the following canonical sum

\[ F(X, Y, Z, W) = \Sigma m(3, 5, 15) \]

Hint: the numbers are odd.

A. (10 points) using the 74LS153 dual one-of-four multiplexer. This chip contains two multiplexers that share the common selectors \( S_1 \) and \( S_0 \). Section A is enabled by setting \( GA \) low, section B by setting \( GB \) low. For your answer, let \( YB \) be the \( F \). Label the chip inputs with \( X, Y, Z, W \), or their complements, or the logic constants 0 or 1. Enable only the section you are using.

B. (10 points) using the 74LS138 1-of-8 decoder / demultiplexer. Label all 6 chip inputs with \( X, Y, Z, W \), their complements, 0, or 1. Connect the appropriate \( Ys \) to the appropriate 3-input chip, crossing out the unused chips. The '138 chip has 3 enable inputs, \( G1, G2A' \) and \( G2B' \). \( G1 \) must be High and the others Low for the chip to be enabled. When the chip is not enabled, all \( Ys \) are High. The address inputs \( C, B, A \) (\( C \) is MSB, \( A \) is LSB) select which \( Y_i \) is Low. E.g., \( CBA = 011 \) makes \( Y3 = \text{Low} \). All other \( Ys \) are High.