An Evidential Reasoning Approach for Recognizing Shape Features

Qiang Ji, Michael M. Marefat, and Paul J. Lever
Dept. of Electrical and Computer Engineering
University of Arizona, Tucson, AZ 85721
qiangji@ece.arizona.edu

Abstract

This paper introduces an evidential reasoning based approach for recognizing and extracting manufacturing features from solid model description of objects. A major difficulty faced by previously proposed methods for feature extraction has been the interaction between features. In interacting situations, the representation for various primitive features is non-unique, making their recognition very difficult. We develop an approach based on generating and combining geometric and topological evidences for recognizing interacting features.

The essence of our approach is in finding a set of correct and necessary virtual links through aggregating the available geometric and topologic evidences at different abstraction levels. The identified virtual links are then augmented to the cavity graph representing a depression of an object so that the resulting supergraph can be partitioned to obtain the features of the object. The main contributions of our approach include introducing the evidential reasoning (Dempster-Shafer theory) to the feature extraction domain and developing the theory of principle of association to overcome the mutual exclusiveness assumption of the Dempster-Shafer theory.

1. Introduction

Traditional CAD boundary description of a solid object in terms of low level entities like faces, edges, and vertices is not conducive to manufacturing automation. Most machine operations require high level semantic features such as pockets, slots, and holes instead of these low level geometric entities. Therefore, in order to achieve true manufacturing automation, several different mechanisms have been proposed to automatically extract the high level semantic features from the low level entities in solid model representations [1,2,5,6]. While these mechanisms can successfully recognize decoupled or isolated primitive features, they have achieved very limited success in identifying and describing compound depressions generated by the interaction of several semantic features [3].

The difficulty in extracting interacting features arises primarily from the ambiguous and non-unique feature representation schemes [3]. The reason is that regardless of the actual workings of the mechanism (say expert rules, syntactic patterns, or graph isomorphism), the employed mechanism searches for a particular combination of geometric and topologic relationships to recognize individual primitive features. However, due to the representation ambiguities, these geometric and topologic relationships are not unique, and therefore the feature recognition mechanism produces brittle behavior.

Let us illustrate the point by means of an example. Graph-based mechanisms, typically, represent a part (or its depression) by a graph (for example a node for each face, and a link between two nodes whose corresponding faces share an edge), and each primitive feature by its corresponding template graph. Subsequently, they use a method like graph isomorphism to identify instances of the features in a given part. For example, figure 1 (a1 and b1) shows two parts with two perpendicular pockets of identical size and orientation. The cavity graph representations of the depressions for both parts are shown in Fig. 1(a2) and Fig.1 (b2). Furthermore, figure 1 shows that although the rectangular pocket formed by the highlighted nodes 1,2,3,4,5 in figure 1(a2) may be extracted by the pocket template (see the first row in fig.2), the rectangular pocket formed by the same set of highlighted nodes in figure 1(b2) cannot be correctly identified as the pocket primitive due to lack of link between nodes 3 and 1. The reason is that graph representations, such as the one shown, attempt to capture generic topologic and geometric relationships indicative of different classes of features, but instances of the features may not follow the captured relationships (especially when a feature interacts with other features).

The set of topologic and geometric relationships which describes a class of features is not unique. One method to overcome this non-uniqueness is to attempt to enumerate all possible topologic and geometric relationships which describe instances of a given class of features under all different circumstances. Such an attempt, however, would either be impossible or exceedingly difficult because of the difficulty in enumerating all distinct configurations, which are either infinite or very large.

Instead of the above brute force enumeration method, the approach that we propose in this paper is to combine evidences supporting or rejecting existence of instances of different features in a well defined manner to determine features present in a depression. In such a framework, each evidence may not by itself be sufficient for recognizing a feature, but it generates a belief (or a
weight), which is a measure of confidence that relates the evidence (a topologic or geometric relationship) to a feature. It is the collection of evidences and their consistent impacts upon each other that produces a description of the features in a depression.

The primitive features of interest in this work are common polyhedral machinable features like pockets, blind slots, prismatic holes, and steps. The primitive features used in this paper have faces which are concave to each other, however extension to features with both convex and concave relationships can be addressed similarly. Figure 2 shows instances of some families of primitive features considered in this research along with their cavity graph representations.

2. Problem Formulation And Solution Overview

2.1 Brief Overview of Graph-Based Approach

Let us more closely consider the graph-based approach to feature recognition and extraction. The following definitions are needed for this purpose:

Definition 1: A cavity graph $G$ for a depression is an ordered five tuple $(N(G), L(G), \psi_G, l_G, m_G)$ such that it is connected and:

1) $N(G)$ is a nonempty set of nodes of $G$, such that for each face $f_i$ of the object there is exactly one node in $G$ and also unifiable nodes of $G$ are unified [3].
2) $L(G)$ is the set of links of $G$ such that $l_G (k) = \text{concave}$ $\forall k \subseteq L(G)$, that is every link has a concave label.
3) $\psi_G : L(G) \rightarrow N(G) \times N(G)$, and $l_G : L(G) \rightarrow \{\text{concave}, \text{convex}\}$ are the incidence define the incidence function and the link labels as before, and

Figure 2. Some families of primitive features and their cavity graph representations. Faces are represented by nodes and concave adjacency relationships between faces are represented by links. Node labels are the dominant directions of surface normals of the corresponding faces.

4) $m_G : N(G) \rightarrow \{B, -B, +X, -X, +Y, -Y\}$ is a function which labels the nodes of $G$. This labeling of the nodes describes the relative spatial orientation of the faces with respect to each other in a qualitative manner [3].

Since depressions are usually associated with concavities in a component, the cavity graphs, which consists solely of concave links, are intended to model the depressions of an object. Figure 1(b2) shows the cavity graph for the depicted part which has two perpendicularly interacting pockets (a part may have more than one cavity graph).

As briefly stated before, in a typical graph-based approach [2, 3], a part or its depressions are represented by graphs, and subsequently a matching between this representation and the feature templates is used for recognition. For example, in the cavity graph shown in figure 1(a2), the highlighted nodes and links correspond to a pocket template (see the first row in figure 2), which is formed by faces 1, 2, 3, 4, 5. In general, this method can be stated as follows [3]: given a cavity graph $G$, find a decomposition, $D = \{g_1, g_2, ..., g_n\}$, such that:

(i) $g_i$ is a primitive template,
(ii) $g_1 \cup g_2 \cup ... \cup g_n = G$
(iii) $g_i$ are maximal, that is $g_i \supsetneq g_j$ for $i > j$
Therefore, in this method, a partitioning of the cavity graph(s) is used to obtain a part's features. The partitions may overlap, but the maximality condition in (iii) above ensures that for example a pocket is not mistaken for two blind-slots, etc.

2.2 Virtual Links

In the previous section, we discussed the use of partitioning to determine the features of a part. However, a correct partitioning of the cavity graphs may still not result in the valid templates for the features of the part, because of the non-uniqueness in representations. For example, neither one of the two subgraphs produced from the partitioning the cavity graph shown in figure 1(b2) corresponds to a pocket (see pocket representation in figure 2). In order to identify and extract the two pockets for the part in figure 1(b1), the cavity graph in 1(b2) must be augmented with two links, 3-1 and 6-2, and then correctly partitioned. The new cavity graph produced by augmenting the original cavity graph with these two links is shown in figure 3(a), and the subgraphs which result from its partitioning are shown in figure 3(b). It is clear that the subgraphs in 3(b) correctly identify the pockets in the depression of the given part.

![Diagram](image)

**Figure 3. Cavity graphs with virtual links.** (a) shows the cavity graph for the example part shown in fig.1(b2) augmented with the virtual links 3-1 and 6-2. The augmented cavity graph is readily partitioned into two maximal constituents, (b1) and (b2), each of which represents one of the pockets in the depression of the part.

**Definition 2: Virtual Links** Suppose a depression with cavity graph $G$ consists of the features $f_1, f_2, \ldots,$ and $f_n$, with the corresponding representations $g_1, g_2, \ldots,$ and $g_n$; then the set of links of $(g_1 \cup g_2 \cup \ldots, \cup g_n)$ - $G$ are referred to as virtual links.

In other words, virtual links are set of links which are not present in the cavity graph of a depression, but whose augmentation results in a supergraph which is isomorphic to the union of the representations for the involved features, and hence can be partitioned to obtain these features. In general, the number of required virtual links as well as their identity for a given depression are not known in advance. Therefore the problem of combining different geometric and topologic evidences to determine the features can be stated as follows in terms of virtual links and partitioning.

**Problem Statement:** Given a depression, generate and combine different appropriate evidences based on the existing geometric and topologic relationships to determine the necessary virtual links, augment the existing cavity graph with the determined virtual links, and then partition the resulting cavity graph to extract the features as described in Figure 4.

![Diagram](image)

**Figure 4. Schematic diagram of the proposed approach for evidential reasoning and graph partitioning to identify and extract features**

3. Feature Extraction Using Dempster-Shafer Theory

3.1 The Dempster-Shafer Theory

The Dempster-Shafer (D-S) approach offers a systematic and consistent mechanism to represent and propagate uncertainties in a coherent manner by aggregating the available geometric and topologic evidences. According to the D-S theory, the set of all possible outcomes (the sample space) in a random experiment is called the frame of discernment (FOD) denoted by $\Theta$. The subsets of $\Theta$ are referred to as events or hypotheses. A major assumption in D-S theory is that elements of $\Theta$ must be mutually exclusive and exhaustive to each other. This assumption implies that one and only
one element in $\Theta$ could be correct. Later, we will demonstrate the limitations of this assumption and propose a method to overcome these limitations.

According to the D-S theory, a number in the range [0,1] inclusive is used to indicate the belief in a hypothesis (or a set of hypotheses) given a piece of evidence. This number, referred to as basic probability assignment (bpa), expresses the degree to which an evidence confirms a hypothesis. It measures the belief assigned to the hypothesis from the evidence.

Given two bpa's $m_1(\cdot)$ and $m_2(\cdot)$ discerned in the same frame, their combined effect can be computed using the Dempster's rule of combination as shown in equation 1.

$$m(C|m_1@m_2(C))=K \sum_{A \cup B=C} m_1(A)m_2(B)$$

where $K=1- \sum_{A \cup B \neq C} m_1(A)m_2(B)$

The combination rule states that the combination of $m_1$ and $m_2$ apportions the total amount of belief among the subsets of $\Theta$ by assigning $m_1(A)m_2(B)$ to the set resulting from intersection of sets A and B.

3.2. Identifying Virtual Links One at a Time

A major assumption in D-S theory is that elements of $\Theta$ must be mutually exclusive and exhaustive. Hence, given a frame of discernment containing all potential virtual links, the assumption means that one and only one potential virtual links can be a virtual link. This assumption is too restrictive for this research since it is possible to have multiple virtual links. To overcome this limitation, we propose an approach that employs an iterative feedback loop during the evidence aggregation process, with one virtual link being determined at a time.

This approach continues iteratively until graph partitioning of the latest cavity graph results in subgraphs that represent the union of the involved primitives. The example part shown in Figure 1(b) will be used to explain the proposed approach.

3.2.1 Construction of the Frame of Discernment

To identify the virtual links, a frame of discernment $\Theta$ is needed. In this research, the $\Theta$ should contain all the potential virtual links. Hence, the elements of $\Theta$ consist of the links in the complement of the cavity graph. For example, for the cavity graph shown in Fig. 1(b2), the six links in the complement of the cavity graph form the following frame:

$$\Theta=\{f_1f_3, f_1f_6, f_2f_3, f_2f_6, f_3f_6, f_4f_5\}$$

3.2.2 Evidence Generation

To identify virtual links, evidences are required. Evidences, which describe topologic and geometric relationships, support or disconfirm hypotheses in the frame of discernment through a measure of confidence or a basic probability assignment (bpa). In this approach, we have developed face-based evidences for employment in the above frame of discernment (eq. 2).

Face-based evidences consider particular geometry and topology between a pair of faces and relate the particular observed geometric and topologic relationship to the possible existence of a virtual link between the given pair of faces. For example, it is more probable to have a concave intersection and hence a virtual link between a pair of planar faces which are nearly perpendicular, than between a pair of planar faces which are nearly parallel. Therefore, observation of surface perpendicularity in the CAD solid model description of the faces can be used to assign a higher belief to the hypothesis for a virtual link between them. Similarly, convexity between two faces is an evidence that strongly disconfirms the existence of a virtual link between the involved faces. Specifically, the bpa assignments are based on the following rules.

- If two faces $f_i$ and $f_j$ have orthogonal principle normals, assign $m((f_i f_j))=0.7$.
- If two faces $f_i$ and $f_j$ are concavely adjacent to each other, assign $m((f_i f_j))=0.7$.
- If two faces $f_i$ and $f_j$ are convexly adjacent to each other, assign $m((f_i f_j))=0.8$.
- If two faces $f_i$ and $f_j$ have almost parallel principle normals, assign $m((f_i f_j))=0.8$.
- 'c' here stands for complement.

Based on above rules, the BPA assignments for the example part shown in Fig. 1(b) can be summarized in Table 1.

| Table 1 BPA assignments by face-based evidences for the example part |
|-------------------------|-----------------|
| $m_1(f_1f_3)=0.7$      | $m_1(\emptyset)=0.3$ |
| $m_2(f_1f_6)=0.7$      | $m_2(\emptyset)=0.3$ |
| $m_3(f_2f_3)=0.7$      | $m_3(\emptyset)=0.3$ |
| $m_4(f_2f_6)=0.7$      | $m_4(\emptyset)=0.3$ |
| $m_5(f_3f_6)=0.7$      | $m_5(\emptyset)=0.3$ |
| $m_6(f_3f_6)=0.8$      | $m_6(\emptyset)=0.2$ |
| $m_7(f_4f_5)=0.7$      | $m_7(\emptyset)=0.3$ |

3.2.3 Experimental Results

Given the frame in eq. 2, the Dempster's rule of combination was then used to aggregate the face evidences in table 1. Fig. 5 gives the aggregated beliefs in each of six potential virtual links.

Links with highest beliefs are shown in bold in figure 5. They are links $(f_3f_1)$ and $(f_6f_2)$. Due to the mutual
exclusiveness assumption, only one of the six link can be a virtual link. Select one of the two (say, \( f_3 f_1 \)) and apply this approach again. The second virtual link \( (f_6 f_2) \) is identified.

\[
\begin{align*}
\text{Links} & \quad \text{Combined BPA's} \\
\{f_3 f_1\} & \quad 0.366 \\
\{f_6 f_2\} & \quad 0.366 \\
\{f_3 f_6\} & \quad 0.110 \\
\{f_2 f_5\} & \quad 0.000 \\
\{f_3 f_2\} & \quad 0.000 \\
\{f_6 f_1\} & \quad 0.000 \\
\end{align*}
\]

Figure 5 Combined BPA's for the example part shown in Fig. 1(b)

The latest cavity graph augmented with the two virtual links is shown in figure 3(a), where the two augmented virtual links are shown by dotted lines. Partitioning the modified graph identifies two pockets, one with base-face 1 and side-faces 2, 3, 4, 5, and the other with base-face 2 and side-faces 1, 4, 5, 6, as part features of this object. This is the correct interpretation for the example part.

3.3 Identifying Multiple Virtual Links

One obvious disadvantage with the above approach is that it cannot determine multiple virtual links simultaneously due to the mutual exclusiveness assumption. It needs several iterations to recover all necessary virtual links. For each iteration, it constructs a frame of discernment, and then performs evidences-gathering and aggregation. This greatly increases the computational complexity. In this section, we propose a method for identifying multiple virtual links simultaneously while still observing the mutual exclusiveness assumption.

3.3.1 Construction of the Frame of the Discernment

Dealing with multiple cause sources, i.e., the simultaneous presence of several correct hypotheses, adds considerable complexity to the inference process and problem formulation due to the mutual exclusiveness constraint. A novel approach was developed in this research for handling multiple virtual links. The main differences between this approach and the earlier approach are the formulation of the frame of discernment and the interpretation of the evidences. Unlike the frame in the earlier approach, which contains only all the potential virtual links (only the singleton hypotheses). In this approach, \( \Theta \) is selected to include all subsets of the set of potential virtual links. For the example shown in Fig.1(b), we may choose \( \Theta \) to contain all subsets of all possible virtual links, i.e., a total of \( 2^6 \) elements in \( \Theta \), as shown below:

\[
\Theta = \{ \{1 f_3\}, \{1 f_6\}, \{2 f_3\}, \{2 f_6\}, \{3 f_6\}, \{4 f_5\}, \\
\{f_6 f_3\}, \{f_6 f_7\}, \{f_3 f_6\}, \{4 f_6\}, \{f_3 f_6\}, \{f_3 f_8\}, \{f_3 f_6\}, \{f_3 f_8\}, \{f_3 f_6\}, \{f_3 f_8\}, \{f_3 f_6\}, \}
\]

One disadvantage with this frame formulation is the computational complexity caused by the huge number of elements in the frame. Given \( n \) singleton hypotheses, there would be \( 2^n \) subsets included in the frame. This problem can be overcome by using domain-specific heuristic knowledge to significantly decrease the scope of the problem, based on the assumption that only certain elements in \( \Theta \) are of semantic interests to us. Those elements can therefore be selected to form a much smaller frame. For this research, the frame pruning is based on the heuristic assumption that an evidence bearing directly on a hypothesis in the frame points to its presence, while non-evidence on a hypothesis in the frame points to its absence. This also applies to negative evidences since a hypothesis with negative evidences should be preserved to evaluate their impacts on other hypotheses. With very limited evidences, this assumption ensures that a large number of hypotheses in the frame would be pruned, resulting in a manageable size of frame of discernment. Consider the frame in equation 3 for example, where a frame of this size is manageable and therefore must be reduced. Based on the pruning assumption and the available evidences, we can prune the elements in above frame to obtain the new frame as shown below. All hypotheses in the new frames have evidences bearing directly on them.

\[
\Theta = \{ \{1 f_3\}, \{1 f_6\}, \{2 f_3\}, \{2 f_6\}, \{3 f_6\}, \{4 f_5\}, \\
\{1 f_3, 2 f_6\}, \{f_1 f_3, f_3 f_6\}, \{f_1 f_3, 2 f_6, f_3 f_6\} \}
\]

3.3.2 Feature Evidences

While the combination of the face-based evidences exerting impact upon singleton subset of \( \Theta \) may sometimes indicate which hypotheses should be preferred over others, the face-based evidences by themselves may be insufficient for deciding the correct choices, because they only carry local information about the object. They are only concerned with the geometry and topology between two faces. Furthermore, the face-based evidences do not allow us to directly exploit the hypotheses at different abstraction levels, because they only interact with hypotheses of the singleton subsets of \( \Theta \).

Another type of evidence, referred to as feature-based evidences, was therefore developed. Feature-based evidences provide and integrate information which is more global and may encompass one or more virtual links. For example, a group of faces may satisfy certain properties which are commonly found in pockets (e.g. in a group of side-faces, forming a loop, each side-face is concavely adjacent to two other neighboring side-faces), then this information about this collection of faces is used by a
feature-based evidence to support the hypothesis for those virtual links involved in such a pocket.

Feature-based evidences may simultaneously support one or more virtual links. Consequently, these evidences may exert their impact upon hypotheses at different abstraction levels. For instance, in terms of our example object shown in figure 1(b), the collection of faces 1, 2, 3, 4, and 5 satisfy most properties of a pocket template including the above stated property that every side-face (2, 3, 4, 5) is concavely adjacent to two other side-faces. As a result, the subgraph representing a pocket by this collection of faces constitutes a pocket feature evidence supporting the hypothesis for a virtual link between 1 and 3. It is clear that in supporting a virtual link between a pair of faces, feature-based evidences consider and evaluate the relationships of those faces with their surrounding faces and edges (topologic entities). Thus, they represent and apply information at a higher level of abstraction.

As required by the earlier approach, where each evidence is associated with a bpa to indicate the belief in a hypothesis (or a set of hypotheses) given an evidence, a bpa is also required for each feature evidence. Since feature evidences are positive evidences, bpa’s are always assigned directly to links supported by the evidences rather than to their complements. The numerical values of bpa’s are determined empirically. Two values of 0.9 and 0.6 are used. While 0.9 is assigned to feature evidences containing only one potential virtual link, 0.6 is assigned to evidences containing more than one potential virtual link. The rational is that there is less ambiguity associated with evidences that have fewer potential virtual links and therefore higher belief should be assigned to such hypotheses.

3.3.3 Evidence Interpretation through the Principle of Association

The frame of discernment shown in eq. 1 contains both singleton and non-singleton elements. Further, the multi-link (non-singleton) elements in the frame represent the conjunction rather than disjunction of its constituents. The available evidences must, therefore, be interpreted differently. A theory of principle of association was developed through this research to assign beliefs to hypotheses in a coherent and consistent manner. The principle of association relates an evidence to all hypotheses that are implied by the evidence. For example, any evidence that confirms the link 1-3 signifies its support for any hypotheses, be it singleton or composite, that contain the link 1-3 or a proposition set of \{f_1 f_3\} or \{f_1 f_3, f_2 f_6\} or \{f_1 f_3, f_2 f_6, f_3 f_6\} for our example part. For this research, this principle of association can be summarized as follows:

- An evidence disconfirming a link or a set of links actually commits its belief to a proposition that contains all disjoint elements of the frame that do not have the link or the set of links.

Based on these principles, the evidences in our example can redirect their beliefs to proper hypotheses.

3.3.4 Experimental Results

In this section, we will show an example of the implementation of this new approach to determine multiple virtual links simultaneously. The example part is shown in Fig. 1(b). The frame of discernment is obtained by including all the subsets of the potential virtual links, followed by pruning the initial frame to eliminate the impossible hypotheses as shown in equation 4. Both face and feature evidences are applicable to this approach. Therefore, by applying both face evidences and feature evidences to the frame in equation 4, by using above principle of association, and then by combining them using equation 1, we obtain the combined bpa’s for the hypotheses shown in Figure 6.

- An evidence disconfirming a link or a set of links actually commits its belief to a proposition that contains all disjoint elements of the frame that do not have the link or the set of links.

Based on these principles, the evidences in our example can redirect their beliefs to proper hypotheses.

3.3.4 Experimental Results

In this section, we will show an example of the implementation of this new approach to determine multiple virtual links simultaneously. The example part is shown in Fig. 1(b). The frame of discernment is obtained by including all the subsets of the potential virtual links, followed by pruning the initial frame to eliminate the impossible hypotheses as shown in equation 4. Both face and feature evidences are applicable to this approach. Therefore, by applying both face evidences and feature evidences to the frame in equation 4, by using above principle of association, and then by combining them using equation 1, we obtain the combined bpa’s for the hypotheses shown in Figure 6.

\[
\begin{align*}
\tau(f_1 f_3, f_2 f_6) = 0.532 & \\
\tau(f_1 f_3, f_2 f_6) = 0.039 & \\
\tau(f_2 f_6, f_3 f_6) = 0.09 & \\
\tau(f_2 f_6, f_3 f_6) = 0.009 & \\
\tau(f_2 f_6, f_3 f_6) = 0.0141 & \\
\tau(f_2 f_6, f_3 f_6) = 0.231 & \\
\end{align*}
\]

Figure 6 Combined BPA’s for the example part shown in Fig. 1(b)

Since hypothesis \{f_1 f_3, f_2 f_6\} has the highest belief committed to it, it is selected and as a result, the most probable virtual links are link 1-3 and link 2-6. This result is the same as the results obtained through the earlier approach. By comparison, it is clear that this approach has the advantage of identifying all virtual links simultaneously, greatly reducing the computational complexity.

4. Conclusion

The main contributions of this research work can be summarized as introducing a method to represent and model knowledge with uncertainty in feature recognition, developing different classes of evidences based on the geometric and topologic relationships at different abstraction levels for effective reasoning, developing the principle of association, which enables us to identify multiple virtual links simultaneously by overcoming the mutual exclusiveness assumption of the Dempster-Shafer theory, and experimenting and investigating the efficacy of these methods. Development of methods such as those described here are important for overcoming the non-uniqueness in topologic and geometric description of
various features in different interactions. The non-uniqueness of feature descriptions in different interactions has been one of the major hindrances in automated identification and extraction of features from CAD solid models.

Although the proposed methods can successfully extract the interacting features for many parts, much work still remains to be done. One immediate goal for us is to generalize the developed methodology to identify and extract features with curved surfaces from components with non-planar surfaces.

References


