Russel Ohl – Inventor of a p-n junction

The most important professor's function
Is to explain the workings of a p-n **Junction**

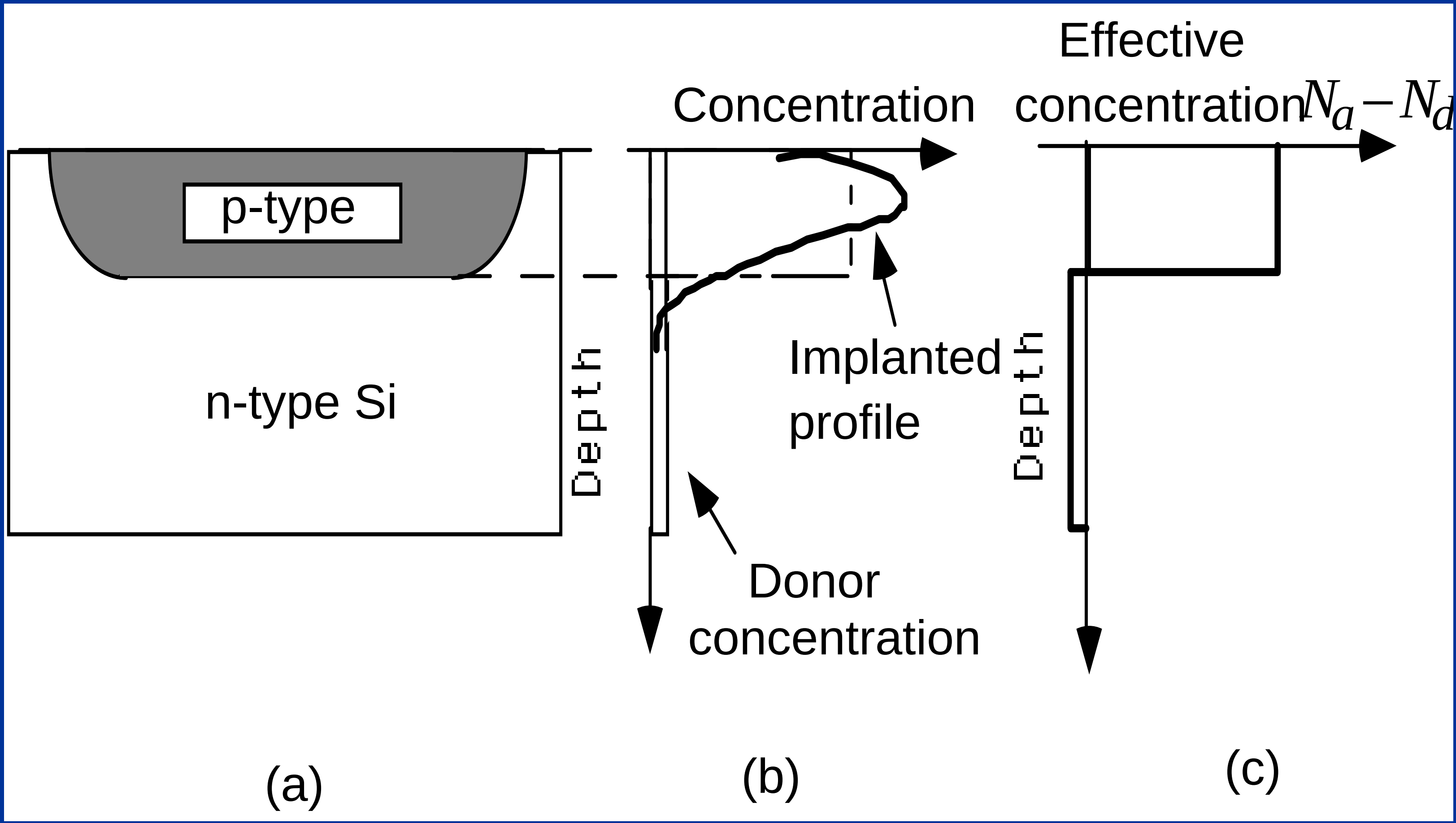
From http://nina.ecse.rpi.edu/shur/Alphab.htm

1940

Professor Karmalkar
ITT Madras

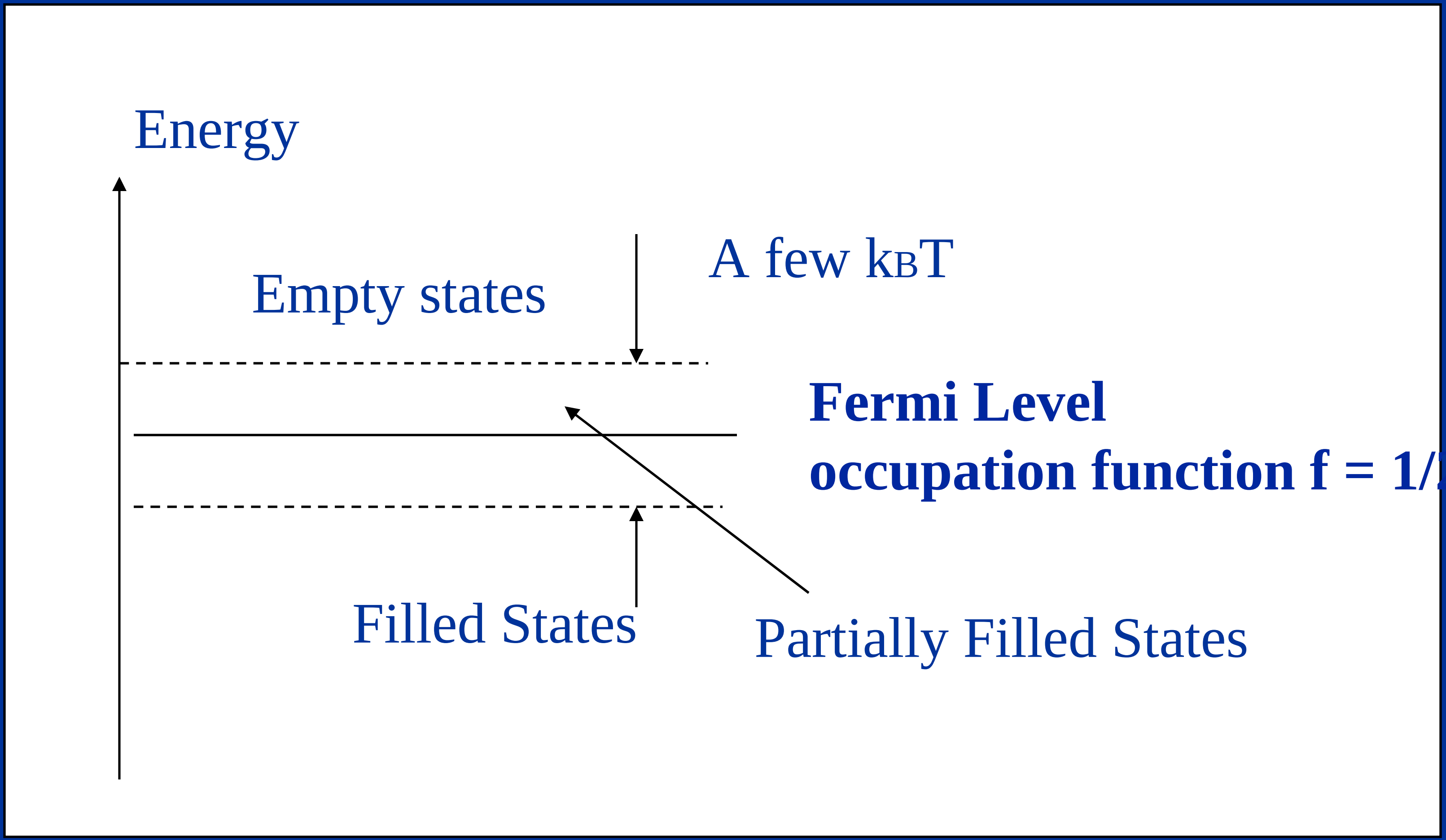
http://www.youtube.com/watch?v=lMoJUqDlSQs

p-n Junction



Boron implantation into n-type Si to produce a p region

Fermi Level


Energy
Empty states
A few k_BT
Fermi Level
occupation function f = 1/2
Filled States
Partially Filled States

Fermi Level in Equilibrium

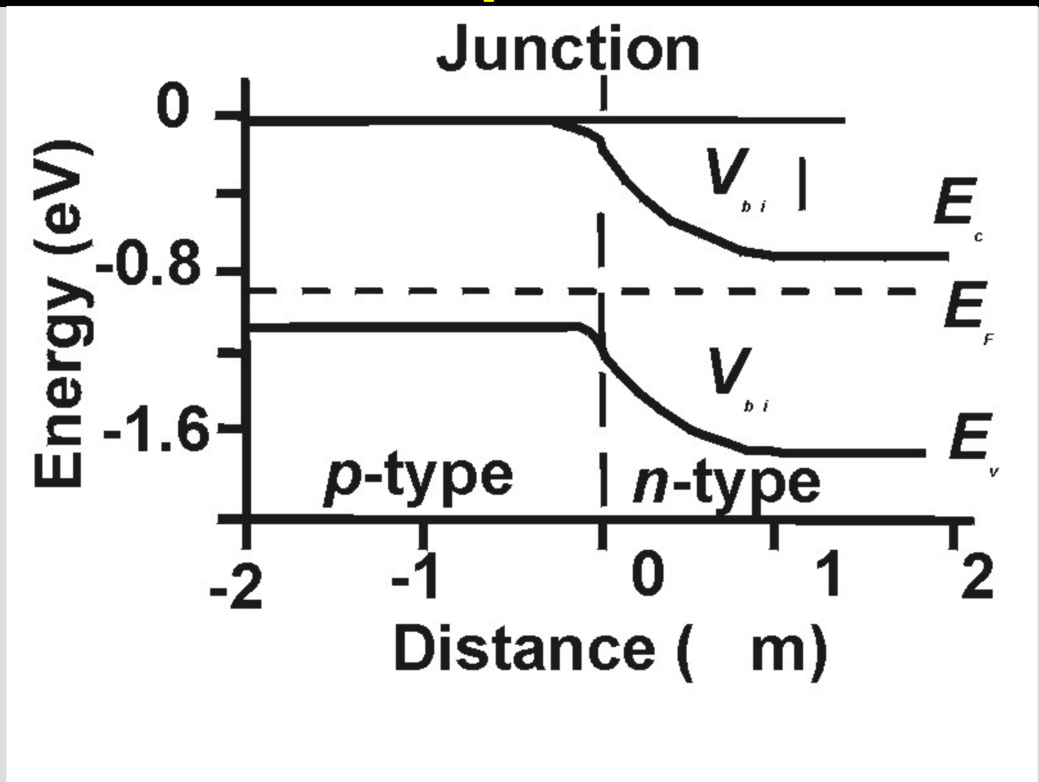
In equilibrium, the electron and hole current densities must be zero, and the quasi-Fermi levels of electrons and holes coincide with the position of the Fermi level, E_F

$$j_p = \mu_p p \frac{\partial E_{Fp}}{\partial x} = \mu_p p \frac{\partial E_F}{\partial x} = 0$$

$$j_n = \mu_n n \frac{\partial E_{Fn}}{\partial x} = \mu_n n \frac{\partial E_F}{\partial x} = 0$$

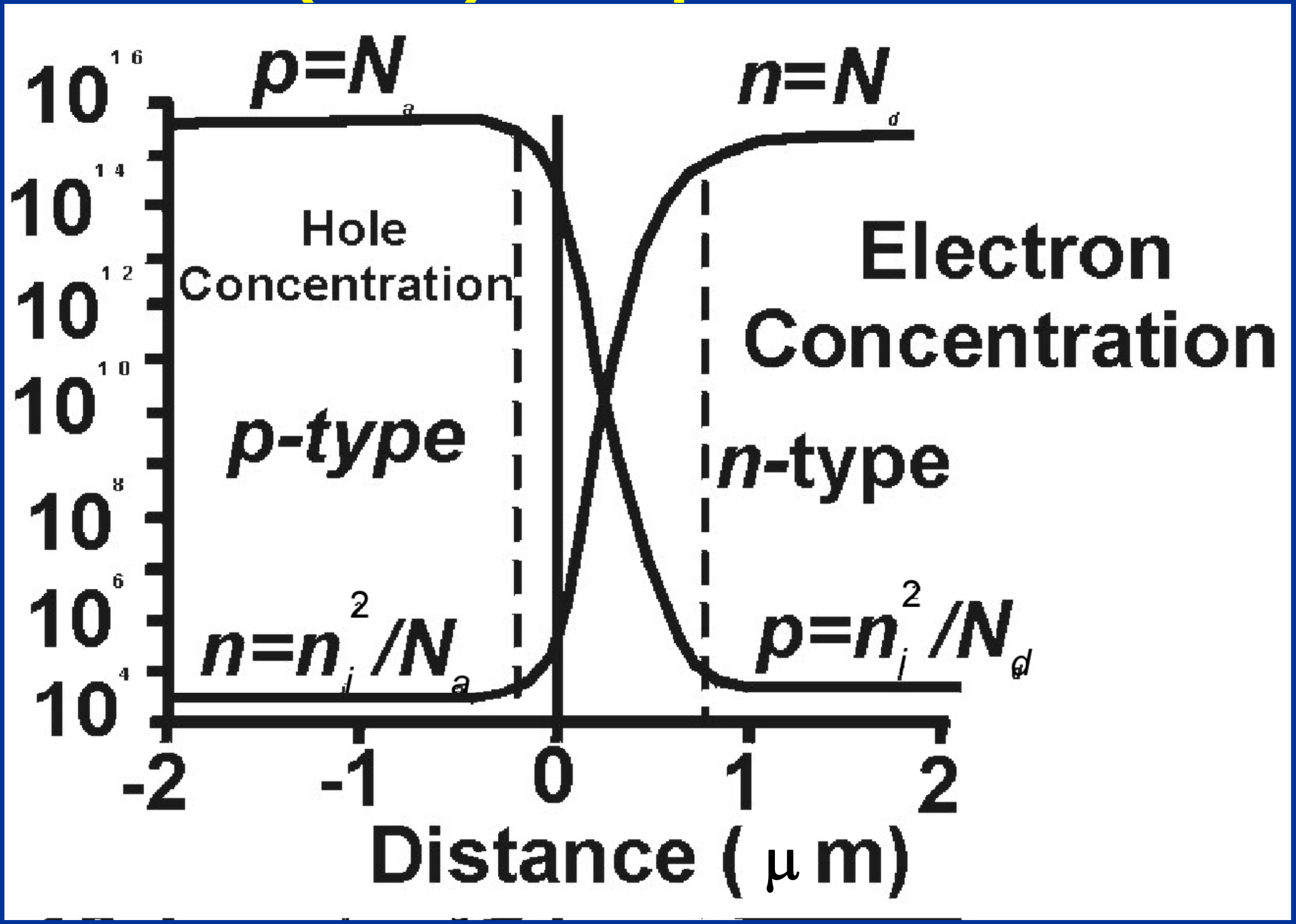
Where j_n, j_p, n, p, μ_n, μ_p, E_{Fn}, and E_{Fp} are electron and hole current densities, carrier concentrations, mobilities, and quasi-Fermi levels, respectively.

Energy Band Diagram of Si p-n junction at Equilibrium



Acceptor and donor densities 5x10^{15} cm^{-3} and 10^{15} cm^{-3}. T = 300 K.

Electron and Hole Concentrations (cm^{-3}) at Equilibrium



Acceptor and donor densities $5\times10^{15}cm^{-3}$ and $10^{15}cm^{-3}$. T = 300 K.

Depletion Approximation

Charge density in the *n*-type depletion region

$$\rho_n \approx qN_d$$

Charge density in the p-type depletion region

$$\rho_p \approx -qN_a$$

Charge densities of free carriers in the depletion region are much smaller

Electron and Hole Concentration Profiles in n-type Region

$$n = N_d \exp\left(\frac{\phi(x)}{V_{th}}\right) = N_d \exp\left(-\frac{E_c(x)}{qV_{th}}\right)$$

$$p = N_a \exp\left[-\frac{V_{bi} + \phi(x)}{V_{th}}\right] = N_a \exp\left[\frac{-qV_{bi} + E_c(x)}{qV_{th}}\right]$$

Built-in voltage $V_{bi} = V_{th} \ln\left(\frac{N_d N_a}{n_i^2}\right)$

Built-in Voltage

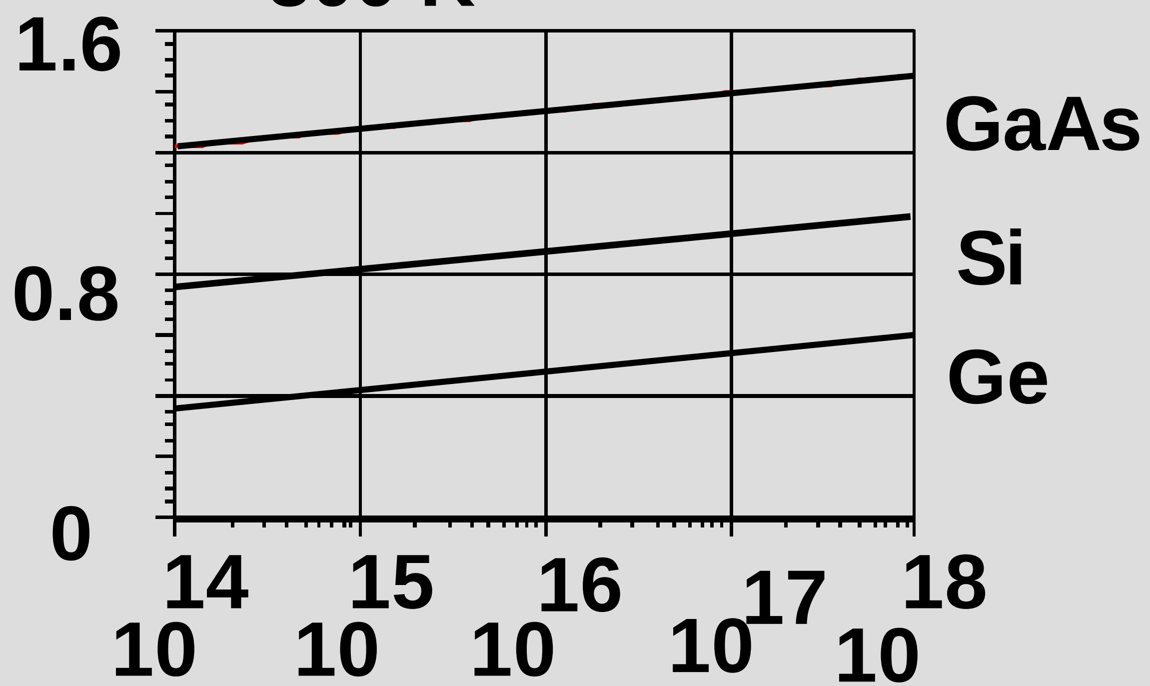
Since, $n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2k_B T}\right)$

$$V_{bi} = E_g + V_{th} \ln\left(\frac{N_d N_a}{N_c N_v}\right)$$

Built-in voltage is approximately proportional to the energy gap.

Built-in Voltage (cont.)

Built-in voltage for n^+-p junctions versus acceptor concentration (cm^{-3})

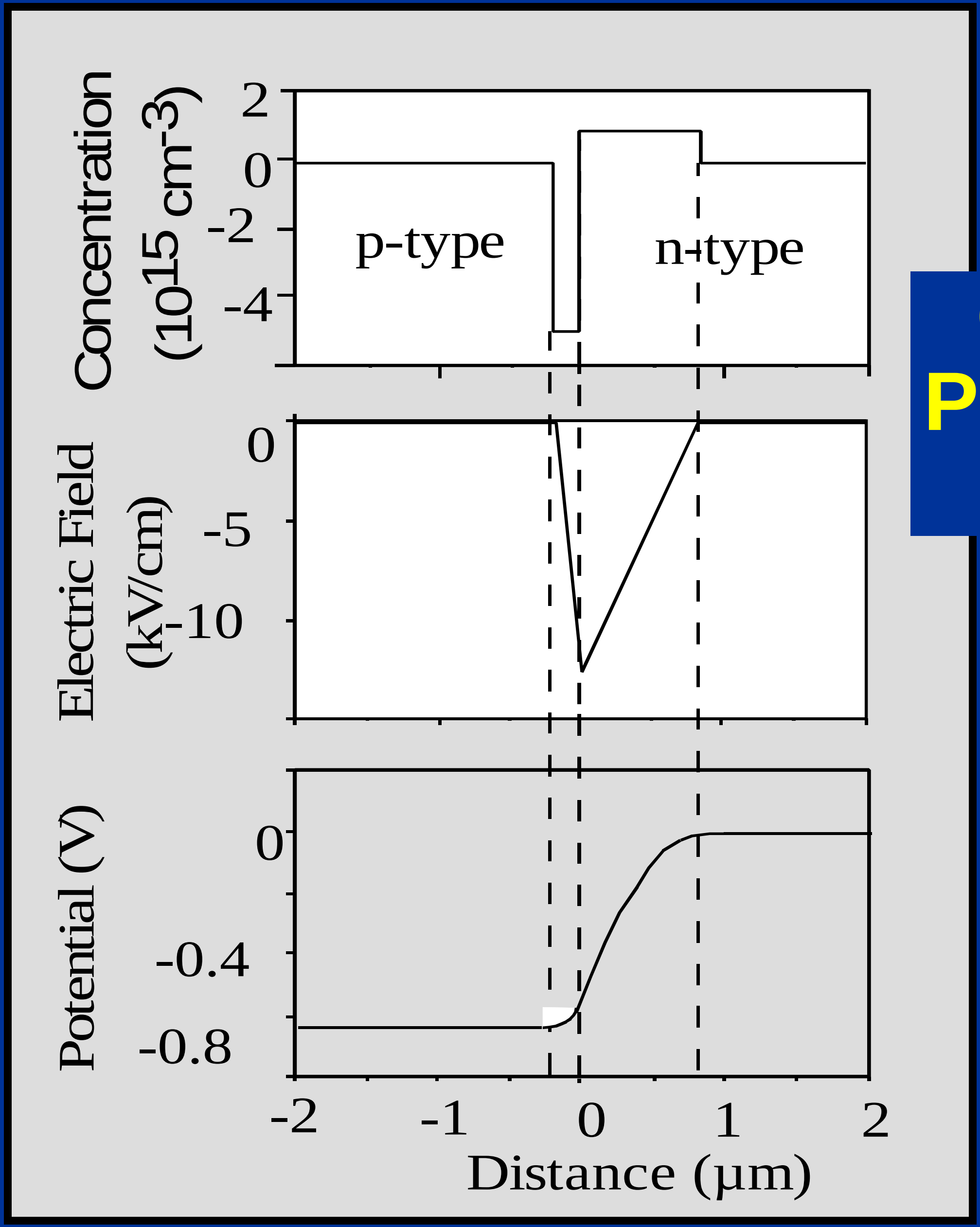


Charge, Field, Potential Profiles

Using the depletion approximation, we obtain

$$\frac{dF}{dx} = \begin{cases} -\dfrac{qN_a}{\varepsilon_s}, for -x_p < x < 0 \\ \dfrac{qN_d}{\varepsilon_s}, for\, 0 < x < x_n \end{cases}$$

$$F = \begin{cases} -F_m\left(1+\dfrac{x}{x_p}\right), for -x_p < x < 0 \\ -F_m\left(1-\dfrac{x}{x_n}\right), for\, 0 < x < x_n \end{cases}$$



Charge, Field, Potential Profiles (cont.)

Depletion Width

$$\frac{x_n}{x_p} = \frac{N_a}{N_d} \qquad \frac{qN_d x_n^2}{2\varepsilon_s} + \frac{qN_a x_p^2}{2\varepsilon_s} = V_{bi}$$

$$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_d\left(1 + N_d / N_a\right)}}$$

$$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_a\left(1 + N_a / N_d\right)}}$$

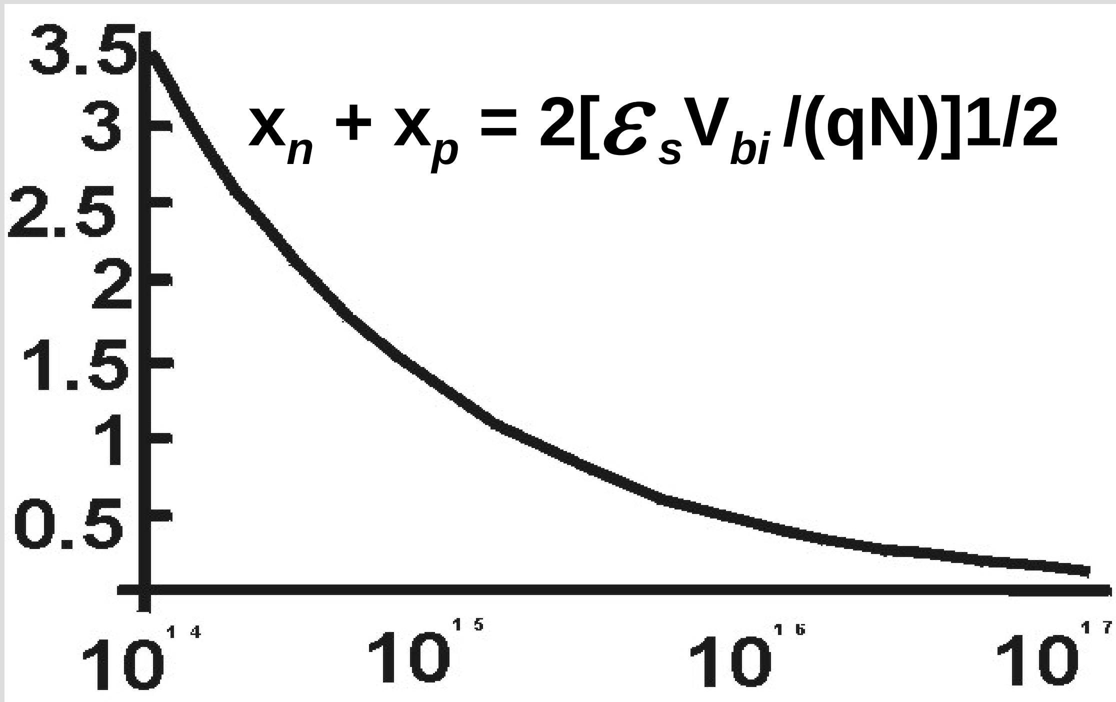
Example

Calculate and plot the total depletion width, $x_n + x_p$, as a function of doping for a Si *p-n* junction with the donor concentration in the *n*-type region, N_d, equal to the acceptor concentration in the *p*-region, N_a (such a junction is called a symmetrical junction). Temperature T = 300 K. The dielectric permittivity of Si is 1.05×10^{-10} F/m. The intrinsic carrier concentration is approximately 10^{10} cm^{-3}. Vary the doping density from 10^{14} cm^{-3} to 10^{17} cm^{-3}.

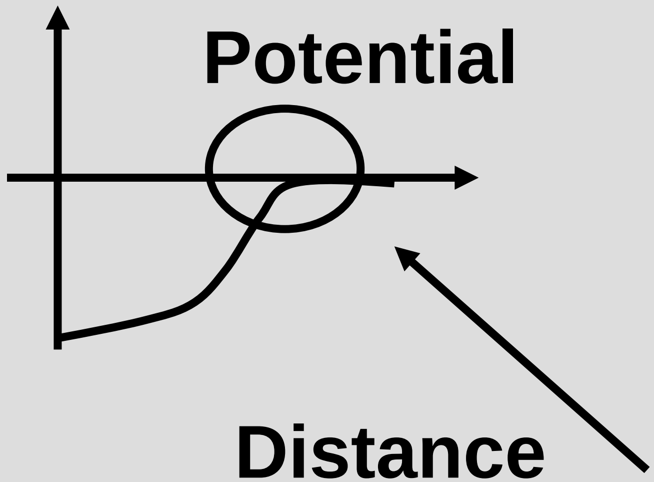
Solution

For a symmetrical junction, $V_{bi} = 2V_{th}\ln(N/n_i)$

Depletion width versus doping (cm^{-3})



Debye Length - Boundary Region



$\phi = 0$

Neutral n-region

Let us consider the region where $\phi < kT/q$

$$\frac{d^2\phi}{dx^2} = -\frac{qN_d}{\varepsilon_s} + \frac{qN_d}{\varepsilon_s}\exp\left(\frac{\phi}{V_{th}}\right)$$

Debye Length

Expanding the exponent into a Taylor series

$$\frac{d^2\phi}{dx^2} \approx \frac{qN_d\phi}{\varepsilon_s V_{th}} = \frac{\phi}{L^2_{Dn}}$$

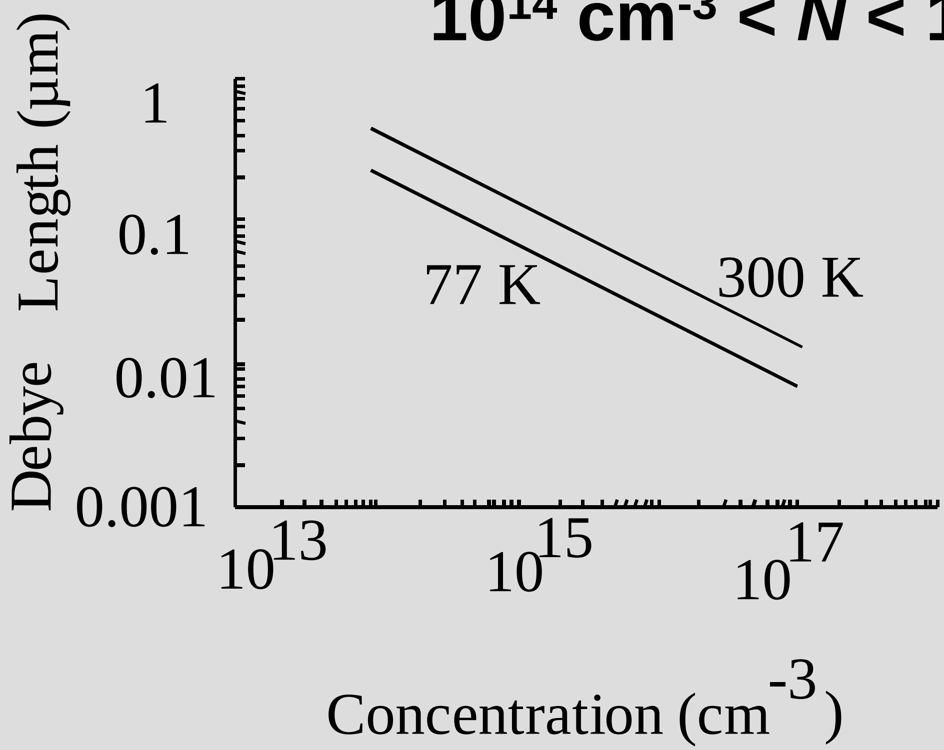
where

$$L_{Dn} = \sqrt{\frac{\varepsilon_s V_{th}}{qN_d}}$$

Example

Calculate and plot the Debye length in GaAs versus doping density at *T* = 77 K and 300 K

10^{14} cm^{-3} < *N* < 10^{17} cm^{-3}



Dielectric permittivity

ε = 1.14x10^{-10} F/m

Summary of Equations Describing p-n Junctions at Zero Bias

Maximum electric field in a *p-n* junction	$F_m = \frac{qN_d x_n}{\varepsilon_s} = \frac{qN_a x_p}{\varepsilon_s}$
Built-in voltage	$V_{bi} = V_{th} \ln\left(N_d N_a / n_i^2\right)$
Width of depletion regions	$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_d\left(1 + N_d / N_a\right)}}$, $x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_a\left(1 + N_a / N_d\right)}}$
Debye lengths	$L_{Dn} = \sqrt{\frac{\varepsilon_s V_{th}}{qN_d}}$, $L_{Dp} = \sqrt{\frac{\varepsilon_s V_{th}}{qN_a}}$

p-n JUNCTIONS Under BIAS

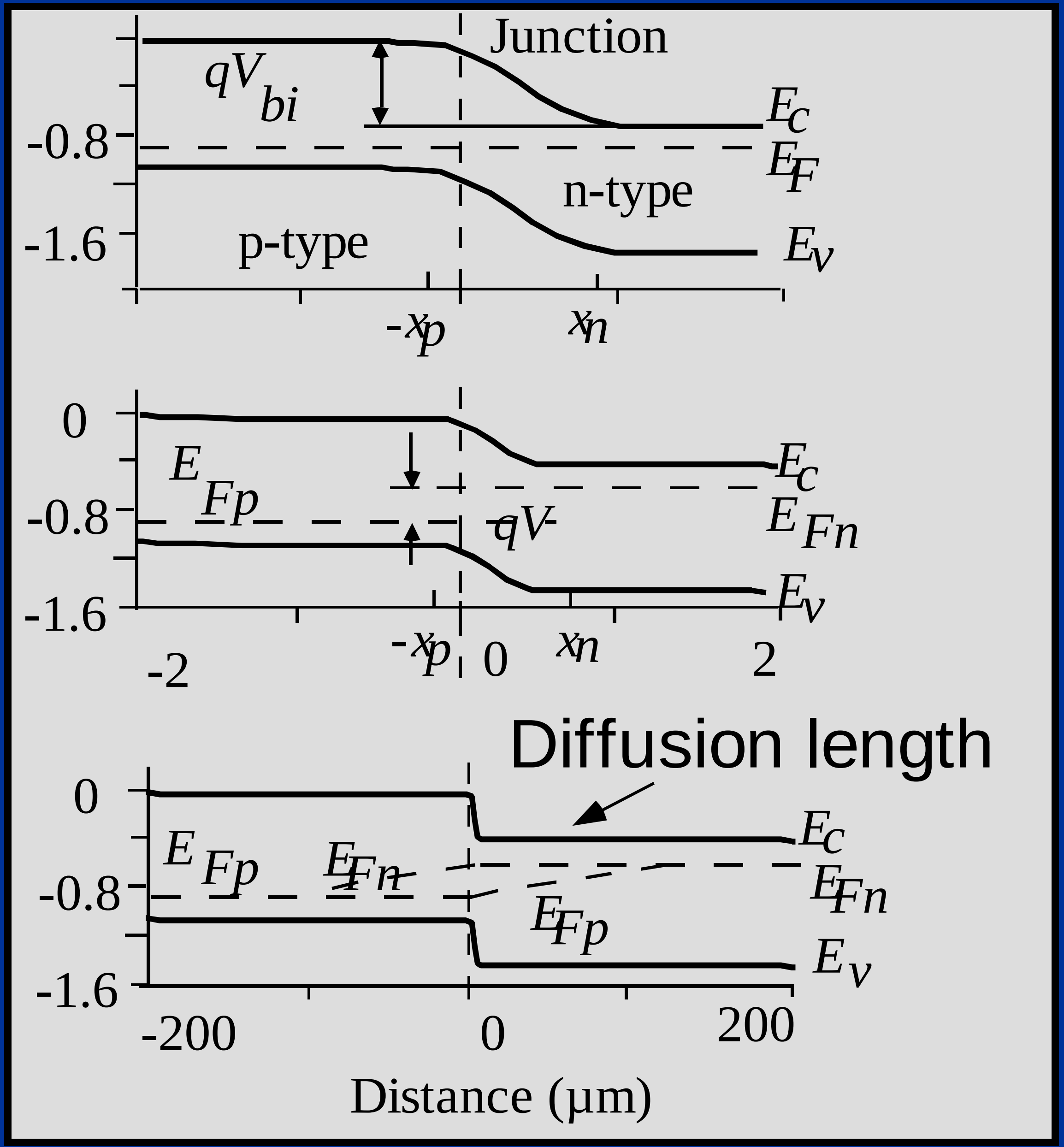
For small voltages, the net current in a p-n junction is still much smaller than the drift and diffusion currents in the depletion region evaluated separately, and, consequently, the electron and hole quasi-Fermi levels defined by

$$n = N_c \exp\frac{E_{Fn} - E_c}{k_B T}$$

$$p = N_v \exp\frac{E_v - E_{Fp}}{k_B T}$$

Remain nearly constant throughout the depletion region.

Band Diagrams



Law of the Junction

$$qV = E_{Fn} - E_{Fp}$$

$$pn = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{k_B T}\right) = n_i^2 \exp\left(\frac{V}{V_{th}}\right)$$

Example

The intrinsic carrier concentration in GaAs at 300 K is approximately 10^6 cm^{-3}. Estimate the forward bias required to create the electron-hole density of 10^{17} cm^{-3} in the depletion region at the point where p = n.

Solution

From the law of the junction

$$V = V_{th} \ln \frac{pn}{v^2{}_i} = 0.02584 \times \ln \frac{10^{34}}{10^{12}} = 1.308V$$

Diffusion Length of Minority Carriers

Diffusion Equation $D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_{pl}} = 0$

Solution of the Diffusion Equation

$$p_n(x) - p_{no} = A\exp\left(\frac{x - x_n}{L_p}\right) + B\exp\left(-\frac{x - x_n}{L_p}\right)$$

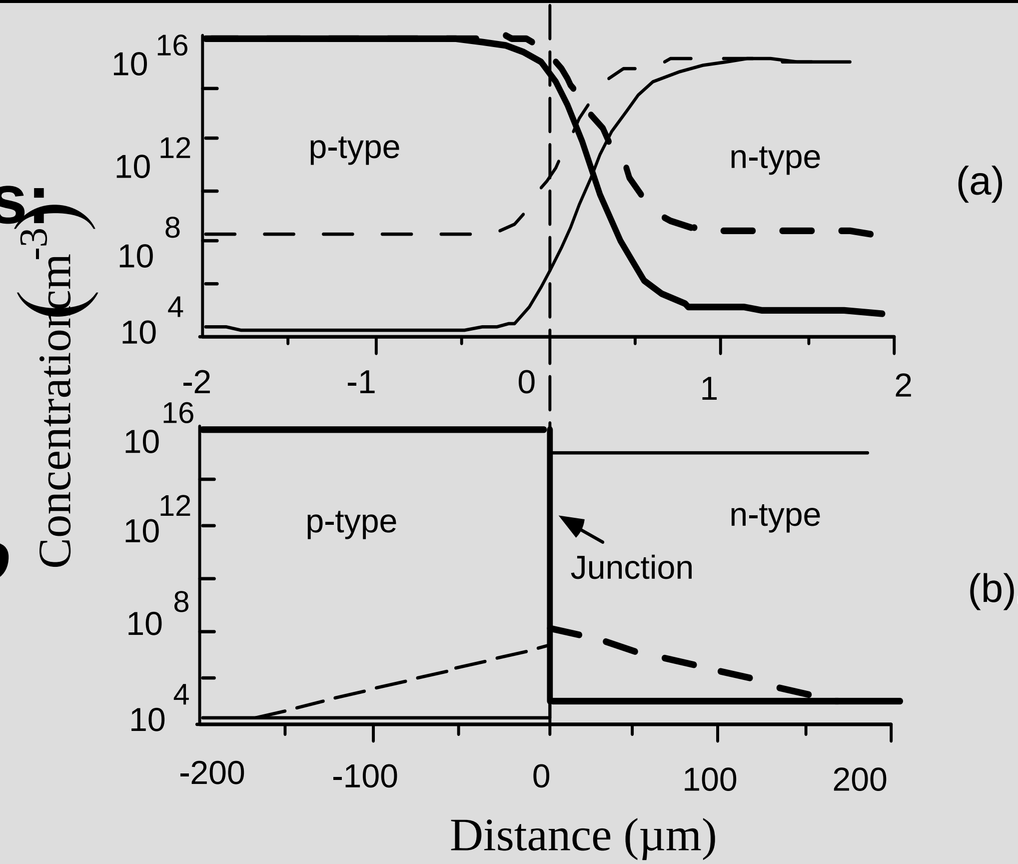
Hole Diffusion Length $L_p = \sqrt{D_p \tau_{pl}}$

Concentration Profiles

Boundary conditions:

$$p_n(x_n) = p_{no} \exp\left(\frac{V}{V_{th}}\right)$$

$$p_n(x \to \infty) = p_{no}$$



Solution:

$$p_n(x) - p_{no} = p_{no}\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right]\exp\left(-\frac{x - x_n}{L_p}\right)$$

Example

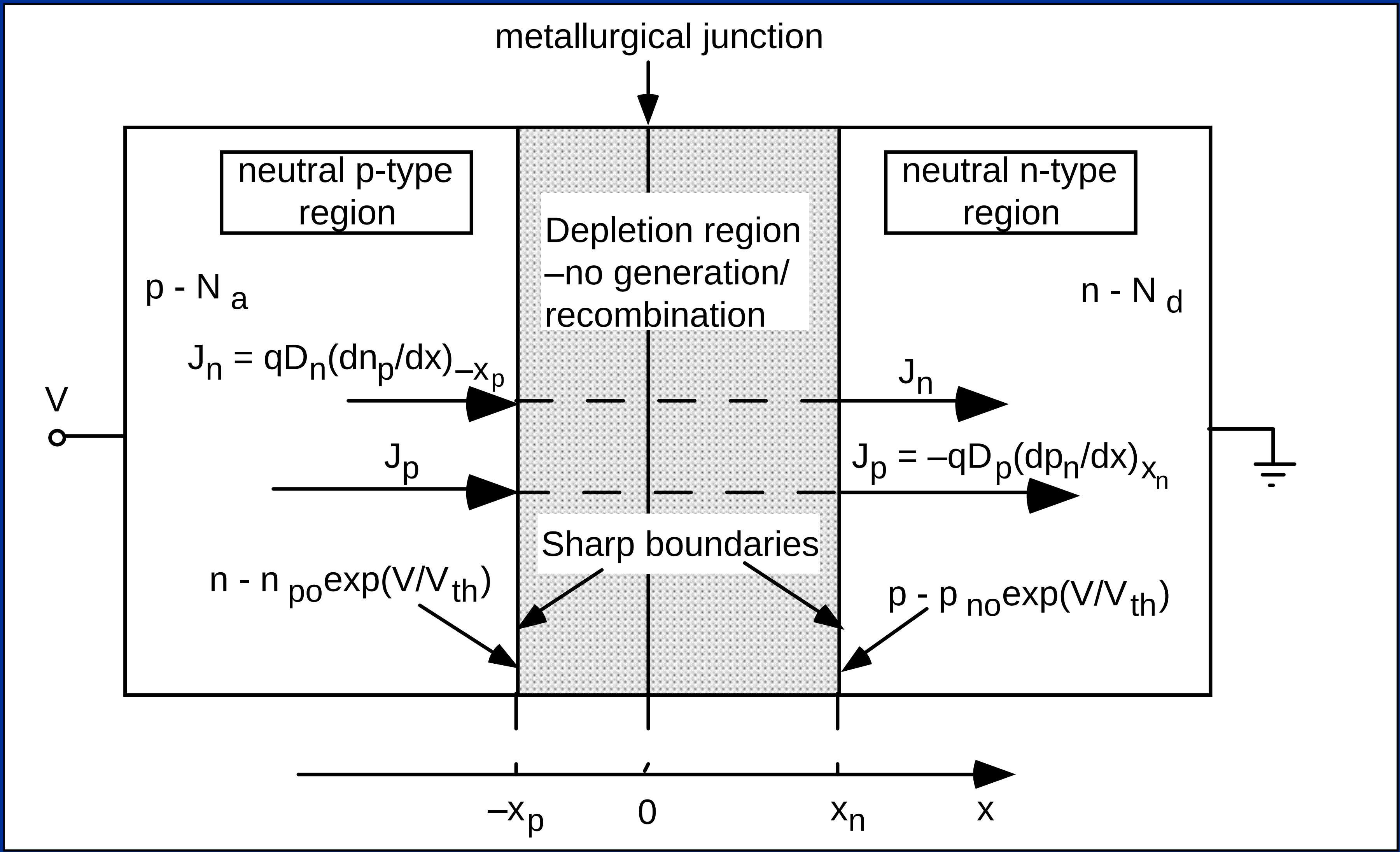
The slope of the semilog dependence of the hole concentration on a distance in the *n*-type region of a *p-n* diode at room temperature (*T* = 300 K) is 1 decade/120μ m. The hole mobility is 250 cm^2/V. What is the hole lifetime?

Solution

The slope in the semilog scale is equal to $1/[L_p \ln(10)]$. Hence L_p = 120/ln(10)=52.1 m. Using the Einstein relation, we find D_p = V_{th} = 250x0.02584 = 6.46 (cm^2/s). Hence, the hole lifetime,

$$\tau_{pl} = \frac{L^2{}_p}{D_p} = \frac{\left(52.1 \times 10^{-4}\right)^2}{6.46} = 4.20 \times 10^{-6}(s)$$

I-V Characteristics of p-n Junctions



Assumptions

- All applied voltage drops across depletion region
- Low-level injection only
- Sharp depletion boundaries
- No generation-recombination processes inside depletion region

Procedure

- Find minority carrier concentration at both boundaries (Use the Law of the Junction)
- Find distribution of minority carriers inside neutral regions (Diffusion equation)
- Find minority carrier diffusion current at boundaries (Gradients of concentrations)
- Total current density is the sum of the minority carrier diffusion currents at the two boundaries ($J = J_n + J_p$)

For the Ideal p-n Junction

The ideal diode equation:

$$J = J_s\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right]$$

Saturation current:

$$J_s = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}$$

Diffusion lengths:

$$L_n = \sqrt{D_n \tau_n} \qquad L_p = \sqrt{D_p \tau_p}$$

Ideal Diode Equation

$$j_p \approx j_{pD} = -qD_p \frac{\partial p_n}{\partial x} = \frac{qD_p p_{no}}{L_p}\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right]\exp\left(-\frac{x - x_n}{L_p}\right)$$

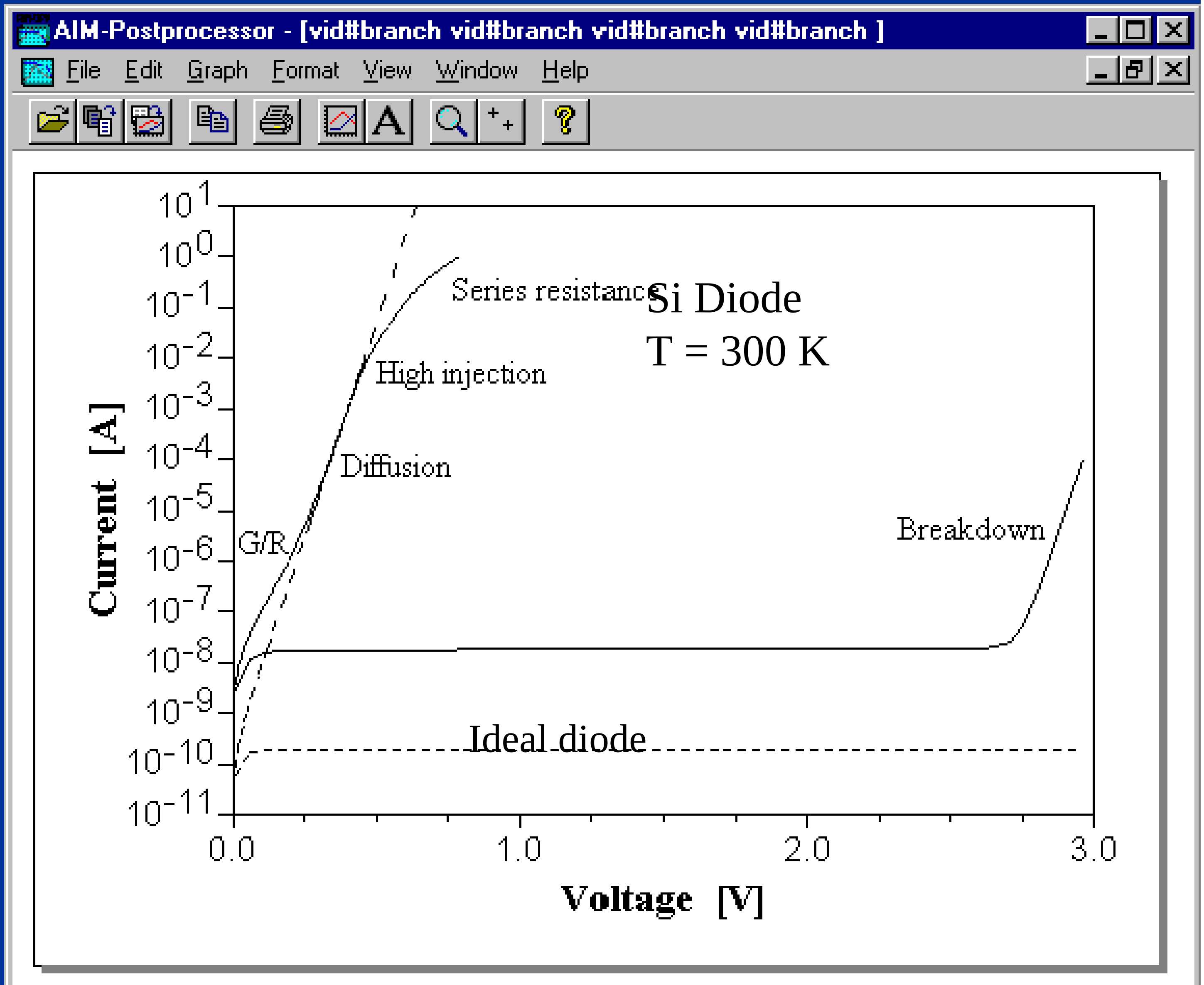
$$j \approx j_{Dp}\Big|_{x=x_n} + j_{Dn}\Big|_{x=-x_p}$$

$$I = I_s\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right] \qquad I = j\,S$$

The diode saturation current is

$$I_s = S\left(\frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n}\right)$$

Non-Ideal Diode Effects



From T. Fjeldly, T. Ytterdal, and M. S. Shur, Introduction to device modeling and circuit simulation, Wiley (1997)

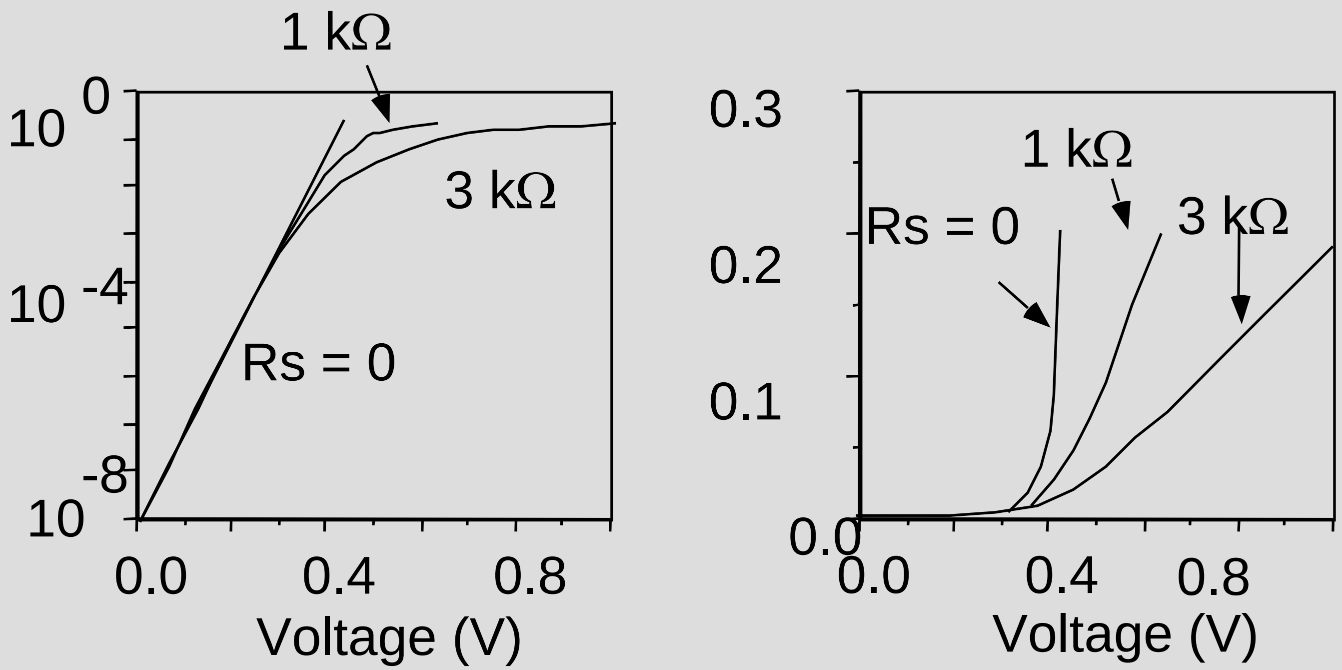
A. Series Resistance

$$I = I_s \left[\exp\left(\frac{V - IR_s}{V_{th}} \right) - 1 \right]$$

B. High Injection

- At high forward bias ($V \Rightarrow V_{bi}$), the injected minority carrier concentration becomes comparable to the majority carrier concentration
- The neutrality condition demands a similar increase in both electron and hole concentrations
- This reduces the growth in the minority carrier concentration and in the current

Effect of Series Resistance



Current-voltage characteristics of p-n diode *Iseff*= 10^{-8} mA. The characteristics are shown in semilog and linear scales. (Assumed η = 1)

B. High Injection

Law of the junction: $\begin{cases} pn = p(N_d + p) = n_i^2 \exp(V/V_{th}) \\ \Rightarrow \quad p \to n_i \exp(V/2V_{th}) \text{ when } p \geq N_d \end{cases}$

Diode current: $I \propto \exp(V/2V_{th})$

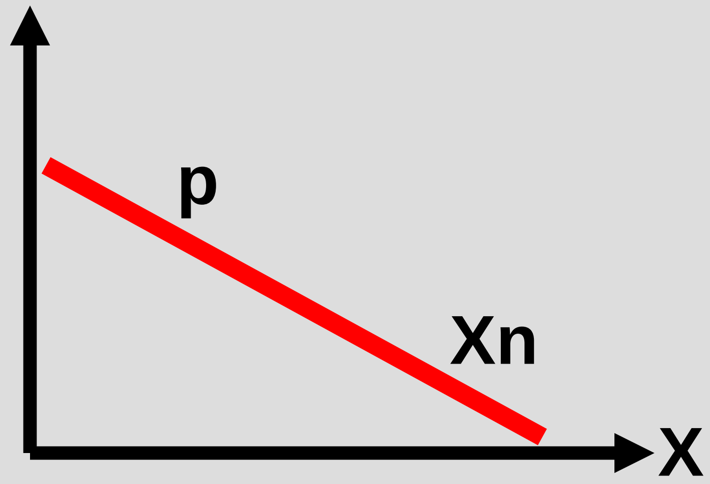
C. G/R Processes in the Depletion Region

G/R processes seek to restore thermal equilibrium: $pn \Rightarrow n_i^2$

Reverse biased $pn < n_i^2$

$$J_{gen} = q \int_{-x_p}^{x_n} |G - R| dx = \frac{q n_i x_d}{\tau_{gen}}$$

Short p^+-n Diode



$X_n << L_p$

Current-Voltage Characteristic

When the length of the *n*-section, *Xn*, is much smaller than the hole diffusion length *Lp*, the solution of the diffusion equation is given by

$$p_n(x) \approx p_{no} + p_{no}\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right]\left(\frac{X_n - x}{X_n - x_n}\right)$$

and the hole distribution in the *n*-type region is a linear function of *x*. In this case, the current density is given by

$$j \approx -qD_p \frac{\partial p_n}{\partial x}\bigg|_{x=x_n} = \frac{qD_p P_{no}}{X_n - x_n}\left[\exp\left(\frac{V}{V_{th}}\right) - 1\right]$$

This current density is $L_p/(X_n\text{-}x_n)$ larger than for a long p^+-*n* diode

Generation Current

Under thermal equilibrium, the thermal generation of electron-hole pairs is balanced by their recombination (*G* = *R*). Under a reverse bias, there are very few electron-hole pairs in the depletion region, and the recombination rate, *R*, is nearly zero. However, the thermal generation processes continuously supply the electron hole pairs into the depletion region. The thermal rate can be estimated as

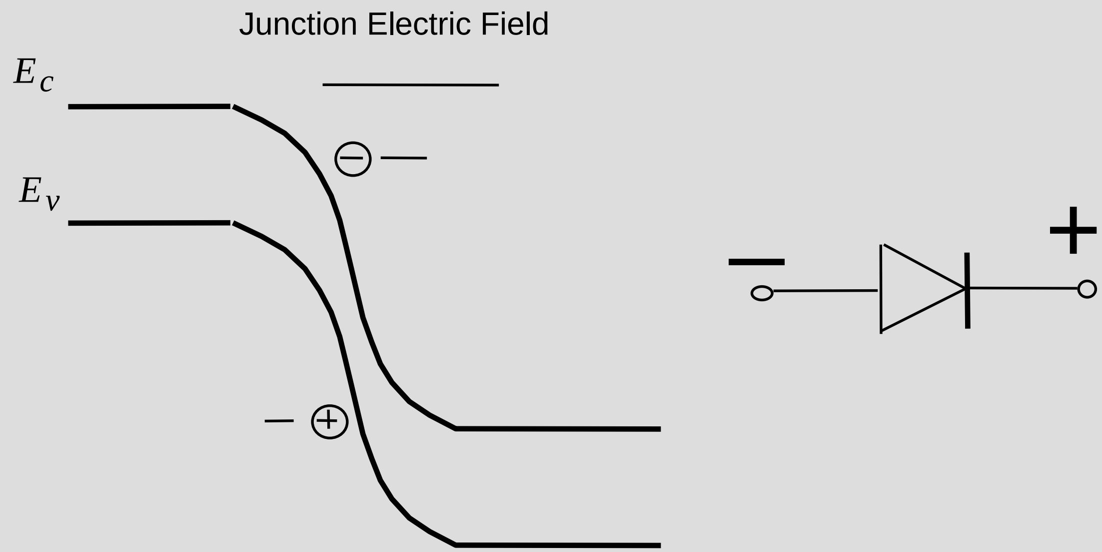
$$G_{thermal} = \frac{n_i}{\tau_{gen}}$$

where τ_{gen} is the effective generation time of electron hole pairs in depletion region.

Generation Current (cont.)

The total electron charge (equal to the hole charge) supplied into the depletion region per unit area per second is equal to $qG_{thermal}x_d = qn_i x / \tau_{gen}$ where $x_d = x_n + x_p$ is the width of the depletion region. Since the generation times typically vary between a microsecond and a nanosecond, orders of magnitude higher that t_{tr}, the generated carriers are swept away almost instantaneously leading to the generation current density

Electrons and Holes Swept by Reverse Bias



Band diagram of *p-n* diode under reverse bias. Arrows show the direction of electron and hole drift in the junction electric field. Also shown is the diode circuit symbol with voltage polarity corresponding to reverse bias.

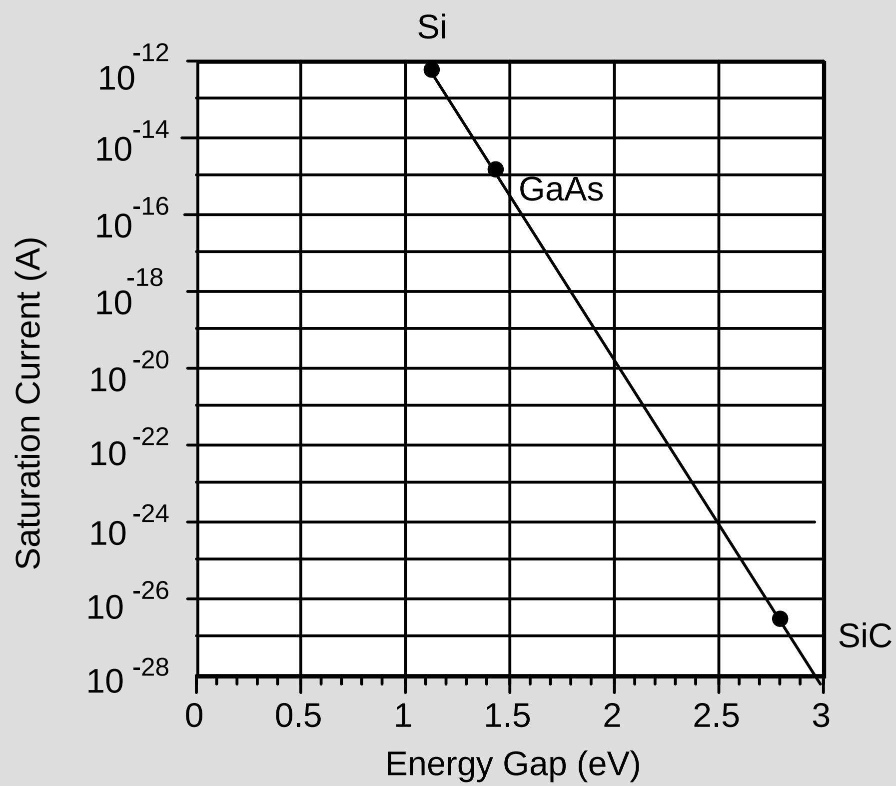
Diffusion and Generation Currents

The total reverse current density, j_R, is given by:
$j_R = j_s + j_{gen}$ where the diffusion component

$$j_s = \left(\frac{qD_p}{N_d L_p} + \frac{qD_n}{N_a L_n} \right) n_i^2 \qquad j_{gen} = \frac{q n_i x_d}{\tau_{gen}}$$

Since j_s is proportional to n_i^2 *and* j_{gen} is proportional to n_i is sufficiently small. In practice, this is often the case for Si, GaAs, and wider gap semiconductors at room temperature and lower temperatures.

Generation Current in Different Semiconductors



Generation current (per 1μm3 of the depletion region volume) versus energy gap (at room temperature).

Recombination Current

Under forward bias conditions (V>0) excess electrons and holes are injected into the depletion region where some of them recombine. The recombination current density is equal to the total electron charge per unit area recombining in the depletion region in one second:

$$j_{rec} = q \int_{-x_p}^{x_n} U_R dx$$

Recombination Current (cont.)

(This electron charge is equal to the total hole charge per unit area recombining in the depletion region.) Here U_R is the net recombination rate. For a simple model accounting only for one impurity (trap) energy level near the middle of the energy gap, this integral can be evaluated, leading to the following expression:

$$j_{rec} = j_{recs} \exp\left(\frac{V}{2V_{th}}\right) \text{ where } j_{recs} \approx \frac{\pi}{2}\frac{qn_iV_{th}}{\tau_{rec}F_{max}}$$

In practical devices , $j_{rec} = j_{recs} \exp\left(\frac{V}{m_r V_{th}}\right)$

Here m_r may differ from two.

Empirical Diode Equation

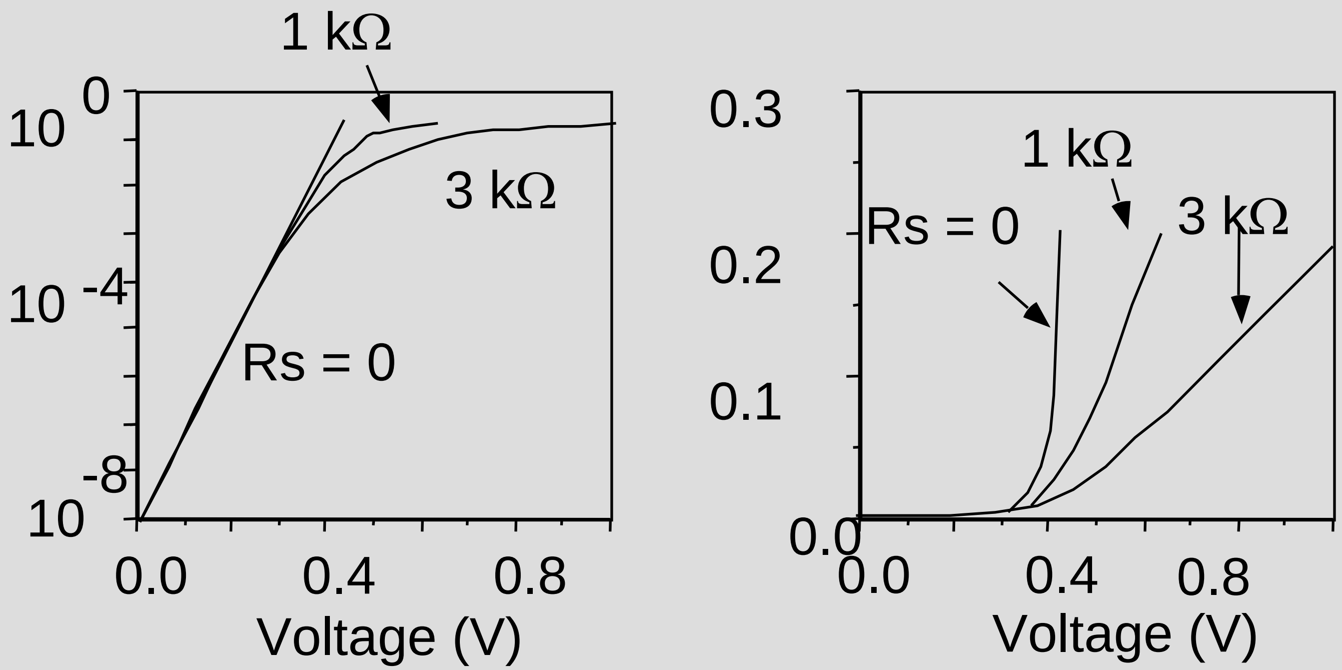
In a real semiconductor diode, the parasitic series resistance, R_s, of the device contacts and the semiconductor neutral regions may play an important role, and this equation has to be modified to include the voltage drop across the series resistance:

$$I = I_s\left[\exp\left(\frac{V - IR_s}{\eta V_{th}}\right) - 1\right]$$

or

$$V = IR_s + \eta V_{th} \ln\left(\frac{I}{I_s}\right)$$

Effect of Series Resistance (repeat again)



Current-voltage characteristics of p-n diode *Iseff* = 10^{-8} mA. The characteristics are shown in semilog and linear scales. (Assumed η = 1)

Diode Model in SPICE

$$I_S(T) = I_S(T_0)\left(\frac{T}{T_0}\right)^{\frac{k}{\eta}} \exp\left(\frac{E_g}{k_B T_0}\right)\exp\left(-\frac{E_g}{k_B T}\right)$$

Here E_g is called activation energy (with a default value set to be equal to the energy gap), T is the device temperature T_0, is a nominal device temperature (at which the device parameters are specified in the circuit simulator; usually T_0 = 300 K), k is an empirical temperature exponent, k=3 and h=1 for *p-n* diodes when the diffusion saturation current is dominant.

$$I_{leakage} = G_{\min} V$$ **Parasitic leakage current**

Example

The default value of G_{min} in SPICE is 10^{-12}1/•. The doping level of the *n*-region of a Si p^+-*n* diode N_d=10^{15} cm-3, the generation time τ_{gen} = 10^{-8} s, the intrinsic carrier density n_i =10^{10} cm^{-3}, the hole diffusion length L_p = 100μm, the hole diffusion coefficient D_p = 10 cm^2 /V, the length of the neutral *n*-type region X_n - x_n is 10 μ m, the dielectric permittivity ε_s = 1.05x10^{-10} F/m, the built-in voltage V_{bi} = 0.6 V, the diode cross section S = 10^{-2} cm^{-2}. Compare the saturation diffusion current, generation current and parasitic leakage current at 10 V reverse bias (using G_{min} = 10^{-12} 1/Ω)

Solution Part 1

$$I_{leakage} = 10^{-12}\left(\frac{1}{\Omega}\right) \times 10(V) = 10^{-11} A$$

Since $p_{no} = \frac{n_i^2}{N_d} = \frac{10^{20}}{10^{15}} = 10^5\left(cm^{-3}\right) >> n^{po}$,

and *Xn - xn* << *Lp*, using eq. (4-3-21) and converting to the SI units, we find

$$I_{Ds} \approx \frac{qD_p p_{no} S}{X_n - x_n} = \frac{1.602 \times 10^{-19} \times 10 \times 10^{-4} \times 10^{11} \times 10^{-6}}{10 \times 10^{-6}}$$

$$= 1.602 \times 10^{-12} (A)$$

Solution (Part 2)

For a *p+-n* diode from eqs. (4-3-24) and (4-3-25), we find

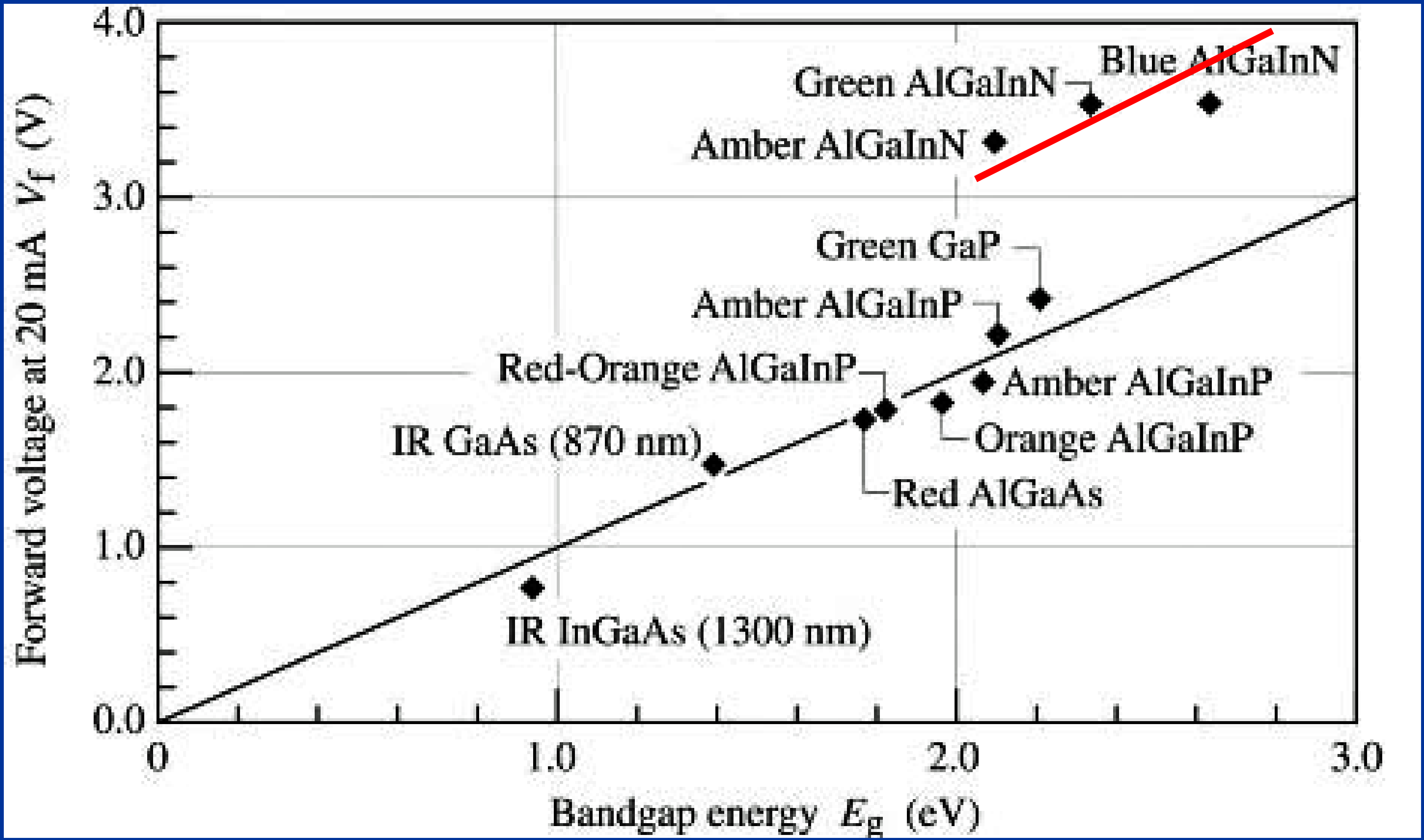
$$x_n = \sqrt{\frac{2\varepsilon_s\left(V_{bi} - V\right)}{qN_d}} = \sqrt{\frac{2 \times 1.05 \times 10^{-10} \times [0.6 - (-10)]}{1.602 \times 10^{-19} \times 10^{21}}}$$

$$I_{gen} = \frac{q n_i x_d S}{\tau_{gen}} = 1$$

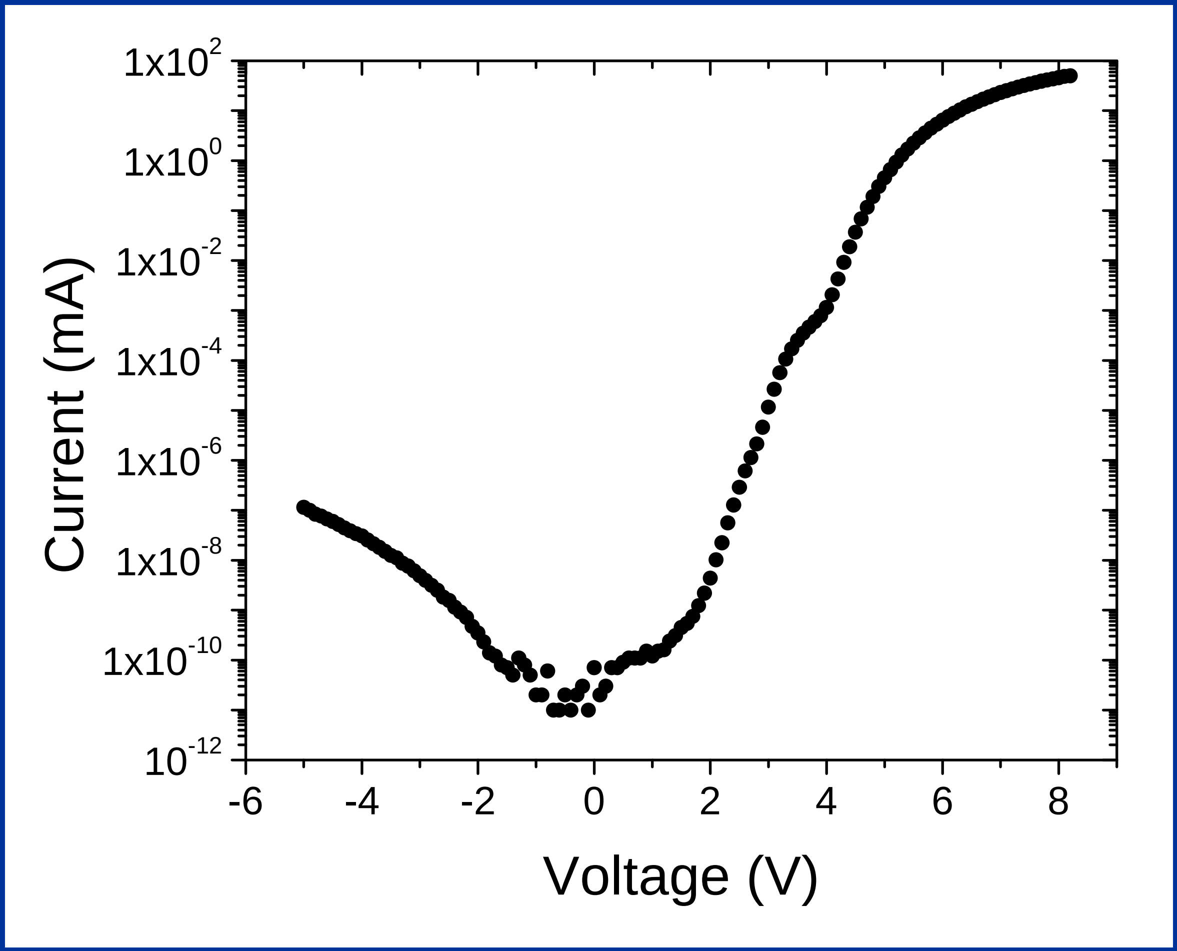
$$\frac{602 \times 10^{-19} \times 10^{16} \times 3.73 \times 10^{-6} \times 10^{-6}}{10^{-8}} = 5.97 \times 10^{-7} (A)$$

In this example, the leakage current determined by the default SPICE parameter is smaller. The generation current here is dominant

Forward LED Voltages (after Krames (2000))



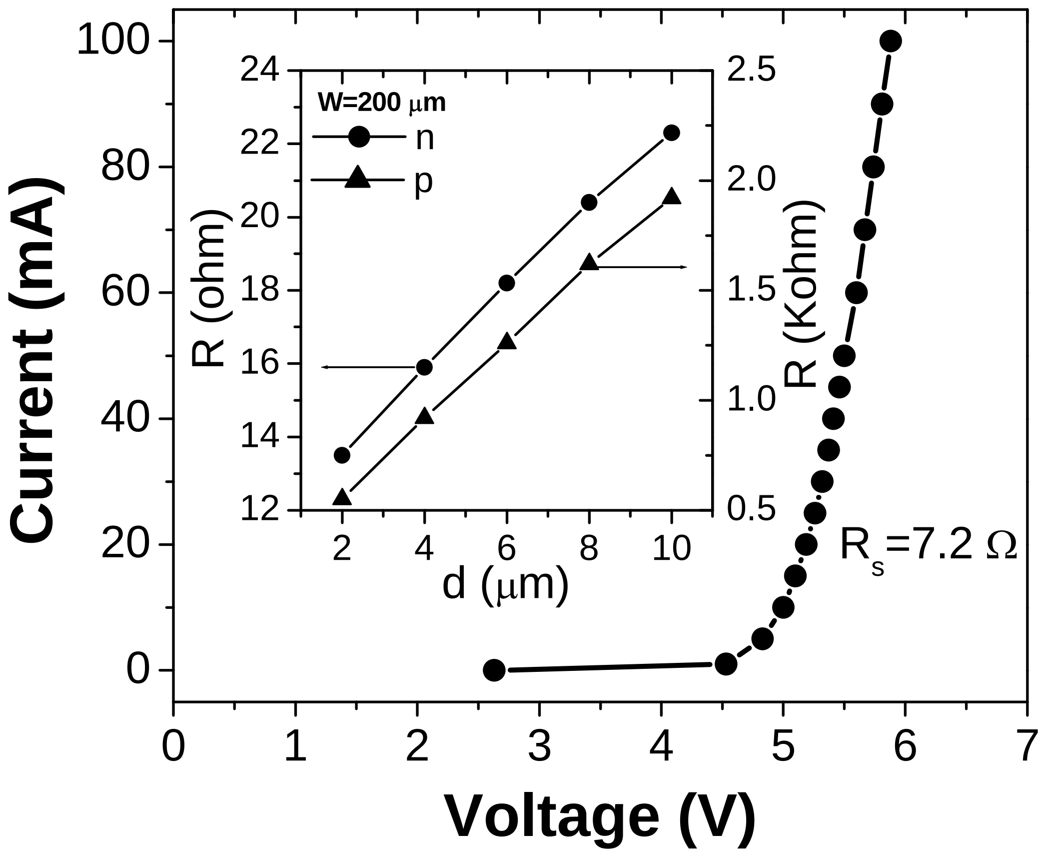
IV characteristic for a 100 ×100 μm² 265 nm LED



From Y. Bilenko, A. Lunev, X. Hu, J. Deng, T. M Katona, J. Zhang, R. Gaska, M. S Shur, W. Sun, V. Adivarahan, M. Shatalov, and A. Khan,JJAP, Express Letter, Vol. 44,

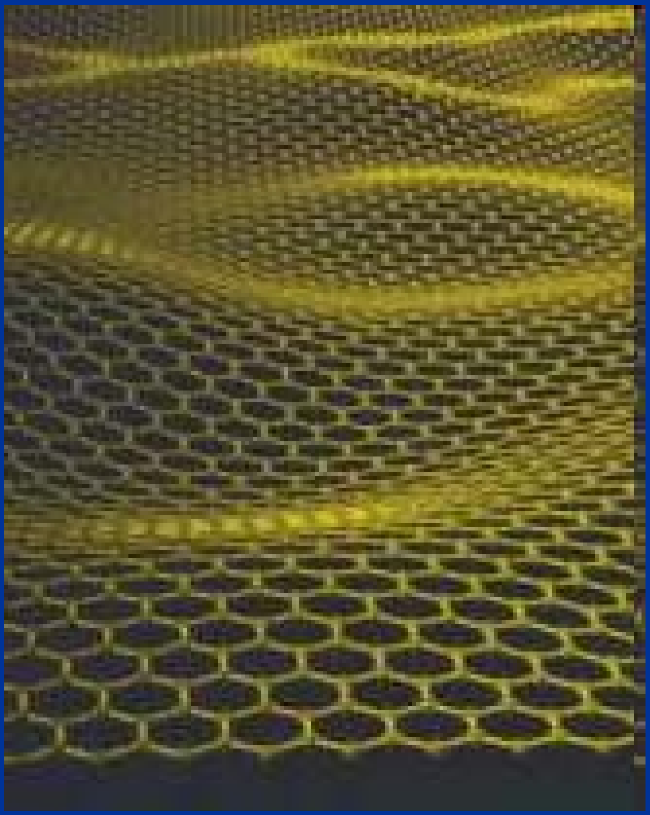
No. 3, pp. L98-L100 (2005)

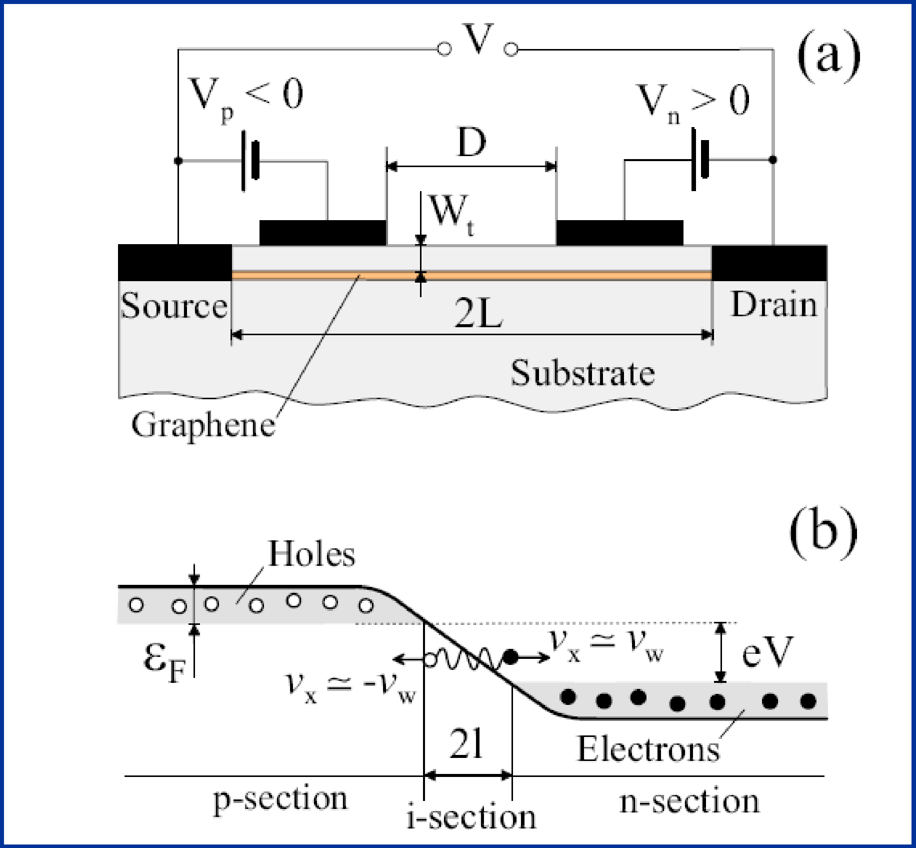
IV curve for the two 0.0001 cm^2 280 nm parallel packaged UV LEDs. Inset showing the p- and n-TLM results



From J. P. Zhang, X. Hu, Y. Bilenko, J. Deng, A. Lunev, M. S. Shur, R. Gaska, M. Shatalov, J. W. Yang, M. A. Khan, Appl. Phys. Lett., Vol. 85, pp. 5532-5534, No 23, 2004

Graphene p-n junction





From V. Ryzhii, M. Ryzhii, V. Mitin and M. S. Shur Graphene Tunneling Transit-Time Terahertz Oscillator Based on Electrically Induced p-i-n Junction

From http://images.iop.org/objects/physicsweb/news/thumb/12/3/23/Graphene.jpg