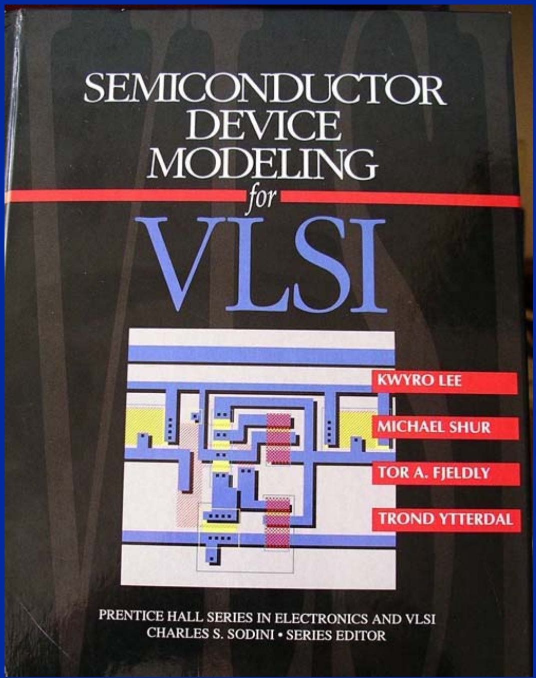
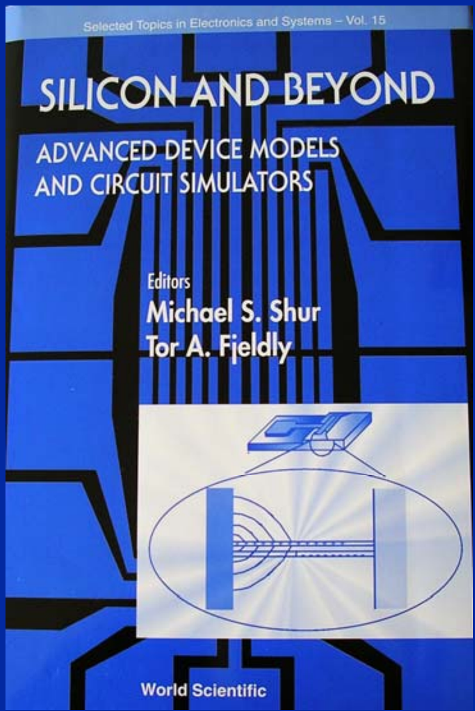
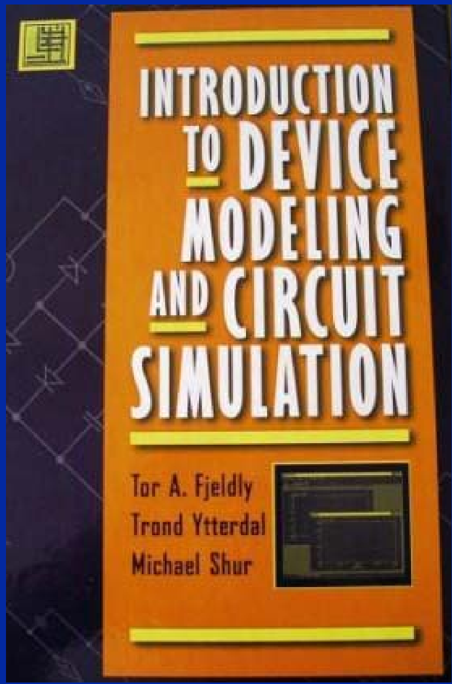
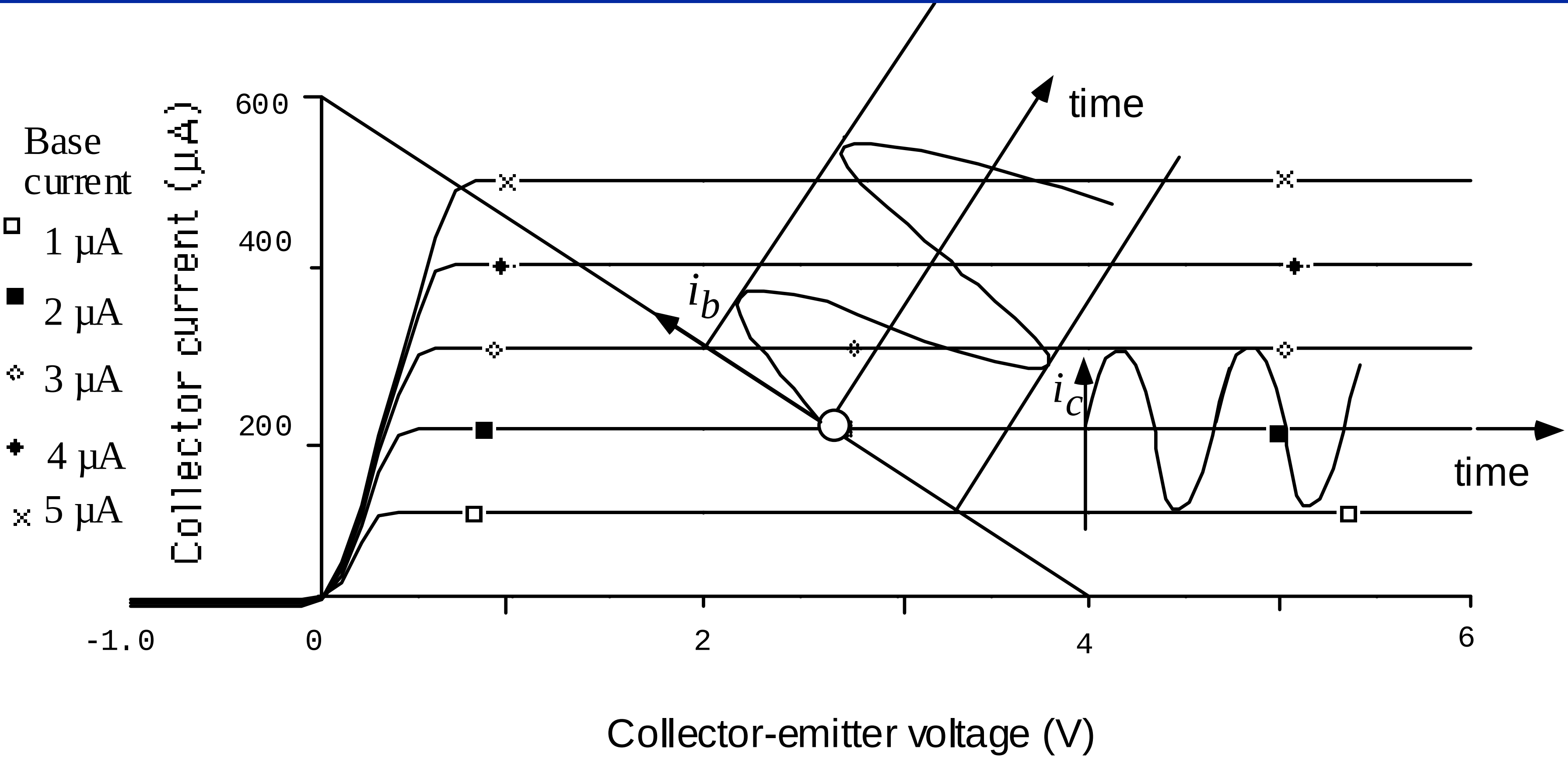
Small Signal Response



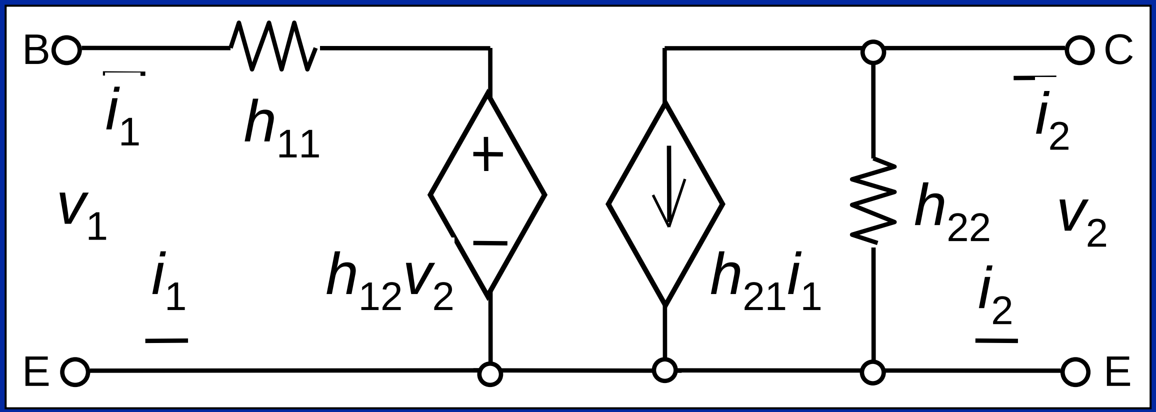




Small signal operation



h parameter small-signal eq.circuit



$$v_1 = h_{11}i_1 + h_{12}v_2$$

$$i_2 = h_{21}i_1 + h_{22}v_2$$

Short-circuit input impedance : $h_{11} \equiv h_i = (v_1/i_1)\big|_{v_2=0}$

Open circuit reverse voltage ratio : $h_{12} \equiv h_r = (v_1/v_2)\big|_{i_1=0}$

Short-circuit forward current ratio : $h_{21} \equiv h_f = (i_2/i_1)\big|_{v_2=0}$

Open circuit output admittance : $h_{22} \equiv h_o = (i_2/v_2)\big|_{i_1=0}$

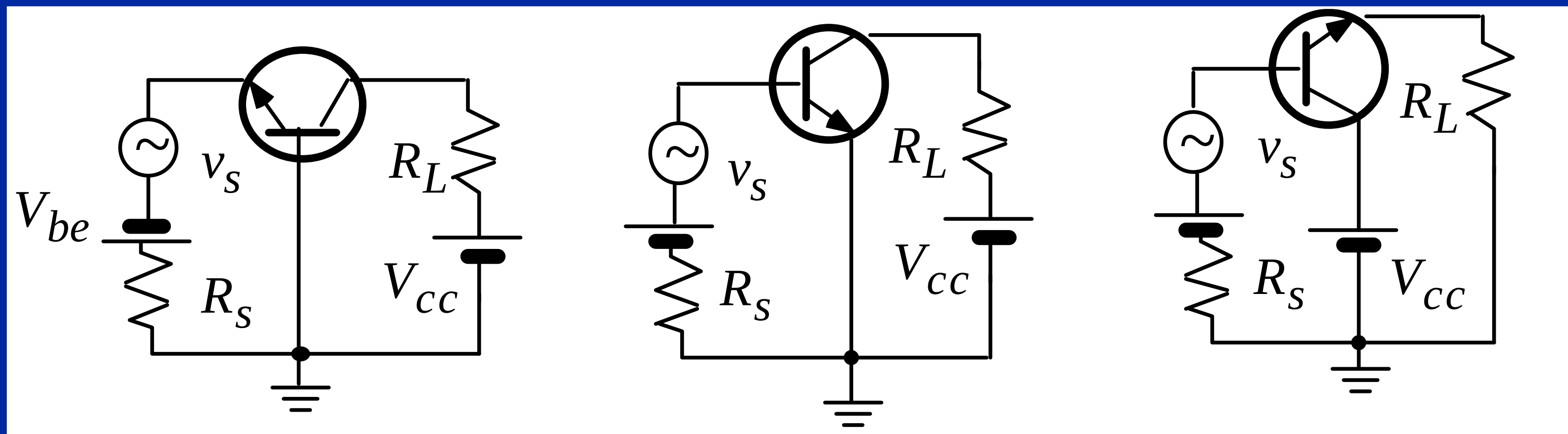
From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York , 1998

Notation for h-parameters

The parameter h_{11} (h_i) is called the **short-circuit input impedance**, h_{12} is called the **open circuit reverse voltage ratio** (h_r), h_{21} is called the **short-circuit forward current ratio** (h_f), and h_{22} is called the **open-circuit output admittance** (h_o). In the alternate notation (h_i, h_r, h_f, and h_o,), a second subscript is often used to denote the transistor configuration. For example the *h*-parameters for the common-emitter transistor circu configuration are denoted as h_{ie}, h_{re}, h_{fe}, and h_{oe}. The total of *h*-parameters is twelve (four for each transistor configuration). In their data sheets, transistor manufacturers usually provide only common-emitter *h*-parameters.

shurm@rpi.edu

Deriving the relationship between h-parameters Common Emitter and Common Collector



Common base · Common emitter · Common collector

$$v_{bc} = h_{ic} i_b + h_{rc} v_{ec}$$
$$i_c = h_{fc} i_b + h_{oc} v_{ec}$$

$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$
$$i_c = h_{fe} i_b + h_{oe} v_{ce}$$

Derivation

For $v_{ec} = 0$, $v_{bc} = v_{be}$. Hence

$\partial v_{be}/\partial i_b = \partial v_{bc}/\partial i_b$ and $h_{ie} = h_{ic}$

$\partial i_c/\partial i_b = \partial\left(-i_e - i_b\right)/\partial i_b$ and $h_{fc} = -h_{fe} - 1$

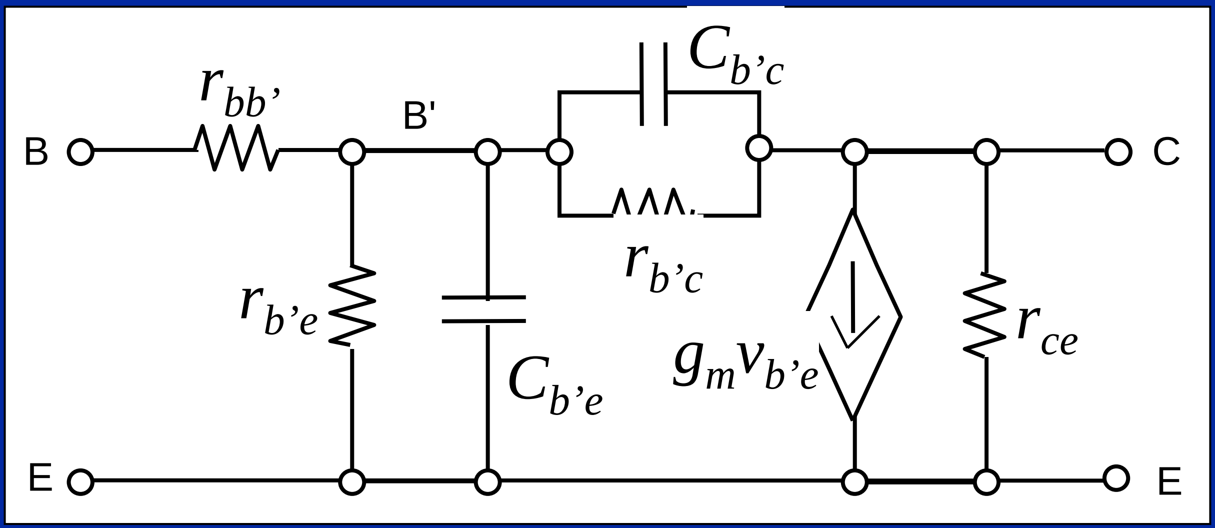
For $i_b = 0$, $v_{be} = const$ and $i_e = -i_c$. Hence $h_{rc} = \partial v_{bc}/\partial v_{ec}\big|_{i_b=0} = 1$

$\partial i_c/\partial v_{ec} = -\partial i_e/\partial v_{ec} = \partial i_e/\partial v_{ce}$. Hence $h_{oe} = h_{oc}$

h parameters

Common-emitter *h*-parameters	Common-base *h*-parameters	Common-collector *h*-parameters
h_{ie}	$h_{ib} = h_{ie}/(h_{fe} + 1)$	$h_{ic} = h_{ie}$
h_{re}	$h_{rb} = h_{ie}h_{oe}/(h_{fe} + 1) - h_{re}$	$h_{rc} = 1$
h_{fe}	$h_{fb} = - h_{fe}/(h_{fe} + 1)$	$h_{fc} = - h_{fe} - 1$
h_{oe}	$h_{ob} = h_{oe}/(h_{fe} + 1)$	$h_{oc} = h_{oe}$

Hybrid-π Equivalent Circuit



Often used in CE configuration.

Relates well to physical (SPICE) parameters.

$$g_m = \frac{\partial I_c}{\partial V_{b'e}} \approx \frac{I_c}{V_{th}}$$

$$r_{b'e} = \frac{\partial V_{b'e}}{\partial I_b} = \frac{\partial V_{b'e}}{\partial I_c/\beta_N} = \frac{\beta_N}{g_m}$$

$$r_{ce} = \frac{\partial V_{ce}}{\partial I_c} \approx \frac{|V_A|}{I_c} \approx \frac{|V_A|}{g_m V_{th}}$$

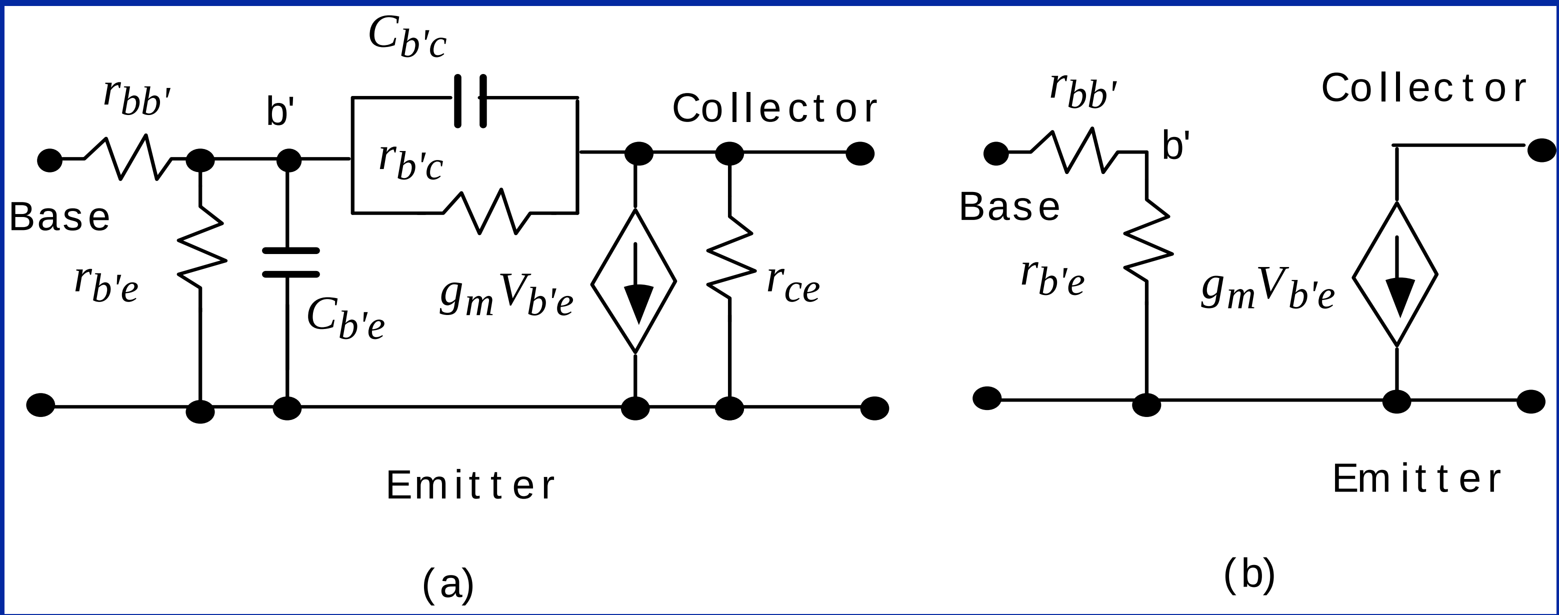
$$r_{b'c} = \frac{\partial V_{cb'}}{\partial I_b} \approx \frac{\partial V_{ce}}{\partial I_c/\beta_N} = \beta_N r_{ce} = \frac{\beta_N |V_A|}{g_m V_{th}}$$

$$C_{b'e} = C_{de} + C_{dife} = C_{de}\left(V_{b'e}\right) + g_{mF}\tau_F$$

$$C_{b'c} = C_{dc} + C_{difc} = C_{dc}\left(V_{b'c}\right) + g_{mR}\tau_R$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York , 1998

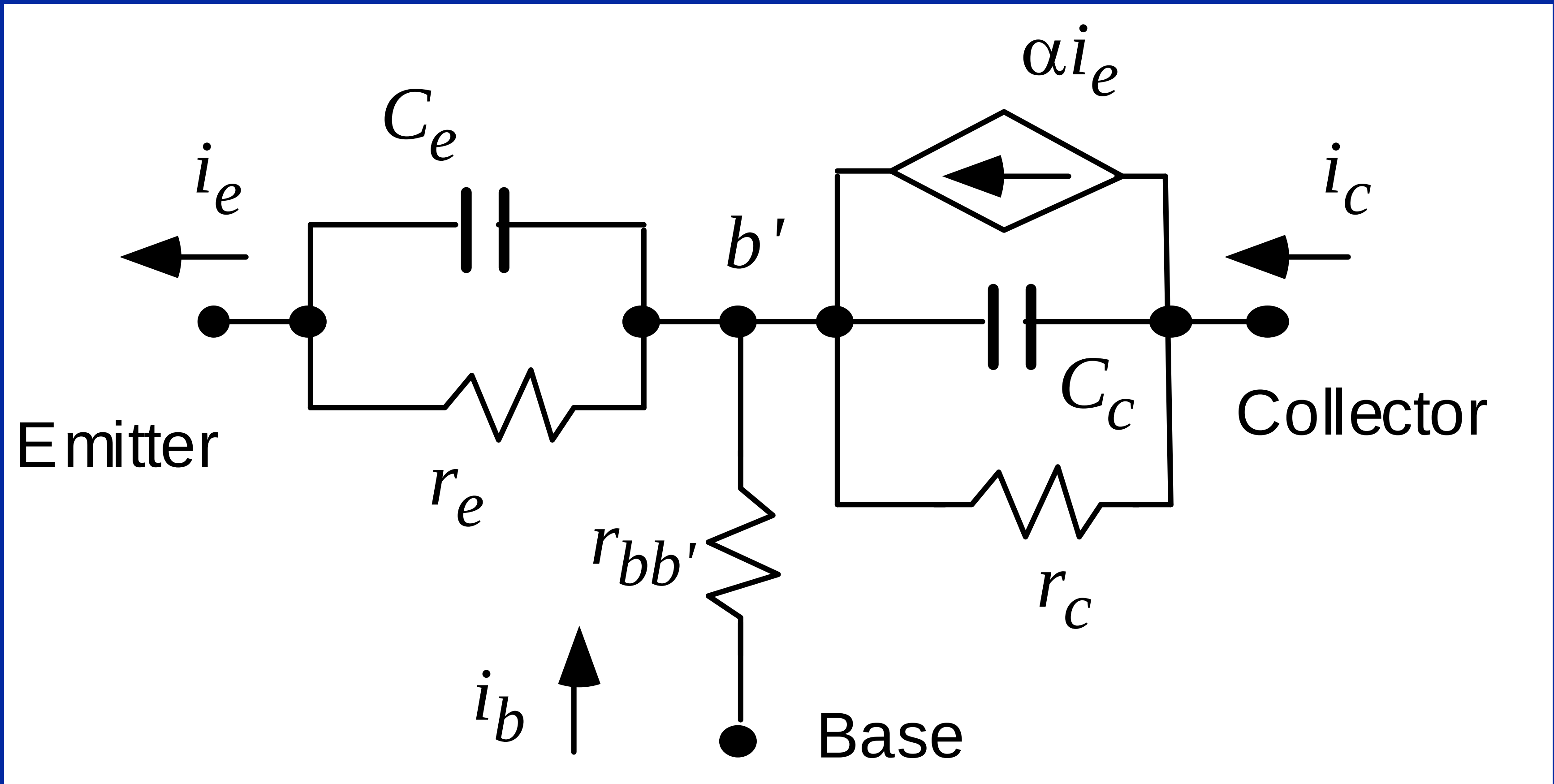
Full (a) and simplified (b) hybrid-π equivalent circuit



$$g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}}$$

$$r_{b'e} \approx \frac{i_c}{i_b}\frac{1}{g_m} \approx \frac{h_{fe}}{g_m} = \frac{\beta}{g_m}$$

T equivalent circuit



π, T and h

Parameter	Expressed through *h*-parameters for common-emitter configuration	Expressed through *h*-parameters for common-base configuration
r_e	h_{re}/h_{oe}	$h_{ib} - h_{rb}\left(1 + h_{fb}\right)/h_{ob}$
$r_{bb'}$	$h_{ie} - h_{re}\left(1 + h_{fe}\right)/h_{oe}$	h_{rb}/h_{ob}
r_c	$\left(1 + h_{fe}\right)/h_{oe}$	$\left(1 - h_{rb}\right)/h_{ob}$
α	$h_{fe}/\left(1 + h_{fe}\right)$	$-h_{fb}$
β	h_{fe}	$-h_{fb}/\left(1 + h_{fb}\right)$

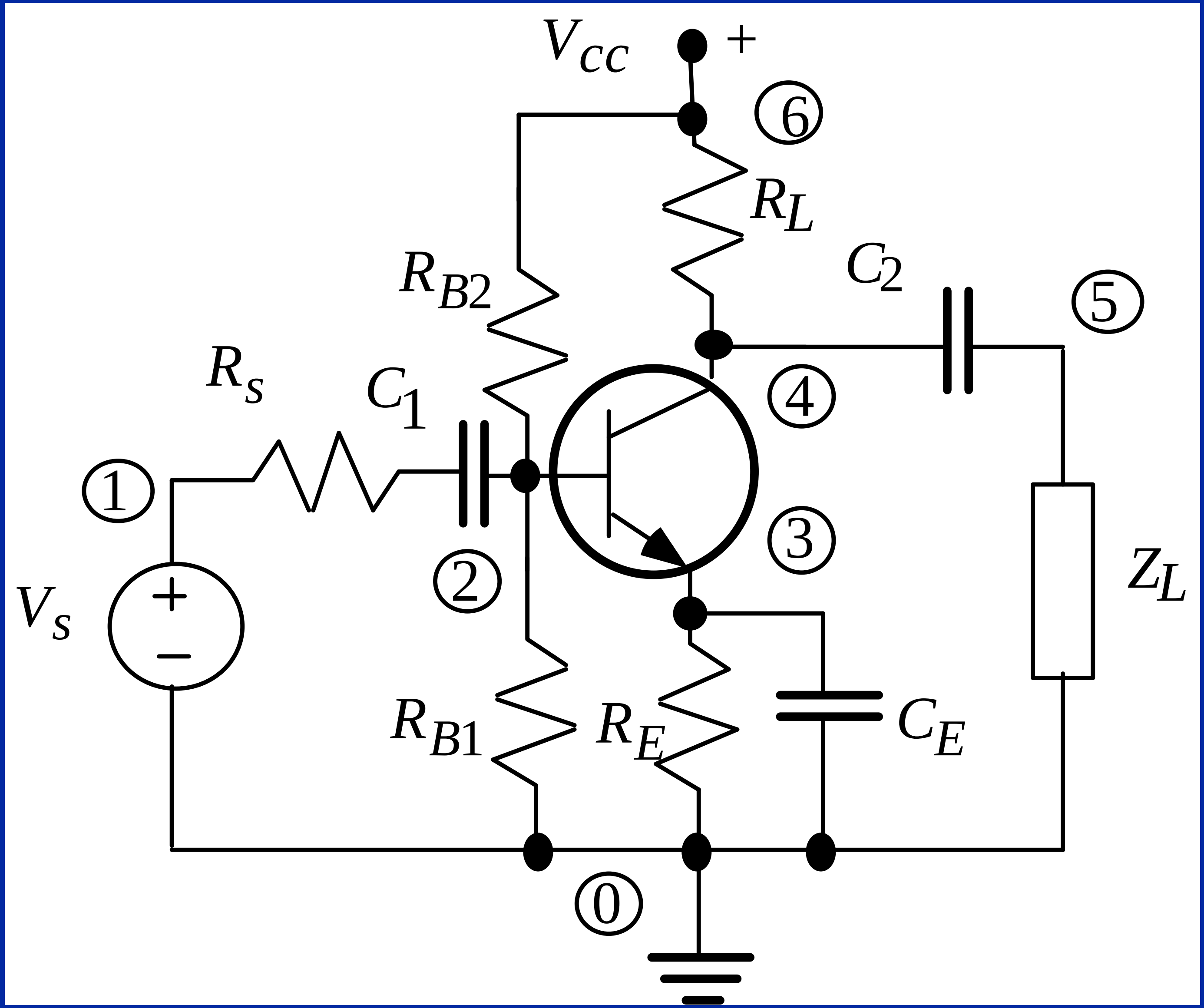
h-parameter	Relation to parameters of hybrid π-equivalent circuit
h_{oe}	$1/\left(r_{b'c} + r_{b'e}\right) + 1/r_{ce} + g_m r_{b'e}/\left(r_{b'c} + r_{b'e}\right)$
h_{ie}	$r_{bb'} + r_{b'e} r_{b'c}/\left(r_{b'e} + r_{b'c}\right)$
h_{fe}	$g_m r_{b'e} r_{b'c}/\left(r_{b'c} + r_{b'e}\right)$

Important relationships

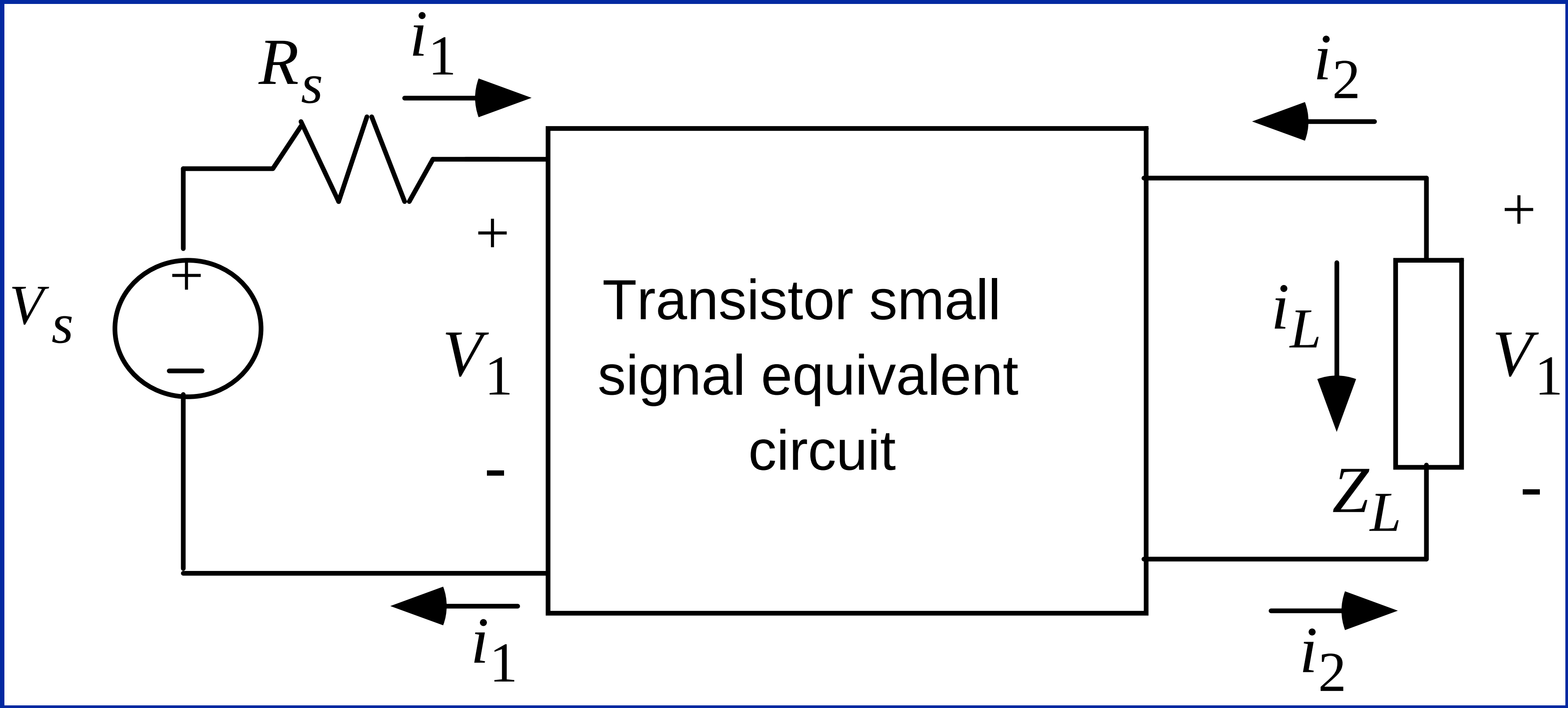
Definition of *h*-parameters	$v_1 = h_{11} i_1 + h_{12} v_2$ $\quad i_2 = h_{21} i_1 + h_{22} v_2$
Dynamic resistance of the forward-biased emitter-base junction	$r_e = \frac{\partial V_{b'e}}{\partial I_e} \approx \frac{V_{th}}{I_e}$
Transconductance	$g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}}$
Resistance in the hybrid-p equivalent circuit	$r_{b'e} \approx \frac{i_c}{i_b} \frac{1}{g_m} \approx \frac{h_{fe}}{g_m} = \frac{\beta}{g_m}$

shurm@rpi.edu

Biasing



Midband



R_s
i_1
i_2
+
V_s
+
V_1
-
Transistor small
signal equivalent
circuit
i_L
+
V_1
-
Z_L
i_1
i_2

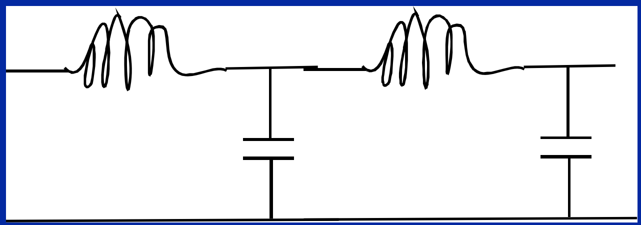
Gains and Impedances

	Definition	Relation to *h*-parameters
Input impedance	$Z_i = v_1/i_1$	$Z_i = h_i - h_f h_r / (h_o + Y_L)$
Output admittance	$Y_o = i_2/v_2$	$Y_o = h_o - h_f h_r / (h_i + R_s)$
Voltage gain	$A_v = v_2/v_1$	$A_v = A_i Z_L / Z_i$
Voltage gain	$A_{vs} = v_2/v_s$	$A_{vs} = A_v Z_i/(Z_i + R_s)$
Current gain	$A_i = i_L/i_1 = - i_2/i_1$	$A_i = h_f / (1 + h_o Z_L)$
Current gain	$A_{is} = - i_2/i_s$	$A_{is} = A_i R_s/(Z_i + R_s)$
Power gains	$A_p = A_v A_i$, $A_{ps} = A_{vs} A_{is}$	

Motorola 2N2219A transistor

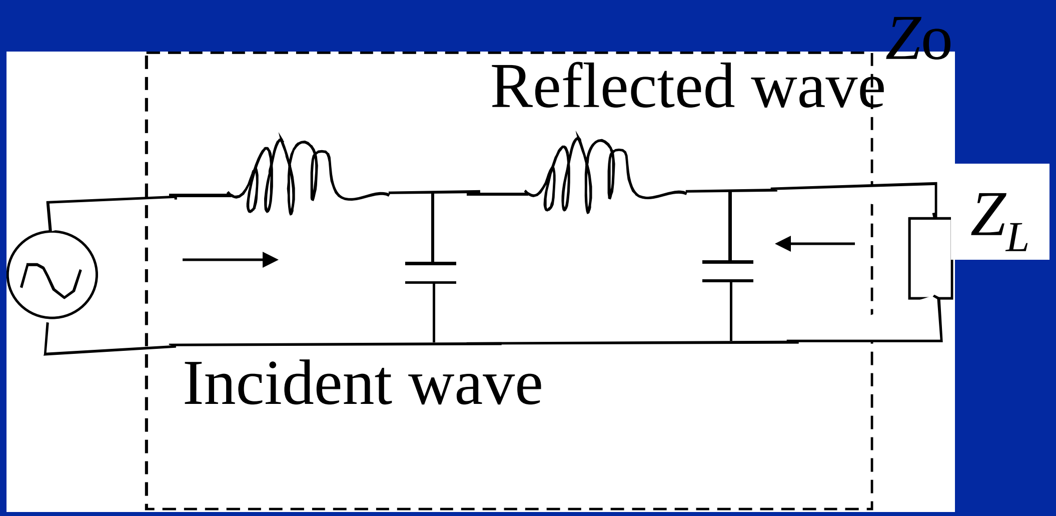
	Common emitter	Common base	Common collector
h_i (ohm)	1250	16.4	1250
h_r	4×10^{-4}	1.1×10^{-5}	1
h_f	75	-0.987	-76
h_o (µmho)	25	0.329	25
Z_i (kohm)	1.19	0.0164	146
Y_o(µmho)	15.77	0.334	24350
A_i	- 71.43	0.986	72.38
A_{is}	- 44.72 (33 dB)	0.977	0.978
A_v	- 119.76 (41.6 dB)	120.1	0.991
A_{vs}	- 44.72 (33 dB)	0.977	0.978

Lossless Transmission Line



$Z_o = (L/C)^{1/2}$

$V = V_i + V_r$



$I = (V_i - V_r)/Z_o$

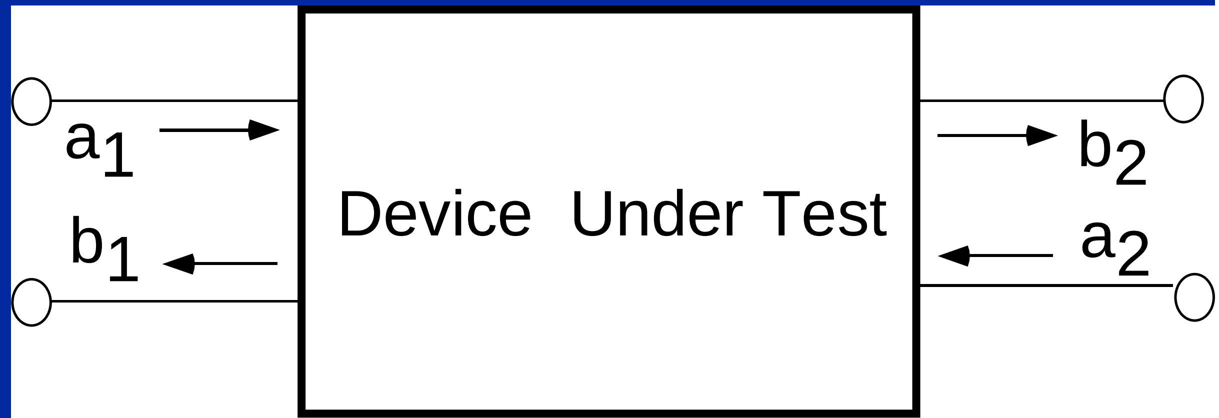
$\Gamma = (Z_L - Z_o)/(Z_L + Z_o)$

shurm@rpi.edu

S-parameters

$$V_{r1} = s_{11} V_{i1} + s_{12} V_{i2}$$

$$V_{r2} = s_{21} V_{i1} + s_{22} V_{i2}$$



Now divide both sides by $Z_o^{1/2}$

$$b_{r1} = s_{11} a_{i1} + s_{12} a_{i2}$$

$$b_{r2} = s_{21} a_{i1} + s_{22} a_{i2}$$

where

$$b_{r1} = V_{r1} / Z_o^{1/2}$$

$$b_{r2} = V_{r2} / Z_o^{1/2}$$

$$a_{r1} = V_{i1} / Z_o^{1/2}$$

$$a_{r1} = V_{i21} / Z_o^{1/2}$$

a_1^2 is the incident power on port 1
b_1^2 is the reflected power on port 1

Definitions

$$s_{11} = \frac{b_1}{a_1}\bigg|_{a_2 = 0}$$

is called the **input reflection ratio**,

$$s_{12} = \frac{b_1}{a_2}\bigg|_{a_1 = 0}$$

is called the **reverse transmission ratio**,

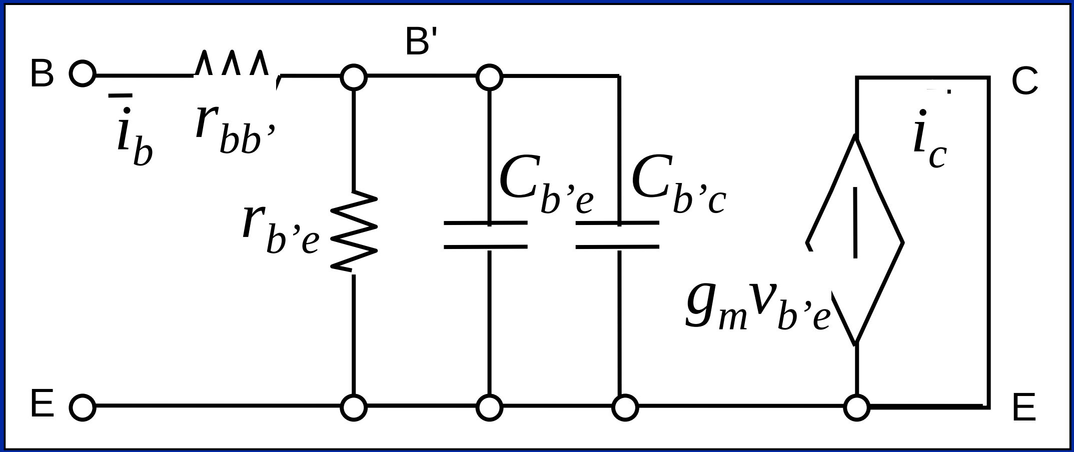
$$s_{21} = \frac{b_2}{a_1}\bigg|_{a_2 = 0}$$

is called the **forward transmission ratio**, and

$$s_{22} = \frac{b_2}{a_2}\bigg|_{a_1 = 0}$$

Cut-off Frequency f_T

Simplified hybrid-π equivalent:



Base ac input current:

$$i_b = v_{b'e}\left[\frac{1}{r_{b'e}} + j\omega\left(C_{b'e} + C_{b'c}\right)\right]$$

CE current gain versus ω:

$$\beta_\omega = \frac{i_c}{i_b} = \frac{\beta_N}{1 + j\omega/\omega_\beta}$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York , 1998

Cut-off Frequency fT (Cont.)

Beta cut-off frequency:

$$\omega_\beta = 2\pi f_\beta = \frac{1}{r_{b'e}\left(C_{b'e} + C_{b'c}\right)} = \frac{g_m}{\beta_N\left(C_{b'e} + C_{b'c}\right)}$$

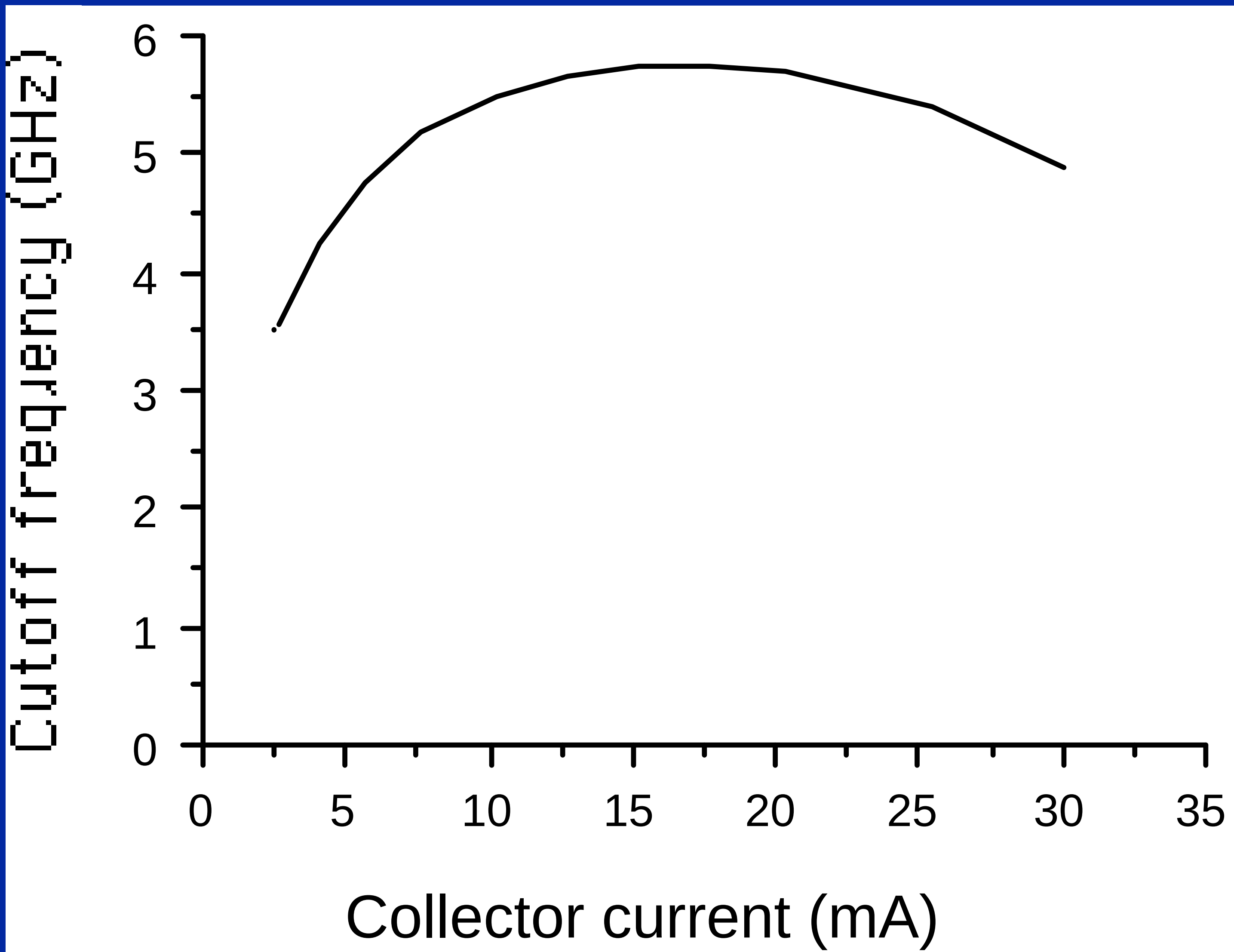
Unity gain cut-off frequency ($|\beta_\omega| = 1$):

$$f_T = f_\beta\sqrt{\beta_N^2 - 1} \approx \frac{g_m}{2\pi\left(C_{b'e} + C_{b'c}\right)}$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York , 1998

shurm@rpi.edu

f_T (GHz)



Cutoff Frequency for FET and BJT

$$f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

$$g_m = \frac{\partial I_d}{\partial V_g} = q\frac{\partial n_s}{\partial V_g} v_s W \approx C_i \frac{v_s}{L}$$

$$f_T = 1/2\pi t_{tr}$$

where $t_{tr} = L/v_{eff}$ is the transit time of electrons in the channel. Assuming that v_{eff} ,to be of the order of 5x10^4 m/s (which is about one half of the electron saturation velocity in Si) we obtain a characteristic transit time for a MOSFET, on the order of t_{tr}(ps) - 20 L (μm) and f_T (GHz) ~ 8/L (μm). In fact, the measured switching times may be quite a bit larger because the transistor response is slowed down by the parasitic and fringing capacitances, C_p, which add to the gate capacitance:

$$f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd} + C_p\right)}$$

Compare with BJT:

$$f_T \approx \frac{g_m}{2\pi\left(C_e + C_{b'c}\right)} = \frac{1}{2\pi\tau_{eff}}$$

fT for FETs (more accurate)

$$wT = \frac{gm}{(Cgs + Cgd)\,(1 + gds\,(Rd + Rs)) + Cgd\,gm\,(Rd + Rs)}$$

wT = 2 Pi fT

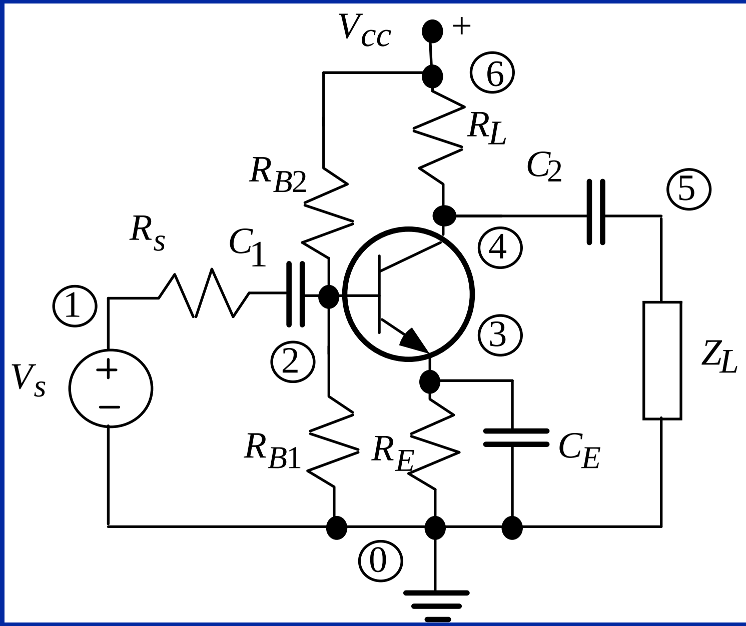
f_{max}

The frequency at which the power gain of the transistor is equal to unity under optimum matching conditions for the input and output impedances is called the **maximum oscillation frequency**, f_{max}. Using a simplified π-equivalent circuit where we neglect $r_{b'c}$ and r_{ce}, we obtain

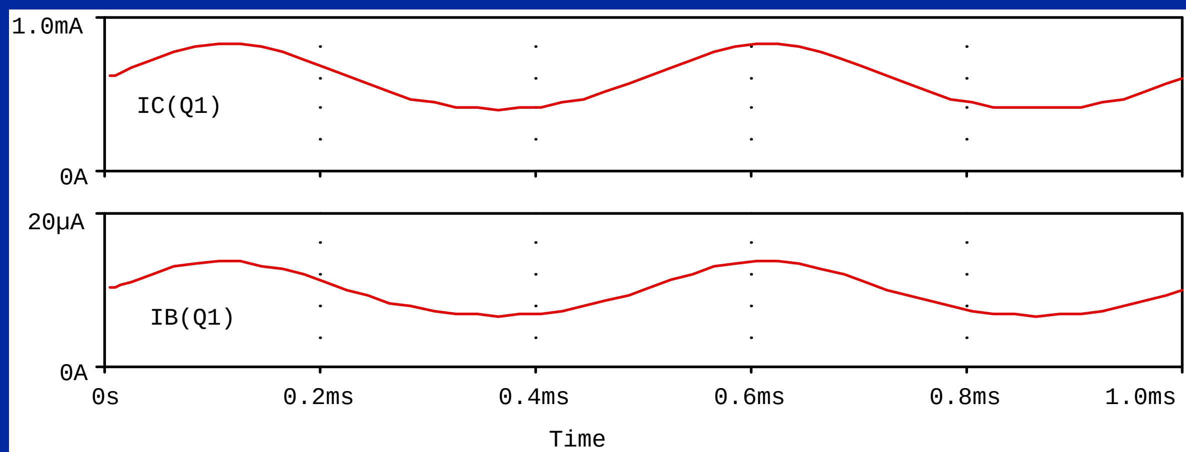
$$f_{max} = \sqrt{\frac{f_T}{8\pi r_{bb'} C_{b'c}}}$$

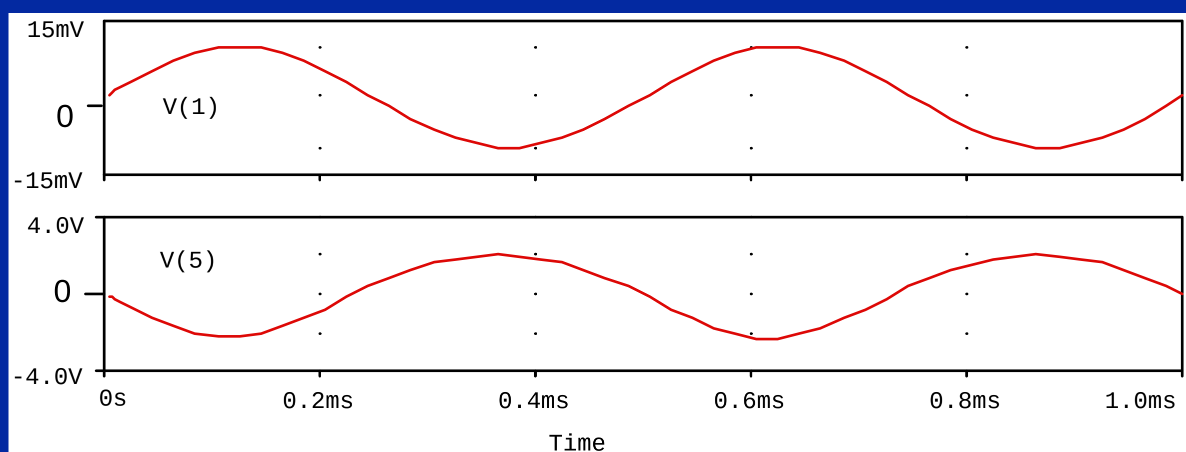
Solution

```
Small Signal BJT Amplifier
Vcc 6 0 DC 15V
Vs 1 0 ac 1V sin(0 0.01V 2KHz)
C1 1 2 20u
RB1 6 2 120k
RB2 2 0 20k
RC 6 4 10k
RL 5 0 2MEG
RE 3 0 2K
CE 3 0 15u
C2 4 5 20u
Q1 4 2 3 BJT
.model BJT NPN (bf=60 rb=100 cjc=10p)
.ac dec 10 0.1 100Meg
.tran  0.1ms 1ms .005ms
.probe
.end
```

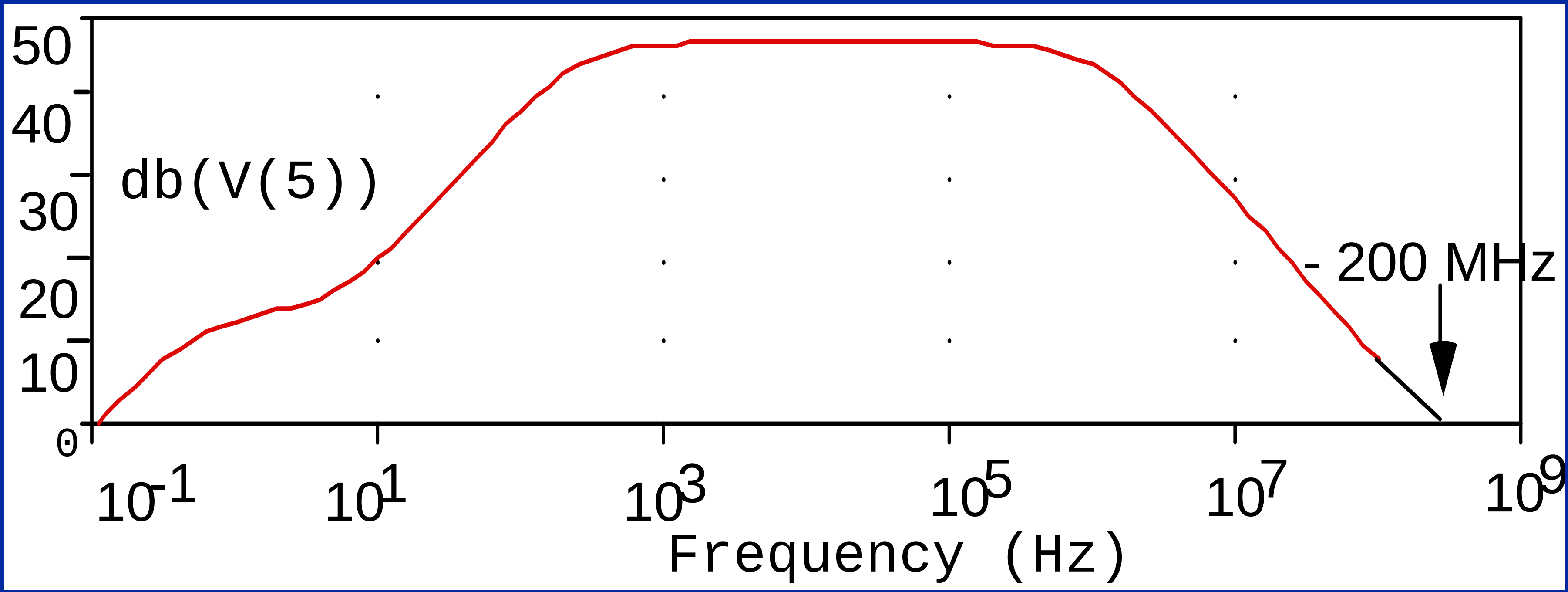


Input and output current and voltages for one stage common emitter BJT amplifier





Voltage gain for one stage common emitter BJT amplifier



Summary

Common-base current gain	$\alpha_\omega = \alpha/(1 + j\,\omega/\omega_\alpha)$ where $\omega_\alpha = 2\pi f_\alpha = 1/(C_e r_e)$
Common-emitter current gain	$\beta_\omega = \beta/(1 + j\,\omega/\omega_\beta)$ where $\omega_\beta = 2\pi f_\beta = g_{b'e}/(C_{b'e} + C_{b'c})$
Cutoff frequency	Crude estimate: $f_T \approx \frac{g_m}{2\pi(C_e + C_{b'c})} \approx \frac{g_m}{2\pi C_e}$ More accurate equation: $f_T \approx \frac{1}{2\pi\tau_{eff}}$ where $\tau_{eff} = \tau_e + \tau_c + \tau_{cT}$, $\tau_e = (C_e + C_{b'c} + C_p)/g_m$, $\tau_{cT} \approx x_{dcb}/v_{sn}$, $\tau_c = r_{cs}$,