# Multi-Sensor Multi-Target Tracking with Out-of-Sequence Measurements*

**Keshu Zhang X. Rong Li Huimin Chen**
Department of Electrical Engineering, University of New Orleans, New Orleans, LA 70148
{kzhang1,xli,hchen2}@uno.edu
**Mahendra Mallick**
Orincon Defense, 4770 Eastgate Mall, San Diego, CA 92121-1970
mmallick@orincon.com

**Abstract** – *In multi-sensor target tracking systems, measurements from the same target can arrive out of sequence, called the out-of-sequence measurements (OOSMs). The resulting problem – how to update the current state estimates with the "old" measurements – has been solved optimally and sub-optimally for one-lag as well as multi-lag OOSM update. In general, the existing algorithms assume perfect target detection and no clutter in the received measurements. The real world has, however, possible missed target detection and random clutter in the possible OOSMs and thus the filter has to handle the measurement origin uncertainty. In this paper, we incorporate the probabilistic data association (PDA) into the two OOSM update algorithms ALG-I and ALG-II proposed previously. We present the algorithms ALG-I and ALG-II in new forms with economic storage and efficient computation based on the nonsingularity assumption of some special matrices. Simulation results show that PDA with the two OOSM update algorithms have compatible RMS errors to the in-sequence PDA filter.*

**Keywords:** Target tracking, out-of-sequence measurement, linear minimum mean square estimation (LMMSE), PDA

## 1 Introduction

In a multi-sensor multi-target tracking system, observations obtained by multiple sensors are usually sent to a fusion center for processing. Out-of-sequence measurements (OOSMs) can arise at the fusion center due to communication delay and varying preprocessing time for different sensor platforms. This can lead to situations where measurements from the same target arrive out of sequence. One possible scenario is that the fusion center receives measurements from a local sensor at time $t_k$ and updates the state estimate and corresponding covariance of a track. After the update, it receives delayed measurements from another local sensor at a prior time $t_d$ ($t_{k-l} \leq t_d < t_{k-l+1}$, $l = 1, 2, \cdots$). The previous results for OOSM update are formulated for a Kalman filter to update the state at time $t_k$ by using the "older" measurement from time $t_d$. In this problem, the measurement at each sampling time is assumed to be target originated and no clutter or interference from other targets is considered. We call the above setting an OOSM update problem. There are some optimal methods [1], [2], [3] and suboptimal methods [4], [5], [6], [7], [8] for one-lag as well as multi-lag OOSM update. Two general algorithms ALG-I and ALG-II proposed in [3] can solve the one-lag as well as the multi-lag OOSM update problems in a globally optimal or suboptimal (optimal with limited information) manner without any restrictions. They are optimal in the LMMSE sense. We will show that ALG-1 and ALG-II have simpler forms with less storage requirement under some additional assumptions which hold for nearly all target tracking applications.

However, almost all real world tracking problems involve nonlinear measurements. Therefore, there is no optimal filtering algorithm exists. In multi-sensor multi-target tracking problems, measurements received at the fusion center can originate from targets or clutter, i.e., false alarms. The filter handles the measurement origin uncertainty via the so-called data association algorithm. The existing optimal criterion for OOSM update within the Kalman filter framework is no longer valid for the target tracking problem with measurement origin uncertainty. In this case, the OOSM update needs to include certain data association algorithm. However, the optimal data association (in the Bayesian sense) relies on all measurements from the beginning up to the current time. With limited storage, for example only based on the state

*Research supported by ONR Grant N00014-00-1-0677, NSF Grant ECS-9734285, and NASA/LEQSF grant (2001-4)-01.

estimate without storing the measurements, it is impossible to have the optimal data association. There exist data association algorithms, such as probabilistic data association (PDA) for a single target in clutter and joint probabilistic data association (JPDA) or multiple hypothesis tracking (MHT) for multiple targets in clutter, that solve the measurement-to-track association sub-optimally. In this setting, it is hard to propose a meaningful criterion to update the OOSMs optimally for multi-target tracking in clutter [9]. In this paper, we provide one solution by incorporating the PDA technique with the OOSM update for tracking a single target in clutter. Through performance comparison between the PDA with OOSM update and the in-sequence PDA filter, we find that the performance degradation of the PDA with the OOSM update is relatively small. We also find that the PDA with the OOSM update has better performance than just ignoring those OOSMs. The generalization of incorporating the JPDA with the OOSM filter update for multi-sensor multi-target tracking in clutter is briefly discussed.

# 2 Estimation with OOSMs under Perfect Target Detection and No Clutter

## 2.1 Problem Formulation

The dynamic and measurement models for a single target are given by

$$x_j = F_{j,j-1}x_{j-1} + w_{j,j-1} \tag{1}$$
$$z_j = H_j x_j + v_j \tag{2}$$

where $F_{j,j-1}$ is the state transition matrix from time $t_{j-1}$ to $t_j$ and $w_{j,j-1}$ is the integrated process noise for this interval. The process noise $w_{j,j-1}$ and the measurement noise $v_j$ are white, mutually uncorrelated, with zero mean and variances

$$\mathrm{var}(w_{j,j-1}) = Q_{j,j-1}, \ \ \mathrm{var}(v_j) = R_j$$

Suppose time $t_d$ is in the sampling interval $t_{k-l} \le t_d < t_{k-l+1}$, where $l = 1, 2, \cdots$, which means that the OOSM $z_d$ is $l$ lags behind.

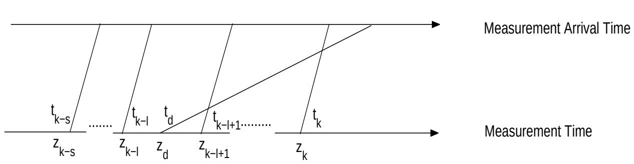


Figure 2.1: The OOSM $z_d$ arrives after the last processed measurement $z_k$.

Similar to (1), we have

$$\begin{aligned} x_k &= F_{k,d}x_d + w_{k,d} \\ z_d &= H_d x_d + v_d \end{aligned}$$

The OOSM update problem is as follows: At time $t_k$ the LMMSE estimate is

$$\hat{x}_{k|k} = E^*[x_k|z^k], \quad P_{k|k} = \mathrm{MSE}[\hat{x}_{k|k}]$$

where

$$\begin{aligned} \hat{x} &= E^*(x|z) = \bar{x} + C_{xz}C_z^{-1}(z - \bar{z}) \\ P &= \mathrm{MSE}(\hat{x}) = E(\tilde{x}\tilde{x}') \\ C_{xz} &= \mathrm{cov}(x,z), \ \ C_z = \mathrm{cov}(z), \ \tilde{x} = x - \hat{x} \end{aligned}$$

In the above, $z^k = \{z_i\}_{i=1}^k$ is the measurement sequence up to $t_k$. For OOSM filtering, we deal with the problem that an earlier measurement at time $t_d$ arrives after the state estimate $\hat{x}_{k|k}$ and the covariance $P_{k|k}$ have been calculated. We want to update this estimate with the earlier measurement $z_d$, that is, to calculate the LMMSE estimator

$$\hat{x}_{k|k,d} = E^*[x_k|\Omega_k, z_d], \quad P_{k|k,d} = \mathrm{MSE}[\hat{x}_{k|k,d}]$$

where $\Omega_k$ stands for the available information for update with the OOSM $z_d$ . In [3], we have considered three cases of information storage for different prior information about $t_d$. Following most suggestions, we will only consider update algorithms with required information storage for the case of knowing the maximum delay $s$ of OOSM, where there is no prior information about the OOSM $z_d$ occurrence time $t_d$ before it arrives, but we know the maximum delay $s$ for the OOSM, i.e., $t_{k-s} \le t_{k-l} \le t_d < t_{k-l+1} \le t_k$.

## 2.2 Algorithm I — Globally Optimal Update (ALG-I)

Based on the linear dynamic model, it follows from recursive LMMSE estimation that the globally optimal update can be written as

$$\begin{aligned} \hat{x}_{k|k,d} &= E^*[x_k|z^k, z_d] && (3) \\ &= \hat{x}_{k|k} + K_d(z_d - H_d\hat{x}_{d|k}) = \hat{x}_{k|k} + K_d\tilde{z}_{d|k} \\ P_{k|k,d} &= P_{k|k} - K_d S_d K_d' && (4) \end{aligned}$$

where

$$K_d = U_{k,d}H_d' S_d^{-1}, \ S_d = H_d P_{d|k} H_d' + R_d, \ U_{k,d} = C_{x_k,\tilde{x}_{d|k}}$$

Let

$$\hat{x}_{d|n} = E^*(x_d|z^n), \ P_{d|n} = \mathrm{MSE}(\hat{x}_{d|n}), \ U_{n,d} = C_{x_n,\tilde{x}_{d|n}}$$

Based on the recursion for $\{\hat{x}_{d|k}, \ P_{d|k}, \ U_{k,d}\}$ derived in [3], it can be shown that when $P_{n+1|n+1}, \ P_{n+1|n}$

and $R_{n+1}$ are nonsingular (which hold for most target tracking problems), we have

$$\begin{aligned} H'_{n+1} S_{n+1}^{-1} \tilde{z}_{n+1|n} &= P_{n+1|n}^{-1} (\hat{x}_{n+1|n+1} - \hat{x}_{n+1|n}) \\ H'_{n+1} S_{n+1}^{-1} H_{n+1} &= P_{n+1|n}^{-1} - P_{n+1|n}^{-1} P_{n+1|n+1} P_{n+1|n}^{-1} \\ (I - K_{n+1} H_{n+1}) &= P_{n+1|n+1} P_{n+1|n}^{-1} \end{aligned}$$

So the recursion for $\{\hat{x}_{d|k}, P_{d|k}, U_{k,d}\}$ starting from $n = k - l + 1$ can be rewritten as

$$\begin{aligned} \hat{x}_{d|n+1} &= \hat{x}_{d|n} + U'_{n,d} F'_{n+1,n} P_{n+1|n}^{-1} (\hat{x}_{n+1|n+1} \\ &\quad - \hat{x}_{n+1|n}) \\ P_{d|n+1} &= P_{d|n} - U'_{n,d} F'_{n+1,n} P_{n+1|n}^{-1} (P_{n+1|n} \\ &\quad - P_{n+1|n+1}) P_{n+1|n}^{-1} F_{n+1,n} U_{n,d} \\ U_{n+1,d} &= P_{n+1|n+1} P_{n+1|n}^{-1} F_{n+1,n} U_{n,d} \end{aligned} \tag{5}$$

with the initial conditions given by

$$\begin{aligned} \hat{x}_{d|k-l+1} &= \hat{x}_{d|k-l} + P_{d|k-l} F'_{k-l+1,d} P_{k-l+1|k-l}^{-1} \\ &\quad (\hat{x}_{k-l+1|k-l+1} - \hat{x}_{k-l+1|k-l}) \\ P_{d|k-l+1} &= P_{d|k-l} - P_{d|k-l} F'_{k-l+1,d} (P_{k-l+1|k-l}^{-1} \\ &\quad - P_{k-l+1|k-l}^{-1} P_{k-l+1|k-l+1} \\ &\quad P_{k-l+1|k-l}^{-1}) F_{k-l+1,d} P_{d|k-l} \\ U_{k-l+1,d} &= P_{k-l+1|k-l+1} P_{k-l+1|k-l}^{-1} F_{k-l+1,d} P_{d|k-l} \end{aligned} \tag{6}$$

where

$$\hat{x}_{d|k-l} = F_{d,k-l} \hat{x}_{k-l|k-l} \tag{7}$$

$$P_{d|k-l} = F_{d,k-l} P_{k-l|k-l} F'_{d,k-l} + Q_{d,k-l} \tag{8}$$

From (5)-(8), we can see that the necessary information in order to update $\{\hat{x}_{d|k}, P_{d|k}, U_{k,d}\}$ needs to include

$$\Omega_k = \{\hat{x}_{k-l|k-l}, P_{k-l|k-l}, \cdots, \hat{x}_{k|k}, P_{k|k}\}$$

For this situation, we do not have any prior information about the OOSM $z_d$ occurrence time $t_d$, and what we know is the maximum delay $s$ for the OOSM, i.e., $t_{k-s} \le t_{k-l} \le t_d < t_{k-l+1} \le t_k$ . In order to save all necessary information for the update, we should have

$$\Omega_k = \{\hat{x}_{k-s|k-s}, P_{k-s|k-s}, \cdots, \hat{x}_{k|k}, P_{k|k}\}$$

and use (5)-(8) together with (3)-(4) for the OOSM update algorithm. The OOSM update is the traditional Kalman filter by adding the OOSM update algorithm, which is shown in Figure 2.2.

Algorithm I presented above is the globally optimal update [3] with less storage requirement since $P_{n+1|n+1}$, $P_{n+1|n}$ and $R_{n+1}$ are all nonsingular. The storage requirement increases linearly with maximum delay $s$. However, the necessary information only includes the state estimates in the concerned time interval. The storage is the same as Algorithm $Al1$ proposed in [5], but can achieve the best performance within the linear estimator class.

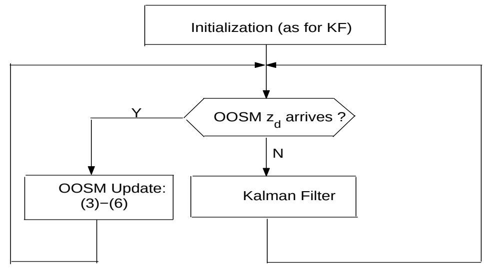


Figure 2.2: Flowchart for Algorithm I

We conclude that Algorithm I has (1) an efficient memory structure; and (2) an efficient computational structure to solve the problem by storing the necessary information instead of retrodiction or augmenting the state [6], [7], [8], [2]. Also, it is globally optimal for $t_{k-l} < t_d < t_{k-l+1}$ as well as $t_d = t_{k-l+1}$.

## 2.3 Algorithm II — Constrained Optimal Update (ALG-II)

Based only on the information $\hat{x}_{k|k}$ and $z_d$ at time when OOSM $z_d$ arrives, the optimal OOSM update is the LMMSE estimator $E^*(x_k|\hat{x}_{k|k}, z_d)$ without prior information [11]. It is in general not globally optimal [i.e., $E^*(x_k|\hat{x}_{k|k}, z_d) \ne E^*(x_k|z^k, z_d)$], but it is optimal conditioned on the available information. As presented in [3], the LMMSE update without prior is given by

$$\begin{aligned} \hat{x}_{k|k,d} &= \tilde{K} z^c = (H^{c\prime} R^{c-1} H^c)^{-1} H^{c\prime} R^{c-1} z^c \\ P_{k|k,d} &= (H^{c\prime} R^{c-1} H^c)^{-1} \end{aligned} \tag{9}$$

where $z^c = \begin{bmatrix} \hat{x}_{k|k} \\ z_d \end{bmatrix}$ and $H^c = \begin{bmatrix} I \\ H_d F_{k,d}^{-1} \end{bmatrix}$ with

$$R^c = \begin{bmatrix} P_{k|k} & (P_{k|k} F_{k,d}^{-1\prime} - U_{k,d}) H'_d \\ H_d (F_{k,d}^{-1} P_{k|k} - U'_{k,d}) & R_d + H_d F_{k,d}^{-1} Q_{k,d} F_{k,d}^{-1\prime} H'_d \end{bmatrix}$$

Algorithm II always gives the optimal update based on the available information. We have the property that, if the update is only one-lag, (9) is the solution given by [7], [1]. In the multi-step update case, (9) is consistent with [4]. In Algorithm II, the estimator contains the term $U_{k,d}$. As for Algorithm I, when $P_{n+1|n+1}$, $P_{n+1|n}$ and $R_{n+1}$ are nonsingular, we have

$$(I - K_{n+1} H_{n+1}) = P_{n+1|n+1} P_{n+1|n}^{-1}$$

So the recursion for $\{U_{k,d}\}$ starting from $n = k - l + 1$ can also be rewritten as

$$U_{n+1,d} = P_{n+1|n+1} P_{n+1|n}^{-1} F_{n+1,n} U_{n,d} \tag{10}$$

with the initial value

$$\begin{aligned} & U_{k-l+1,d} \\ & = P_{k-l+1|k-l+1} P^{-1}_{k-l+1|k-l} F_{k-l+1,d} P_{d|k-l} \end{aligned} \tag{11}$$

If the maximum delay for the OOSM is $s$. We should require

$$\Omega_k = \{P_{k-s|k-s}, \cdots, P_{k|k}\}$$

Similar to Algorithm I, this OOSM update algorithm can be implemented at the arrival time of OOSM $z_d$. The OOSM update is the traditional Kalman filter by adding the OOSM update using (9)-(11). The information storage increases linearly with the maximum delay $s$. The storage is the same as Algorithm $Bl1$ of [5], it can achieve better performance in terms of the MSE errors.

## 2.4 Update with Arbitrarily Delayed OOSMs

In the case of arbitrarily delayed multiple OOSMs, i.e., any OOSM arrives before the next OOSM occurrence time belongs to the single-OOSM update problem, we can solve it by sequentially applying the single-OOSM update. But if some other OOSMs occur during the period between the occurrence time and arrival time of one OOSM, the solution for the optimal update is not so simple. In the following, we only consider the problem of update with two OOSMs. Generalization to update with more than two OOSMs is straight forward.

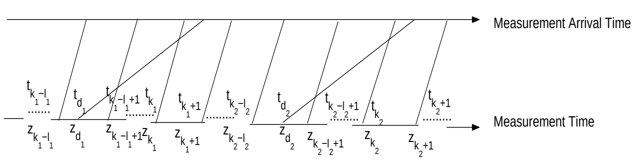


Figure 2.4.1 OOSMs in the maximum delay period

Suppose $z_{d_1}$ and $z_{d_2}$ are two OOSMs observed at $t_{k-l_i} \le t_{d_i} < t_{k-l_i+1}$ with $1 \le l_i < s$, $i = 1, 2$, and arrived during the time period $[t_{k_i}, t_{k_i+1})$. If $z_{d_1}$ arrives before $t_{d_2}$ (see Figure 2.4.1), the state update with $z_{d_2}$ at its arrival time can use the single-OOSM update as before. At $z_{d_2}$ occurrence time, there is no other OOSMs except $z_{d_2}$. So we can directly apply Algorithm I or II for updating with the single-OOSM $z_{d_2}$. If both of them arrive at the same time, although we can update the state estimate with them stacked together, computationally and operationally, it is better to update with the OOSMs sequentially.

If we consider the case that $z_{d_1}$ arrives after $t_{d_2}$ (see Figure 2.4.2), we can not simply apply the single OOSM update algorithm twice. Suppose $z_{d_2}$ arrives before $z_{d_1}$, or we process $z_{d_2}$ before $z_{d_1}$ if both of them arrive at the same time. According to ALG-I and ALG-II for single-OOSM update, we need to update $\hat{x}_{k_2|k_2}$ and $P_{k_2|k_2}$ with $z_{d_2}$ when it arrived. By using a Kalman filter, until time $t_{k_1}$, we have the state estimate sequence $\{\hat{x}_{k_2|k_2,d_2},\ P_{k_2|k_2,d_2},\ \ldots,\ \hat{x}_{k_1|k_1,d_2},\ P_{k_1|k_1,d_2}\}$. At the time when OOSM $z_{d_1}$ arrives, in order to update $\{\hat{x}_{k_1|k_1,d_2},\ P_{k_1|k_1,d_2}\}$, the necessary information for ALG-I or ALG-II needs to include $\{\hat{x}_{k_2-l_2|k_2-l_2,d_2},\ P_{k_2-l_2|k_2-l_2,d_2},\ \ldots,\ \hat{x}_{k_2-1|k_2-1,d_2},\ P_{k_2-1|k_2-1,d_2}\}$ or $\{P_{k_2-l_2|k_2-l_2,d_2},\ \ldots,\ P_{k_2-1|k_2-1,d_2}\}$. Therefore, at the time when the first OOSM $z_{d_2}$ arrives, we not only need to update the current state $\{\hat{x}_{k_2|k_2},\ P_{k_2|k_2}\}$, but also update $\{\hat{x}_{k_2-l_2|k_2-l_2}, P_{k_2-l_2|k_2-l_2}, \ldots, \hat{x}_{k_2-1|k_2-1}, P_{k_2-1|k_2-1}\}$ for ALG-I or $\{P_{k_2-l_2|k_2-l_2}, \ldots, P_{k_2-1|k_2-1}\}$ for ALG-II between OOSM $z_{d_2}$ occurrence time and its arrival time. The update for $\{\hat{x}_{k_2-i|k_2-i},\ P_{k_2-i|k_2-i}\}$ or just $\{P_{k_2-i|k_2-i}\}$ with $i = 1, \ldots, l_2$ is quite simple. It can be implemented with the same procedure for $\{\hat{x}_{k_2|k_2},\ P_{k_2|k_2}\}$ by treating the arrival time of OOSM $z_{d_2}$ as in the time interval $[t_{k-i}, t_{k-i+1}]$ with $i = 1, \ldots, l_2$.

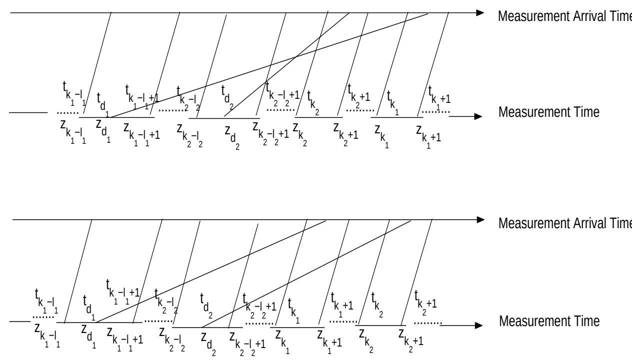


Figure 2.4.2 OOSMs in the maximum delay period

# 3 Estimation with OOSMs in Clutter

## 3.1 Problem Formulation

For multi-sensor multi-target tracking in the presence of clutter, a set of measurements $\mathbf{z}_j$ collected in a scan are sent to the fusion center. Some of them are target originated and others are false measurements. The existing algorithms for tracking a target in the presence of clutter include non-Bayesian and Bayesian techniques [10]. Probabilistic data association filter (PDA) and its extension, joint probabilistic data association filter (JPDA), belong to Bayesian techniques. PDA and JPDA are target-oriented approach. For a known number of targets, PDA (JPDA) evaluates the measurement-to-target association probabilities and combines them into the corresponding state estimates. MHT is a measurement-oriented or track-oriented approach.

Here we limit the discussion to a single target tracking in clutter and assume a measurement set $\mathbf{z}_d$ produced at previous time $t_d$ arrived at the fusion center after the measurement set $\mathbf{z}_k$ produced at the most recent time $t_k$, where $t_k > t_d$. Then we can identify that the measurement set produced at $t_d$ contains OOSMs. We will formulate the PDA incorporating OOSM update for a single target tracking. It is easy to analyze and the result can be generalized to other more complicated cases.

The set of validated measurements is denoted as

$$\mathbf{z}_j = \{z_j^i\}_{i=1}^{m_j}$$

where $z_j^i$ is the $i$-th validated measurement and $m_j$ is the number of measurements in the validated region at time $t_j$. In view of the assumptions listed, the association events

$$\theta_j^i = \begin{cases} \{z_j^i \text{ is the target originated measurement}\} \\ \quad i = 1, \ldots, m_j \\ \{\text{None of the measurements is target} \\ \quad \text{originated}\} \quad i = 0 \end{cases}$$

are mutually exclusive and exhaustive for $m_j \geq 1$. The problem is as follows: an earlier set of measurements $\mathbf{z}_d = \{z_d^i\}_{i=1}^{m_d}$ at time $t_d$ arrives after the state estimate $\hat{x}_{k|k}$ and the covariance $P_{k|k}$ have been calculated. Using the total probability theorem, the state estimate using $\mathbf{z}_d$ is

$$\begin{aligned} \hat{x}_{k|k,d} &= E(x_k|\mathbf{z}^k, \mathbf{z}_d) \\ &= \sum_{i=0}^{m_d} E(x_k|\theta_d^i, \mathbf{z}^k, \mathbf{z}_d) P(\theta_d^i|\mathbf{z}^k, \mathbf{z}_d) \\ &= \sum_{i=0}^{m_d} \hat{x}_{k|k,d}^i \beta_d^i \end{aligned}$$

where $\hat{x}_{k|k,d}^i$ for $i = 1, \ldots, m_d$ is the updated state conditioned on the event that the $i$-th validated measurement $z_d^i$ is target originate and $\beta_d^i = P(\theta_d^i|\mathbf{z}^k, \mathbf{z}_d)$ is the conditional probability of the event — the association probability — and $\hat{x}_{k|k,d}^0 = \hat{x}_{k|k}$, $P_{k|k,d}^0 = P_{k|k}$. Also

$$\begin{aligned} P_{k|k,d} &= E\{[x_k - \hat{x}_{k|k,d}][x_k - \hat{x}_{k|k,d}]'|\mathbf{z}^k, \mathbf{z}_d\} \\ &= \sum_{i=0}^{m_d} E\{[x_k - \hat{x}_{k|k,d}][x_k - \hat{x}_{k|k,d}]'|\theta_d^i, \mathbf{z}^k, \mathbf{z}_d\}\beta_d^i \\ &= \bar{P}_{k|k,d} + \tilde{P}_d \end{aligned}$$

where $\bar{P}_{k|k,d} = \sum_{i=0}^{m_d} \beta_d^i P_{k|k,d}^i$, $\tilde{P}_d = \sum_{i=0}^{m_d} \beta_d^i \hat{x}_{k|k,d}^i (\hat{x}_{k|k,d}^i)' - \hat{x}_{k|k,d}(\hat{x}_{k|k,d})'$. Based on different OOSM update, $\hat{x}_{k|k,d}^i$, $P_{k|k,d}^i$ and $\beta_d^i$ will have different forms.

## 3.2 OOSM Update: PDA with ALG-I

It follows from the proposed globally optimal OOSM update filter Algorithm I, that

$$\begin{aligned} \hat{x}_{k|k,d}^i &= \hat{x}_{k|k} + K_d(z_d^i - H_d\hat{x}_{d|k}) = \hat{x}_{k|k} + K_d\tilde{z}_d^i \\ P_{k|k,d}^i &= P_{k|k} - K_d S_d K_d' \end{aligned}$$

With $\tilde{z}_d = \sum_{i=1}^{m_d} \beta_d^i \tilde{z}_d^i$, we have

$$\hat{x}_{k|k,d} = \sum_{i=0}^{m_d} \hat{x}_{k|k,d}^i \beta_d^i = \hat{x}_{k|k} + K_d \tilde{z}_d$$

Let $P_{k|k,d}^c = P_{k|k} - K_d S_d K_d'$. Then

$$\bar{P}_{k|k,d} = \beta_d^0 P_{k|k} + [1 - \beta_d^0] P_{k|k,d}^c$$

and

$$\tilde{P}_d = K_d \left[ \sum_{i=0}^{m_d} \beta_d^i \tilde{z}_d^i (\tilde{z}_d^i)' - \tilde{z}_d \tilde{z}_d' \right] K_d'$$

The association probability can be derived the same as in-sequence PDA filter,

$$\beta_d^i = \begin{cases} \dfrac{e_i}{b + \sum_{j=1}^{m_d} e_j} & i = 1, \ldots, m_k \\ \dfrac{b}{b + \sum_{j=1}^{m_d} e_j} & i = 0 \end{cases} \tag{12}$$

where $e_i = e^{-\frac{1}{2}(\tilde{z}_d^i)' S_d^{-1} \tilde{z}_d^i}$ and $b = \lambda |2\pi S_d|^{1/2} \frac{1 - P_D P_G}{P_D}$ with gate probability $P_G$ and detection probability $P_D$.

## 3.3 OOSM Update: PDA with ALG-II

It follows from the proposed constrained optimal OOSM update Algorithm II, that

$$\begin{aligned} \hat{x}_{k|k,d}^i &= E^*(x_k|\hat{x}_{k|k}, z_d^i) = \tilde{K} z_i^c \\ P_{k|k,d}^i &= (H^{c\prime} R^{c-1} H^c)^{-1} \end{aligned}$$

where $E^*(x_k|\hat{x}_{k|k}, z_d^i)$ is the LMMSE without prior and $z_i^c = \begin{bmatrix} \hat{x}_{k|k}' & (z_d^i)' \end{bmatrix}'$. With $\tilde{z}_d = \sum_{i=1}^{m_d} \beta_d^i z_i^c = \begin{bmatrix} (1 - \beta_d^0)\hat{x}_{k|k}' & \sum_{i=1}^{m_d} \beta_d^i (z_d^i)' \end{bmatrix}'$, we have

$$\hat{x}_{k|k,d} = \sum_{i=0}^{m_d} \hat{x}_{k|k,d}^i \beta_d^i = \beta_d^0 \hat{x}_{k|k} + \tilde{K} \tilde{z}_d$$

Let $P_{k|k,d}^c = (H^{c\prime} R^{c-1} H^c)^{-1}$. Then

$$\bar{P}_{k|k,d} = \beta_d^0 P_{k|k} + [1 - \beta_d^0] P_{k|k,d}^c$$

and

$$\tilde{P}_d = \tilde{K}[\sum_{i=0}^{m_d} \beta_d^i z_i^c (z_i^c)' - \tilde{z}_d(\tilde{z}_d)']\tilde{K}' + \beta_d^0[1-\beta_d^0]$$
$$\hat{x}_{k|k}\hat{x}_{k|k}' - \beta_d^0 \hat{x}_{k|k}(\tilde{z}_d)'\tilde{K}' - \beta_d^0 \tilde{K}\tilde{z}_d \hat{x}_{k|k}'$$

The association probability has the same form as (12) with $e_i = e^{-\frac{1}{2}(\tilde{z}_d^i)' S_d^{-1} \tilde{z}_d^i}$ and $b = \lambda |2\pi S_d|^{1/2} \frac{1-P_D P_G}{P_D}$, where $\tilde{z}_d^i = z_d^i - H_d E^*(x_d|\hat{x}_{k|k})$ and $S_d = H_d P_{d|k} H_d' + R_d$. Therefore, the only task for the PDA to incorporate the constrained optimal OOSM update is to get the LMMSE $\hat{x}_{d|k} = E^*(x_d|\hat{x}_{k|k})$ and $P_{d|k} = \text{MSE}[\hat{x}_{d|k}]$. Based on the limited information storage $\Omega_k = \{P_{k-s|k-s}, \ldots, P_{k|k}, \mathbf{z}_d\}$, we can only achieve the LMMSE without prior

$$\hat{x}_{d|k} = F_{k,d}^{-1} \hat{x}_{k|k} \tag{13}$$
$$P_{d|k} = F_{k,d}^{-1} \bar{R}_d F_{k,d}^{-1\prime} \tag{14}$$

where

$$\bar{R}_d = Q_{k,d} + F_{k,d} U_{d,k}' + U_{d,k} F_{k,d}' - P_{k|k}$$

Then $\tilde{z}_d^i = z_d^i - H_d F_{k,d}^{-1} \hat{x}_{k|k}$ and $S_d = H_d F_{k,d}^{-1} \bar{R}_d F_{k,d}^{-1\prime} H_d' + R_d$.

PDA with OOSM update is suggested for single target tracking in clutter. The OOSM update filter can only handle the state update problem. OOSM update in the presence of measurement origin uncertainty can not be done easily. This means we can not expect the PDA with OOSM update will achieve the same performance as the in-sequence PDA filter. If we want to have the updated PDA filter have the same performance as the in-sequence PDA, we need also update the associated probability $\beta_j^i$ with $j = k-l+1, \ldots, k$. But $\beta_d^i$ relies on the observation $z_j^i$. So in order to update $\beta_j^i$, all observations from $t_{k-l+1}$ to $t_k$ are needed. However, the information we have at the OOSMs arrival time is limited, such as $\Omega_k = \{\hat{x}_{k-s|k-s}, P_{k-s|k-s}, \ldots, \hat{x}_{k|k}, P_{k|k}, \mathbf{z}_d\}$ or $\Omega_k = \{P_{k-s|k-s}, \ldots, P_{k|k}, \mathbf{z}_d\}$. Even the PDA with globally optimal OOSM update will have difference in performance with the in-sequence PDA. But we can not affirm that the updated PDA filter will always perform poorer than the in-sequence PDA since the PDA filter itself is not optimal. There are no fundamental optimal criteria for us to obtain the optimal OOSMs update within the PDA framework.

If the available information for update is $\Omega_k = \{\hat{x}_{k-s|k-s}, P_{k-s|k-s}, \ldots, \hat{x}_{k|k}, P_{k|k}, \mathbf{z}_k, \mathbf{z}_d\}$ or $\Omega_k = \{P_{k-s|k-s}, \ldots, P_{k|k}, \mathbf{z}_k, \mathbf{z}_d\}$, we can also update $\beta_k^i$ with $i = 1, \ldots, m_i$ in order to obtain a more accurate estimate of $x_k$. The procedure includes update state $\hat{x}_{k-1|k-1}$ to $\hat{x}_{k-1|k-1,d}$, then applying the in-sequence PDA filter to yield $\hat{x}_{k|k,d}$ by recalculating $\beta_k^i$ with $\mathbf{z}_k$. The performance will be the same as PDA with $l-1$ lag OOSM update if we treat the original PDA with OOSM update as an $l$-lag problem.

### 3.4 OOSM Update: Multi-Target Case

In the previous subsections, we have incorporated the PDA with Algorithms I and II for the OOSM update. For multi-target tracking in clutter, we need to consider both the data association issue and the OOSM update. For data association via JPDA or its variants e.g., nearest neighbor JPDA, incorporating the OOSM update using Algorithms I and II is straightforward. For each track, when receiving OOSMs, the Algorithms I and II operate based on the marginal data associated probability obtained via JPDA and the filter updates are decoupled among different tracks once the marginal data association probabilities for the validated measurements are computed by evaluating the joint events using JPDA. All these subtleties are data association issue rather than the OOSM update.

## 4 Simulations

Several simple numerical examples are given in this section to verify the proposed algorithms. Consider a discretized continuous time kinematic system driven by white noise with power spectral density $q$, known as nearly constant velocity model or white-noise acceleration model in target tracking, described by the following linear model

$$x_j = F_{j,j-1} x_{j-1} + w_{j,j-1}$$
$$z_j = H_j x_j + v_j$$

where $x_j = \left[x_j^{(1)}, x_j^{(2)}\right]$, $w_{j,j-1}$ and $v_j$ are zero mean white mutually uncorrelated Gaussian noise with

$$F_{j,j-1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad H_j = [1, 0]$$
$$\text{var}(w_{j,j-1}) = Q = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} q, \ \text{var}(v_j) = R = 1$$

where $T$ is the sampling interval. The prior information is

$$\hat{x}_{0|0} = \bar{x} = [200\text{km}, 0.5\text{km/sec}]'$$
$$P_{0|0} = \begin{bmatrix} R & R/T \\ R/T & 2R/T^2 \end{bmatrix}$$

and the maneuver index is $\lambda = \sqrt{qT^3/R}$.

In order to consider multi-lag delay as well as single-lag OOSM update, we choose a sequence of OOSMs $z_d$. These OOSMs occurred at $d = (l+1)n$ and arrived at $(l+1)n + l$ with $n = 1, 2, \ldots$, corresponding to $l$-lag delayed OOSMs, where $l = 1, 2, \ldots$. For example, if the in-sequence observation sequence is $\{z_1, z_2, z_3, \ldots\}$, then the observation sequence with OOSMs for $l = 1$ is $\{z_1, z_3, z_2, z_5, z_4, \ldots\}$ and the updated states are $x_3, x_5, x_7, \ldots$; the observation series with OOSMs for $l = 2$ is $\{z_1, z_2, z_4, z_5, z_3, z_7, z_8, z_6, \ldots\}$ and the updated states are $x_5, x_8, x_{11}, \ldots$; and so on. Ideal estimates were obtained by the Kalman filter using all target originated observations only (including OOSMs) in the right time sequence.

For clutter generation, we use a Poisson model:

$$\mu_F(m) = e^{-\lambda V} \frac{(\lambda V)}{m!}$$

with $\lambda$ — density in measurement space, $V$ — volume of validation region in measurement space. By choosing $\lambda V \in [0, 1]$, we can simulate different clutter densities. We set $P_D = 1$. Residual based $\chi^2(99.9)$ test is used for testing tracking divergence. In simulation results, there are less than 10% track loss.

We show RMS errors over 1000 monte Carlo runs for the OOSM updated states at $(l+1)n + l$ with $n = 1, 2, \ldots$, where KF — in-sequence Kalman filter without clutter; IS-PDAF — in-sequence PDA filter; IG-PDAF — in-sequence PDA filter ignoring the OOSMs; UD-PDAF1 — PDA with globally optimal OOSM Update; UD-PDAF2 — PDA with constrained optimal OOSM update.

## 4.1 OOSMs with Good Accuracy

We design the OOSM model by choosing $\mathrm{var}(v_j) = R/10$ at $j = (l+1)n$ with $n = 1, 2, \ldots$, which means the OOSMs are more accurate than the in-sequence measurements. In the following, we show the RMS errors for the 1-lag, 2-lag and 4-lag OOSM update problems at time $k = 29$ with $\lambda V \in [0,\ 0.25,\ 0.5,\ 0.75,\ 1]$ in Figure 4.1.1, Figure 4.1.2 and Figure 4.1.3 respectively.

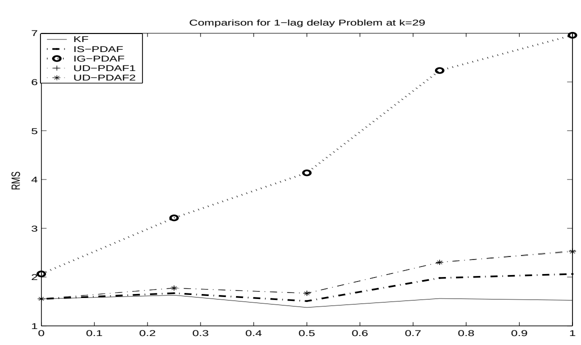


Figure 4.1.1 The RMS errors for 1-lag OOSM upate at time $k = 29$ with $\lambda V \in [0,\ 0.25,\ 0.5,\ 0.75,\ 1]$

As shown in these figures, when there is no clutter, i.e., $\lambda V = 0$, ALG-I has the same performance as the KF, while ALG-II has slightly poorer performance. For target tracking in clutter, the PDA with OOSM update by Algorithm I or II yields RMS errors close to the KF, which indicates that through OOSM update, the performance has significant improvement especially for small-lag OOSMs. For large-lag OOSMs, by ignoring them, the performance does not suffer much even if the OOSMs have much better accuracy. The RMS errors of UD-PDAF1 and UD-PDAF2 are very close to that of the in-sequence PDA filter. It also shows that the performance of IS-PDAF, IG-PDAF, UD-PDAF1 and UD-PDAF2 deteriorates as the clutter becomes heavier.

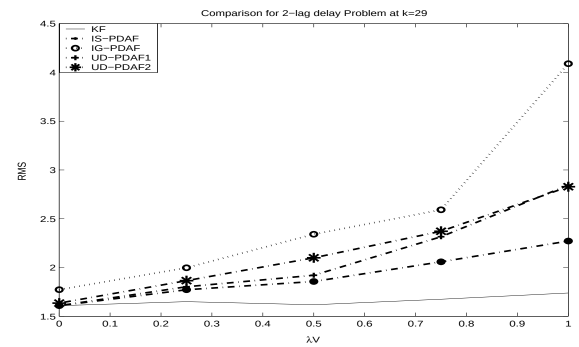


Figure 4.1.2 The RMS errors for 2-lag OOSM upate at time $k = 29$ with $\lambda V \in [0,\ 0.25,\ 0.5,\ 0.75,\ 1]$

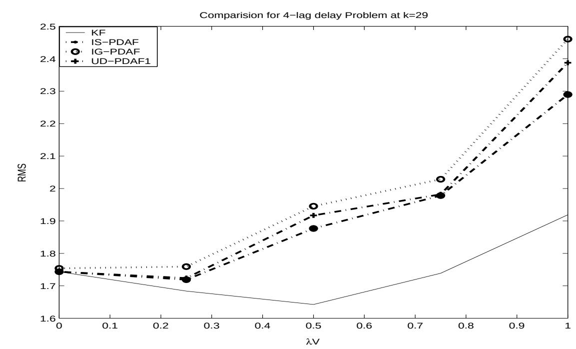


Figure 4.1.3 The RMS errors for 4-lag OOSM upate at time $k = 29$ with $\lambda V \in [0,\ 0.25,\ 0.5,\ 0.75,\ 1]$

## 4.2 OOSMs with Moderate Accuracy

The OOSMs have $C_{v_j} = R$, at $j = (l+1)n$ with $n = 1, 2, \ldots$, which means the in-sequence measurements have the same accuracy as the OOSMs. From Figure 4.2, we can clearly see that by ignoring the OOSMs, the performance still suffers a lot. But the PDA with OOSM update improves the performance, which is close to that of the in-sequence PDA filter.

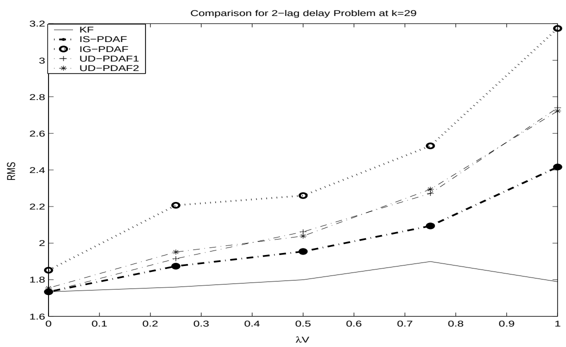


Figure 4.2 The RMS errors for 2-lag OOSM upate at time $k = 29$ with $\lambda V \in [0, 0.25, 0.5, 0.75, 1]$

# 5 Conclusions

In this paper, we first provided a simplified version of the general OOSM update algorithms Algorithm I and II presented in [3] under the assumptions of non-singularity of certain matrices valide for most tracking applications. Then we proposed using PDA with Algorithm I and II for the OOSM update in the presence of clutter. Simulation results show that the PDA with the OOSMs update in clutter performs significantly better than ignoring the OOSMs, especially for small-lag OOSMs. Its performance is close to the in-sequence PDA update for OOSMs with various lags and under mild clutter where the PDA filter has less than 10% track loss. In summary, the PDA incorporating the two OOSM update algorithms has (1) an efficient processing structure; (2) an efficient memory structure; (3) an efficient computational structure. A brief discussion was given concerning how to incorporate the OOSM update algorithms with the JPDA for multi-target tracking in clutter.

# References

[1] Y. Bar-Shalom, *Update with Out-of-Sequence Measurements in Tracking: Exact Solution*, IEEE Trans. on Areaspace and Electronic Systems, Vol 38, No. 3, pp. 769–778, July 2002.

[2] S. Challa, J. E. Robin, and X. Wang, *A Bayesian Solution and its Approximations to Out-of-Sequence Measurement Problems*, Proc. International Conf. on Information Fusion, Annapolis, MD, pp. 919-926, July 2002.

[3] K. S. Zhang, X. R. Li, and Y. M. Zhu, *Optimal Update with Out-of-Sequence Measurements for Distributed Filtering*, Proc. International Conf. on Information Fusion, Annapolis, MD, pp. 1519-1526, July 2002.

[4] M. Mallick, S. Coraluppi, and C. Carthel, *Advances in Asynchronous and Decentralized Estimation*,Proc. IEEE Conf. Aerospace, Big Sky, MT, March 2001.

[5] Y. Bar-Shalom, M. Mallick, H. Chen, and R. Washburn, *One-Step Solution for the General Out-of-Sequence Measurements Problem in Tracking*,Proc. IEEE Conf. Aerospace, Big Sky, MT, March 2002.

[6] S. Thomopoulos, and L. Zhang, *Distributed Filtering with Random Sampling and Delay*,Proc. 27th IEEE Conf. Decision and Control, Austin, TX, Dec. 1988.

[7] R. D. Hilton, D. A. Martin and W. D. Blair, *Tracking with Time-Delayed Data in Multisensor Systems*,Naval Surface Warfare Center,NSWCDD/TR-93/351, Dalgren, VA, Aug. 1993.

[8] S. S. Blackman, and R. F. Popoli, *Design and Analysis of Modern Tracking Systems*,Artech House, Norwood, MA, 1999.

[9] M. Mallick, SJ. Krant, and Y. Bar-Shalom, *Multisensor Multi-target Tracking using Out-of-Sequence Measurements*,Proc. International Conf. on Information Fusion, Annapolis, MD, pp. 135-142, July 2002.

[10] Y. Bar-Shalom, and X. R. Li, *Multitarget-Multisensor Tracking: Principles and Techniques*,YBS Publishing, Storrs, CT, 1995.

[11] X. R. Li, Y. M. Zhu, and C. Z. Han, *Unified Optimal Linear Estimation Fusion—Part I: Unified Models and Fusion Rules*,Proc. International Conf. on Information Fusion, Paris, France, pp. MoC2.10-MoC2.17, July 2000.