Sensor Selection for Active Information Fusion

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Motivation

In active Information fusion, to determine the most informative and cost-effective sensors requires evaluating all possible sensor combinations, which is computationally intractable when information-theoretic criterion is used. We present a method for efficient mutual information computation.

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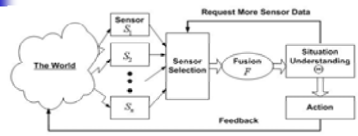
Active Information Fusion



Selectively choose the most decision-relevant sensors while minimizing the cost associated with using the sensors for acquiring information

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Sensor Selection Criterion

Maximize Expected Information Gain and Minimize Sensor Cost

- Expected Information Gain: mutual information

$$I(\Theta;S)=H(\Theta)-H(\Theta\,|\,S)$$

$$=\sum_{\Theta}\sum_{S_1}\cdots\sum_{S_n}\{p(\theta,s_1,\ldots,s_n)\log\frac{p(\theta\,|\,s_1,\ldots,s_n)}{p(\theta)}\}$$

- Cost: computational cost, activation cost, operating cost, etc.

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Computational Challenge

Time complexity for computing mutual information

$$T(n)=\sum_{i=1}^{n}\binom{n}{i}2^{i}\bullet C_I\bullet i$$

where n is total number of sensors, C_I is inference time. Computing higher order mutual information is computationally difficult.

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Definitions

[*Sensor Synergy*]: A measure of synergetic potential between two sensors S_i and S_j in reducing uncertainty of hypothesis Θ

$$r_{ij}=\frac{I(\Theta;S_i,S_j)-\max(I(\Theta;S_i),I(\Theta;S_j))}{H(\Theta)}$$

[*Synergy Matrix*]: Let a sensor set be $\{S_1,\cdots,S_n\}$ sensor synergy coefficient matrix is a matrix defined as

$$R=\begin{bmatrix}0 & r_{12} & \cdots & r_{1n}\\ r_{21} & 0 & \cdots & r_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ r_{n1} & r_{n2} & \cdots & 0\end{bmatrix}$$

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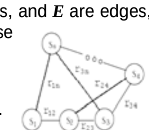
Definitions

[*Synergy Graph*]: Given a sensor synergy matrix, a graph $G=(S,E)$ **,** where $\boldsymbol{S}$ are the nodes, representing the set of sensors, and $\boldsymbol{E}$ are edges, representing the set of pair-wise synergetic links weighted by synergy coefficients r_{ij}, is a sensor synergy graph.



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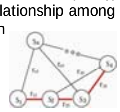
Definitions

[*Synergy Chain*]: Given n sensors in a synergy graph G, If a subset of sensors and sensors in this subset are serially connected, this subset of sensors is referred to as a synergy chain.

[*Markov Synergy Chain*]: Given a synergy chain with n sensors, for all $i=1,\ldots,n-1$, if $rij>0$ for $j=i+1$ and $rij=0$ for $j\neq i+1$, then the synergy chain that describes the synergetic relationship among $\{S1,\ldots,Sn\}$ is a Markov synergy chain

Synergy Chain (Highlighted)



Markov Synergy Chain

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Theorems

- [*Chain Rule*]: Given a Markov synergy chain with a set of sensors $S=\{S_1,\ldots,S_n\}$. For any n, the mutual information for a Markov synergy chain

$$I^M(\Theta;S_1,\ldots,S_n)=I(\Theta;S_1)+\sum_{i=1}^{n-1}(I(\Theta;S_i,S_{i+1})-I(\Theta;S_i))$$

- [*Upper Bound*]: For a synergy chain $\{S_1,\ldots,S_n\}$ in a synergy graph G, $I(\Theta;S_1,\ldots,S_n)$ is upper-bound by the mutual information of the corresponding Markov synergy chain, i.e.,

$$I(\Theta;S_1,\ldots,S_n)\le I^M(\Theta;S_1,\ldots,S_n)$$

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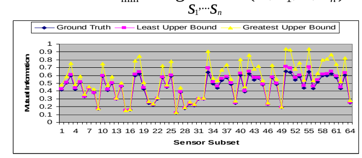
Least Upper Bound

A synergy chain $\{S_1,\ldots,S_n\}$ may correspond to multiple Markov Synergy Chains, depending on sensor orders. There exists a least upper bound, i.e.,

$$I^M_{\min}=\arg\min_{S_1\cdots S_n} I^M(\Theta;S_1,\ldots,S_n)$$



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Sensor Selection Algorithm

- The approximate algorithm is greedy-like, but the searching process is guided by a synergy graph to achieve a near optimal solution; Search space is reduced by pruning synergy graph.
- Computational difficulty in computing higher order mutual information can be circumvented by using its least upper bound.

SENSOR-SELECTION(n)
for each i,j, compute $I(\Theta;S_i)$ and $I(\Theta;S_i,S_j)$
Construct a synergy graph G and prune it
Choose S_{i^*},S_{j^*} such that $U(I(\Theta;S_i,S_j),C(S_i,S_j))$ is maximized for all i and j
$\mathbf{S}\leftarrow\{S_{i^*},S_{j^*}\}$
while $|\mathbf{S}|<n$
  for each $\mathbf{S}'$, where $|\mathbf{S}'|=|\mathbf{S}|+1$, and $\mathbf{S}'$ is a synergy chain on G and $\mathbf{S}\subset\mathbf{S}'$
    Find all Markov synergy chains of $\mathbf{S}'$
    $Iub(\Theta;\mathbf{S}')\leftarrow\arg\min(I(\Theta;\mathbf{S}'))$, where $I(\Theta;\mathbf{S}')$ is computed, and min takes over all Markov synergy chains of $\mathbf{S}'$
  $\mathbf{S}'^*\leftarrow\arg\max(U(Iub(\Theta;\mathbf{S}'),C(\mathbf{S}')))$, where max takes over all $\mathbf{S}'$
  if $U(Iub(\Theta;\mathbf{S}'^*),C(\mathbf{S}'^*))>U(Iub(\Theta;\mathbf{S}),C(\mathbf{S}))$
    $\mathbf{S}\leftarrow\mathbf{S}'^*$
  else break
return $\mathbf{S}$

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Algorithm Evaluation

100 different BN test models with sensors 7, 8, 9 and 10, parameterized with different CPTs. The result averaged among 100 trials

Number of Sensors	Our Approach		Brute-Force
	Closeness To Ground Truth	Run time (Seconds)	Run time (Seconds)
7	98.44%	1.020	63.87
8	98.23%	1.099	355.05
9	97.25%	1.209	2967.36
10	98.11%	1.430	13560.54

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Summary

- The computational difficulty in computing higher order mutual information is circumvented by efficiently computing their least upper bound.
- A graph theoretic approach to effectively eliminate non-promising sensor subsets, therefore significantly reducing the search space.
- The solution is close to the optimal solution.

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