Incentive Mechanism for Truthful Occupant Comfort Feedback in Human-in-the-loop Building Thermal Management

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***Abstract*—Energy inefficiency and underlying occupant discomfort associated with building thermal management systems have led to development of human-in-the-loop control system. Minimizing total energy consumption and maximizing comfort of all the occupants present is the major objective of such system based on human comfort feedback. However, the approaches proposed so far lack a built-in mechanism to elicit truthful comfort feedback from the occupants. In this work, we utilize an incentive based mechanism design framework to elicit true thermal comfort feedback (function) from the occupants present. This requires the building occupants to "purchase" or spend credits to achieve their personalized comfort levels within their zone of occupancy. The comfort pricing policy has been derived as an extension of the Vickrey-Clarke-Groves (VCG) pricing. It ensures *incentive-compatibility* of the mechanism, which implies that an occupant acting in self-interest cannot stand to benefit by declaring their comfort function untruthfully. This would hold irrespective of the thermal comfort choices made by the other occupants present in the building. We further propose as to how this mechanism could be implemented in practice with limited comfort feedback complexity, where the building operator would iteratively learn and refine the occupants comfort function based on simple two-dimensional comfort feedback (preferred temperature setting, and willingness-to-pay value) by the occupant. Simulations using parameters based on our Watervliet experimental facility demonstrates the effectiveness of the proposed mechanism.**

***Index Terms*—Truthful comfort feedback, incentive mechanism, human-centered building environment management, collaborative comfort management, smart building energy management.**

I. Introduction

Energy consumption in buildings accounted for 40% (38.5 Quadrillion Units of BTUs, or Quads) of primary energy consumption in US in 2014. This consumption is much greater than that accounted by other major sectors such as industrial (33%) or transportation (27%) [1]. Building energy consumption represented a cost of approximately $416 billion as of 2012. Buildings can also be held responsible for 38% of the energy related carbon dioxide emissions in the nation. Overall data suggests that 40% of the total energy consumption in US and 20% worldwide can be attributed to residential and commercial building usage [2]. Hence, attaining energy efficiency in buildings not only provides a means to reduce energy consumption and costs, it can also lead to significant reduction in greenhouse gas (GHG) emissions. Despite high energy costs and expensive HVAC upgrades, in many indoor environments occupant comfort levels have not been satisfactory [3], [4].

Lack of real-time information about actual occupancy and level of comfort leads to energy wastage due to unnecessary space conditioning during unoccupied periods, and overly conservative HVAC settings when occupants are present in the space. However, through a human-in-the-loop building thermal management system that incorporates real-time occupant comfort feedback, energy efficiency in buildings can be improved significantly without compromising occupant comfort. However, personalized indoor environment and comfort expectations pose a conflicting situation in multi-occupant shared spaces (such as office floors, student dorms, lab spaces, residential living, etc.), where each occupant has its own individual range of comfortable temperature. External factors such as attire, physical and mental condition, and level of tolerance along with environmental factors such as time of the day, lighting conditions, etc. can further influence an individual's comfort preference [5], [6], [7].

Gathering individual occupant feedback in real-time can be the most effective way of capturing occupant's personal comfort preferences. Arriving at temperature set-points to minimize the discomfort among all occupants of different rooms or zones in a building, considering the thermal correlations, is an important yet challenging problem. The total energy cost also needs to be accounted in the formulation, when trying to determine the optimal temperature set-points in different zones of a building. Hence, achieving temperature settings for a building that attains energy efficiency while satisfying occupant preferences requires knowledge of the true comfort ranges of all occupants in the building. In this paper we focus on achieving true comfort range preference of occupants in a multi-zonal multi-occupant space, where there can be significant thermal correlation between different zones of the shared space, and the individual comfort ranges (discomfort functions) are held privately by each occupant.

A. Related Work

Many approaches for energy efficient building thermal management have appeared in recent literature; these include solutions based on model predictive control (MPC) [8], [9], [10], [11], [12], [13], and others that utilize the building thermal storage to reduce cost under variable electricity rates

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[14], [15], [16], [17]. Studies specifically focusing on human-in-the-loop control approaches or those incorporating occupant feedback have extensively explored occupant thermal comfort modeling as a precursor to building thermal management. Most of the occupant thermal comfort modeling studies can be broadly classified into the following categories: (1) the chamber study model, based on mapping thermal comfort from environmental and personal factors to a 7-level comfort value scale, viz. the predicted mean votepredicted percent dissatisfied (PMV-PPD) [18], [19]; (2) human body physiology based models such as the core to skin model [20], the comfort model for multi-human segments [21], model based on of sensation of human body segments [22]; and (3) adaptive comfort models developed in field study, viz. [23] and [24].

Multiple studies [25], [26], [27] have used the PMV index as the metric for occupant comfort integration in the control formulation. Thermovote [28] utilized a 7-level occupant comfort voting to integrate with the building control logic. Purdon et al. [29] developed a smart phone interface to receive 3-point scale comfort feedback from occupants and determine the direction for temperature drift with a system defined step-size. Zhao et al. [30] conducted a simulation study tying occupant subjective thermal comfort feedback with MPC control algorithm for the active HVAC system against a baseline rule-based control algorithm. Some more recent work considered thermal complaint behavior using one-class classifier [31], [32]. However, most of the existing work do not provide a clear incentive to motivate occupants towards providing truthful feedback on their individual comfort levels. The message complexity involved in obtaining occupant feedback in real-time has also not been discussed.

In a recent work of ours, we presented usage of the pricing signals to drive occupants towards consensus on temperature settings in the shared spaces [33]. The work presented in this paper is broadly related to [33], however it attempts to address several limitations of the previous work. To start with, the linear pricing policy based on dual variable in [33] does not necessarily leads to truthful feedback related to thermal comfort from the occupants. However, the VCG-based pricing policy presented in this paper guarantees truthful reporting of thermal comfort functions as a *dominant strategy*, i.e., an occupant can not gain by declaring its comfort function untruthfully, irrespective of the strategies (truthful or untruthful) followed by all the remaining occupants. Also, the solution presented in [33] was iterative, which could take up to hundreds of iterations (each iteration involving an automated bidding by the occupant agent, say a smart phone) to perform distributed computation of the consensus temperature; this had to be initiated any time the zonal occupancy changed or an occupant agent provided different (new) thermal choice feedback. The solution proposed in this work is more centralized, as the optimal temperature set points can be determined in one single computation by the building operator once the occupancy levels for zones are determined. The building operator maintains thermal comfort profiles (functions) for all the occupants, which is then used to recompute the optimal thermostat control set points any time the zonal occupancy changes. To start with, the occupant specific thermal comfort profile can be initialized to some default function and then updated based on the comfort feedback provided by the occupants.

The major technical contributions of this work, including the novelty proposed can be summarized as follows. 1) We propose a mechanism design framework to obtain true thermal comfort function (range) input from the occupants. This requires for the building occupants to "purchase" their personalized thermal comfort levels through some sort of credit mechanism. The pricing policy for thermal comfort, derived as an extension of Vickrey-Clarke-Groves (VCG) pricing, ensures *incentive-compatibility* of the mechanism, i.e., an occupant acting in self-interest ceases to gain by declaring its thermal comfort preference untruthfully, irrespective of the thermal choices made by all the other occupants. 2) We also discuss as to how this mechanism can be implemented in practice with limited information feedback complexity, whereby the building operator can iteratively learn and refine the occupant comfort function based on simple two-dimensional feedback (such as preferred temperature setting, and willingness-to-pay value) by the respective occupant. 3) We present extensive simulation results using parameters based on our experimental facility to demonstrate that indeed none of the occupants stand to benefit by declaring their thermal preferences untruthfully in a shared multi-occupant setup.

The rest of the paper is structured as follows. The overall approach, optimization objective, and the proposed comfort pricing mechanism is presented in Section II. Section III discusses couple of approaches to estimate occupant thermal discomfort function through limited feedback. Simulation results based on the parameters of our test facility are presented in section IV. Finally, we conclude our work in Section V.

II. Incentive Mechanism Framework And Overall Design

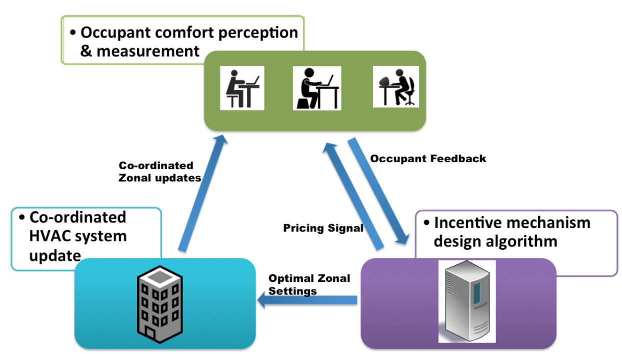

Fig. 1. A high level process flow depiction of the solution proposed in this work.

Figure 1 depicts the high level work-flow of the proposed solution and consists of the following components:
Occupant location and zonal temperature estimation: We make the assumption that all the thermal "zones" are equipped with temperature sensors that can feed the zonal temperature in real time to the central processor[1]. We further assume

[1] A "zone" is defined as a unit of space (a room, for example) that can be described in our model by a single temperature set-point

that the building operator can localize each occupant at a zone-level; this can be attained through bluetooth beacons or other wearable devices.

Thermal comfort feedback from occupants: The building occupants provide their thermal comfort feedback through a smartApp as is convenient to them. The feedback can be provided only occasionally based on when an occupant is uncomfortable at the current thermal setting.

Comfort price calculation and optimal zonal setting: Based on the location of the occupant and the corresponding thermal profile, building operator can calculate the optimal thermal settings to minimize sum of the occupant discomfort and total energy cost. Additionally, the building operator can also calculate a payment for each occupant based on the optimally determined zonal temperature settings and the occupant's corresponding discomfort function. Based on the optimal zonal temperatures calculated, an acceptable control algorithm is then used to attain the desired settings (we have described an adaptive feed-forward control algorithm for this purpose in our previous work [33]).

A. Minimization Objective

For a given building consisting of m zones, we denote the set of occupants located in zone j using the set S_j. Discomfort function of a particular occupant i is represented using D_i, and E denotes the overall energy cost as a function of the heat input vector. Following these notations, a reasonable minimization objective to attain in steady state with the zonal temperature vector y can be:

$$\text{minimize } \sum_{j=1}^{m} \sum_{i \in S_j} D_i(y_j) + E(u), \tag{1}$$

where y_j represents the temperature of zone j, and u is the heat input (control input) vector required to attain the set of corresponding zonal temperatures. An occupant i located in zone j (i.e., $i \in S_j$) would encounter the zonal temperature y_j, and hence its discomfort can be more appropriately represented as $D_i(y_j)$. In this work we assume the discomfort function $D_i(y_j)$ to be convex in its argument y_j, and can be flat over an intermediate range (occupant's comfort range). Further in equation (1), $E(u)$ is assumed to be a convex function of the control input vector u. For the sake of definiteness in this work, we work with $E(u) = u^T \Gamma u$ (where Γ is a positive definite matrix) in the simulations, although this function has to be estimated in practice or can be fed through real time pricing values. For the purpose of simulation in this study, we consider the widely popular RC building thermal model based on electric circuit analogy [34] given by:

$$C\dot{T} = -DR^{-1}D^T T + B_0 T_\infty + Bu, \tag{2}$$

where T is the temperature vector, u is the vector of heat inputs into the m zones of the building, and B is the corresponding input matrix. Further, vectors R and C represent the wall thermal resistors and capacitances respectively, and are both diagonal positive definite matrices. Also, D is the incidence matrix mapping the system capacitances to the resistors, and is of full row rank. $B_0 = -DR^{-1}d_0^T$ is a column vector with non-zero elements denoting the thermal conductances of nodes connected to the ambient. We can now use the steady-state relationship between y and u to optimize (1). This relationship is derived from $y = B^T T$, and setting $\dot{T} = 0$ in (2) gives:

$$u = g(y) \doteq (B^T A^{-1} B)^{-1}(y - B^T A^{-1} B_0 T_\infty), \tag{3}$$

where $A = DR^{-1}D^T$. In case the individual occupant discomfort functions were known to the building operator, the optimal zonal temperature vector y^* could be computed directly. However, there are several complexities associated with such a approach. Firstly, an occupant may not be able to correctly estimate its own discomfort function completely and reporting the entire function to the building operator has information complexity. Secondly, even with the assumption that occupants have knowledge of their discomfort functions the incentive for them to report it truthfully is missing. In practice then, it may be more desirable to have a mechanism through which the building operator can indirectly learn the true discomfort functions of the occupants in real time through minimal information exchange. The pricing policy needs to be such that it can guide the occupants towards a common temperature set-point for every zone in the building. We explore this principle in the following sub-section II-B.

B. Incentive Mechanism Design Framework

Using the well-known VCG mechanism [35] we design an incentive-compatible pricing policy to solve the pricing issue in our problem. Note that the standard VCG mechanism is defined for a fixed-quantity resource. However, in our case the amount of energy (resource) is flexible. The VCG mechanism can be extended naturally to “elastic” resources with convex costs, as we present in this work.

We can write the energy cost $E(u)$ as $G(y) = E\big(g(y)\big)$ using (3), which is convex in y. Let z denote the vector of zonal temperatures as computed by the building operator. Once the actual zonal temperatures reach steady state, we expect them to correspond to their computed values, i.e., $y = z$. Assuming this steady-state condition $y_j = z_j$ for all zones j, the minimization objective in (1) can be rewritten as:

$$\sum_{j=1}^{m} \sum_{i \in S_j} D_i(z_j) + G(z), \tag{4}$$

where function $G(z) = E\big(g(z)\big)$ represents the total energy cost in terms of z. Let z^* denote the value of vector z that maximizes (4). Further, let z^*_{-i} be the value of vector z that maximizes (4) *when occupant (user) i is absent from the building* (i.e., the discomfort function of occupant i is *not* considered in optimizing (4)), all other system parameters being the same. Then the *comfort price* that an occupant i pays, p_i, is computed as

$$p_i = \Big(\sum_{j=1}^{m} \sum_{i' \in S_j \setminus \{i\}} D_{i'}(z^*_j) + G(z^*) \Big) - \Big(\sum_{j=1}^{m} \sum_{i' \in S_j \setminus \{i\}} D_{i'}(z^*_{-i,j}) + G(z^*_{-i}) \Big). \tag{5}$$

The expression in (5) can be interpreted as the opportunity cost that an occupant i imposes on the system. Note that p_i can be written as $p_i = d_i + e_i$, where $d_i = \sum_{j=1}^{m}\sum_{i' \in S_j} D_{i'}(z_j^*) - \sum_{j=1}^{m}\sum_{i' \in S_j \setminus \{i\}} D_{i'}(z_{-i,j}^*)$, and $e_i = G(z^*) - G(z_{-i}^*)$. Here, d_i, the *discomfort cost*, is a measure of the total discomfort caused by the presence of occupant i on all the other occupants. Also, e_i is the *energy cost* imposed by occupant i on the building operator for maintaining the required thermal condition within the building. While the inclusion of e_i in the comfort price is natural, it can be shown that the inclusion of d_i is also necessary to ensure truthfulness of occupant feedback. Note that, for the fixed-resource case the comfort price (VCG pricing policy) would consist only of the discomfort cost (or in other words the opportunistic cost). Note that e_i, d_i and even p_i could be negative for an individual occupant, but the sum of the payments made by all occupants must exceed the total energy cost incurred by the building operator. We illustrate some of these cases through case studies in Section IV.

We now establish the truthfulness property of the mechanism proposed. Assume that the occupants report their discomfort functions to the building operator based on which the optimal temperature vector is calculated by minimizing (4). Focusing on a specific occupant i, that reports a discomfort function $\bar{D}_i$, that may or may not be truthful (i.e., the reporting by occupant i is not guaranteed be the truthful discomfort function D_i). Now, let z^*, p_i denote the optimal zonal temperature vector, and the comfort price paid by occupant i, when the discomfort function is reported truthfully. Further, let $\bar{z}^*$, $\bar{p}_i$ denote the optimal zonal temperature vector, and the comfort price paid by occupant i, when the discomfort function is reported untruthfully as $\bar{D}_i$, i.e., $\bar{D}_i$ $(\neq D_i)$. Then we can establish the following:

Proposition 1: For any discomfort function $\bar{D}_i$ reported by occupant i, $D_i(\bar{z}_i^*) + \bar{p}_i \geq D_i(z_i^*) + p_i$.

Loosely speaking, the above result states that an occupant can not gain (in terms its discomfort suffered plus comfort price paid, $D_i + p_i$) by reporting its discomfort function untruthfully to the building operator. Note that this holds true whether or not the *other* occupants report their discomfort functions truthfully. Proposition 1 establishes the "incentive compatibility" property of our mechanism. The proof of this proposition follows as a special case of that of *Proposition 3* and is therefore omitted for brevity.

III. Estimating Discomfort Functions through Limited User Feedback

The incentive-compatibility result of Section II-B establishes that rational occupants have no incentive to declare their discomfort functions untruthfully. However, asking occupants to declare their exact discomfort functions is unrealistic. We envision a two-phase approach to address this issue, and estimate the occupant discomfort functions through use of limited complexity feedback (bid) messages communicated by the occupants. First is the initialization step, in which a newly registering occupant is asked some optional questions on its preferred temperature range and credits it is willing to pay to get the temperature from some default setting to the occupant's comfort range. Based on this an initial discomfort function could be constructed. If the occupant chooses not to provide such input, a "default" comfort profile (representing that of an average occupant) could be used at this step. The second phase involves continuous refinement of this discomfort function based on occupant comfort feedback collection. This feedback may be provided only occasionally – say when an occupant is feeling uncomfortable. Such limited feedback could then be "merged" with the current comfort profile associated with the occupant.

The two phases as described above are motivated by different approaches to approximating VCG mechanisms with limited bid-complexity. The initialization approach requests occupants to declare their discomfort functions at certain discrete points, using which approximate discomfort functions are constructed via piecewise linear approximation. This "multi-bid" approximation to VCG can be shown to guarantee the social optimality and incentive compatibility properties up to an additive approximation degree determined by the granularity of the discretization; this has been shown in a recent work in another context [37], but we show that similar results hold for our model as well.[2]

Another simplification is using a deadband penalty function approach to approximate an occupant discomfort function. Note, that in this approximation an occupant just needs to convey his/her lower and upper limit temperature points. We use this deadband penalty function approximation for the simulation studies in Section IV. Prior to that, however, we analyze the the two VCG approximation approaches in Sections III-A and III-B.

A. Piecewise linear approximate discomfort function

In this mechanism, the building operator requires each occupant to declare its discomfort at certain specified points in the feasible temperature range. The discomfort can be specified through a willingness-to-pay value, specified in terms of credits the occupant is willing to pay to get the temperature from a a specific temperature to is desired set-point. A piecewise linear approximation of the discomfort function of the occupant is constructed from discrete points. Let this piecewise approximate discomfort function for an occupant i in zone j be denoted by $\tilde{D}_i(z_j)$ (see Figure 2). Based on this piecewise linear discomfort function, we solve the optimization problem in (4) and obtain cost for occupant i by solving (5). Note that the pricing structure is similar to the VCG mechanism, with the $D_i(z_j)$ in VCG mechanism being replaced by its approximation $\tilde{D}_i(z_j)$.

The true discomfort function $D_i(z_j)$ is assumed to be a convex function. As can be seen in Figure 2, $\tilde{D}_i(z_j) \geq D_i(z_j), \forall z_j$. Let, $B = \max_{z_j}(\tilde{D}_i(z_j) - D_i(z_j))$ represent the maximum deviation of the approximate discomfort function $\tilde{D}_i(z_j)$ from the actual discomfort function $D_i(z_j)$ over the

[2] The two-dimensional bid idea is motivated by the *progressive second price auction* mechanism, proposed in [38] for a single fixed resource. A recent work [39] has shown the efficiency of the equilibrium point of such two-dimensional bid-based auctions for a more general network model and elastic resources with convex costs.

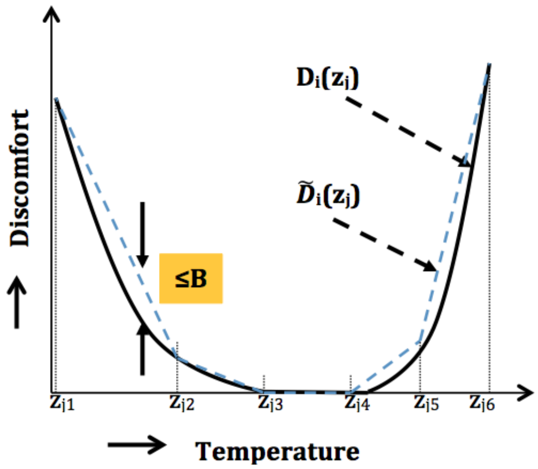

Fig. 2. Piecewise linear approximation of discomfort function for occupant i located in zone j.

temperature range of zone j. Then we have the following results.

Proposition 2: Assume that each occupant declares its discomfort value truthfully at every specified temperature set-point. Then the attained social valuation (i.e. total discomfort plus energy cost) differs from the optimum in (1) by at most NB, where N is the maximum number of occupants at any time.

Proof: Let z^* be the optimal zonal temperature vector corresponding to actual discomfort functions $D(\cdot)$, and $\tilde{z}$ be the optimal zonal temperature vector corresponding to piecewise linear approximation of the discomfort functions $\tilde{D}(\cdot)$. Then, from optimality of the VCG and piecewise linear approximation mechanisms, the following two inequalities hold for all z:

$$\sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j^*) + G(z^*) \leq \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j) + G(z), \quad (6)$$

$$\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\tilde{z}_j) + G(\tilde{z}) \leq \sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(z_j) + G(z). \quad (7)$$

We also have

$$0 \leq \tilde{D}_i(z_j) - D_i(z_j) \leq B. \quad (8)$$

For a total of N occupants distributed over m zones we can further write,

$$\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(z_j) - \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j) \leq NB. \quad (9)$$

Now, using (7), (8) and (9) we can establish that

$$\begin{aligned}\sum_{j=1}^{m}\sum_{i\in S_j} D_i(\tilde{z}_j) + G(\tilde{z}) &\leq \sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\tilde{z}_j) + G(\tilde{z})\\ &\leq \sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(z_j^*) + G(z^*)\\ &\leq NB + \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j^*) + G(z^*).\end{aligned} \quad (10)$$

Note, that the first inequality in (10) follows since $D_i(z_j) \leq \tilde{D}_i(z_j)\ \forall i, j$. The second inequality holds since $\tilde{z}$ is the optimal solution for piecewise linear approximation mechanism, and the third inequality follows from (9). Therefore, we have

$$\sum_{j=1}^{m}\sum_{i\in S_j} D_i(\tilde{z}_j) + G(\tilde{z}) - \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j^*) - G(z^*) \leq NB. \quad (11)$$

This concludes the proof. The bound NB might seem to be a large bound, but on a per-occupant basis it would be just $\frac{NB}{N} = B$.

Proposition 3: The maximum utility gained by any occupant k through untruthful declaration of its discomfort values at the specified set-points is upper bounded by B.

Proof: Let $\tilde{z}$ be the temperature vector when occupant i in zone j reports its piecewise linear discomfort function truthfully, while the other occupants $i' \in S \setminus \{i\}$ declare $\bar{D}_{i'}$ as their piecewise linear discomfort function; $\bar{D}_{i'}$ may or may not be truthfully reported (i.e., it may or may not equal $D_{i'}$). Now, assuming that the reporting by all other occupants $i' \in S\setminus\{i\}$ remain the same, let $\bar{z}$ be the temperature vector occupant i does not declare truthfully; $\bar{D}_i(z_j)$ being the untruthful piecewise linear discomfort function reported by occupant i. Total discomfort and price (dis-utility) incurred by occupant i when it bids truthfully is given as,

$$\pi_i(\tilde{z}_j) = D_i(\tilde{z}_j) + \tilde{p}_i. \quad (12)$$

Similarly, the dis-utility incurred by occupant i when it bids untruthfully can be written as:

$$\pi_i(\bar{z}_j) = D_i(\bar{z}_j) + \bar{p}_i. \quad (13)$$

The total utility gained through an untruthful bid by occupant i can thus be represented as $\Delta\pi_i = \pi_i(\tilde{z}_j) - \pi_i(\bar{z}_j)$, which can be simplified as:

$$\begin{aligned}\Delta\pi_i &= \big(D_i(\tilde{z}_j) + \tilde{p}_i\big) - \big(D_i(\bar{z}_j) + \bar{p}_i\big)\\ &= \big(D_i(\tilde{z}_j) + \tilde{D}_i(\tilde{z}_j) - \tilde{D}_i(\tilde{z}_j) + \tilde{p}_i\big)\\ &\quad -\big(D_i(\bar{z}_j) + \tilde{D}_i(\bar{z}_j) - \tilde{D}_i(\bar{z}_j) + \bar{p}_i\big)\\ &= \big(D_i(\tilde{z}_j) - \tilde{D}_i(\tilde{z}_j)\big) - \big(D_i(\bar{z}_j) - \tilde{D}_i(\bar{z}_j)\big)\\ &\quad +\big(\tilde{D}_i(\tilde{z}_j) + \tilde{p}_i - \tilde{D}_i(\bar{z}_j) - \bar{p}_i\big).\end{aligned} \quad (14)$$

Using the expression for p_i from (5), $\big(\tilde{D}_i(\tilde{z}_j) + \tilde{p}_i\big)$ can be written as:

$$\tilde{D}_i(\tilde{z}_j) + \Big(\sum_{j=1}^{m}\sum_{i'\in S_j\setminus\{i\}} \tilde{D}_{i'}(\tilde{z}_j) + G(\tilde{z})\Big) - \Big(\sum_{j=1}^{m}\sum_{i'\in S_j\setminus\{i\}} \tilde{D}_{i'}(\tilde{z}_{-i,j}) + G(\tilde{z}_{-i})\Big), \tag{15}$$

which simplifies to:

$$\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\tilde{z}_j)+G(\tilde{z})-\Big(\sum_{j=1}^{m}\sum_{i'\in S_j\setminus\{i\}} \tilde{D}_{i'}(\tilde{z}_{-i,j})+G(\tilde{z}_{-i})\Big). \tag{16}$$

Similarly, $\big(\tilde{D}_i(\bar{z}_j) + \bar{p}_i\big)$ can be written as:

$$\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\bar{z}_j)+G(\bar{z})-\Big(\sum_{j=1}^{m}\sum_{i'\in S_j\setminus\{i\}} D_{i'}(\bar{z}_{-i,j})+G(\bar{z}_{-i})\Big). \tag{17}$$

Subtracting (17) from (16) and canceling the constant we have the difference of total occupant discomfort and energy cost at $\tilde{z}$ and $\bar{z}$, $\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\tilde{z}_j)+G(\tilde{z})-\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(\bar{z}_j)+G(\bar{z})$. Since $\tilde{z}$ minimizes $\sum_{j=1}^{m}\sum_{i\in S_j} \tilde{D}_i(z_j) + G(z)$, it follows that this difference term is less than or equal to zero.

From (8), we have $\big(D_i(\tilde{z}_j) - \tilde{D}_i(\tilde{z}_j)\big) \leq 0$, and $\big(\tilde{D}_i(\bar{z}_j) - D_i(\bar{z}_j)\big) \leq B$. Combining these facts, we obtain $\Delta\pi_i \leq B$, which concludes the proof.

B. Deadband penalty discomfort function

The approach described in Section III-A requires the occupants to specify their discomfort values (through willingness-to-pay-value declarations) at certain specified temperature set-points. In practice, however, estimating such values could be difficult for many occupants. A simpler - and perhaps more practical - approach is to require occupants to only report the comfort range (by declaring the lower and upper limits of that range), and fitting a “reasonable” curve to it to construct the (approximate) discomfort function for the occupant. This leads to the deadband penalty function approximation approach, as illustrated by Figure 3. Here, the approximate discomfort function $\hat{D}_i(\cdot)$ is of the form:

$$\hat{D}_i(z_j) = \begin{cases} \alpha(z_j - z_i^U)^2 & \text{if } z_j > z_i^U, \\ 0 & \text{if } z_i^L \leq z_j \leq z_i^U, \\ \alpha(z_j - z_i^L)^2 & \text{if } z_j < z_i^L, \end{cases} \tag{18}$$

where z_i^U and z_i^L are the upper and lower limits (respectively) of the comfort range of occupant i located in zone j, z_j is the temperature of zone j, and α is a scalar (appropriately chosen). This is in accordance with the minimal input required to operate a dual mode thermostat. The choice of the discomfort (penalty) function is motivated by related studies on the PMV-PPD model [40], [41]. While individual discomfort functions may be difficult to obtain, these studies establish that the number of occupants uncomfortable at a particular temperature varies quadratically with increase/decrease in temperature.

We next solve the optimization problem in (4) and obtain cost for occupant i by solving (5) using the discomfort function $\hat{D}$. Note that the pricing structure is still similar to the VCG and piecewise linear approximation mechanism with the $D_i(z_j)$ in VCG mechanism being replaced by $\hat{D}_i(z_j)$. As Figure 3 reveals, $\hat{D}_i(z_j)$ can be either greater or less than the true discomfort $D_i(z_j)$ for the possible values of z_j. Let, $B = \max_{z_j} |\hat{D}_i(z_j) - D_i(z_j)|$ represent the maximum deviation of $\hat{D}_i(z_j)$ from the actual discomfort function $D_i(z_j)$ over the temperature range of zone j. Then we can establish the following results.

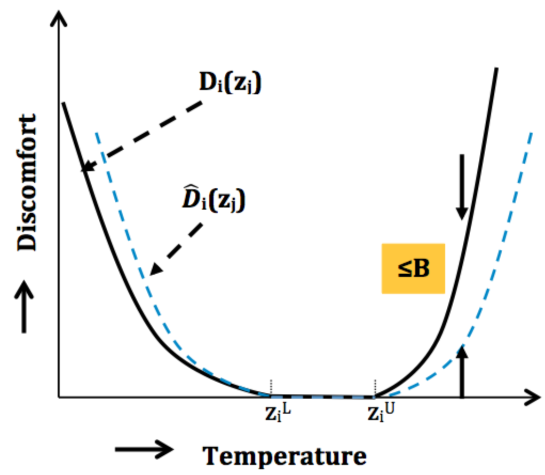

Fig. 3. Deadband penalty function approximation of discomfort for occupant i located in zone j.

Proposition 4: Assume that each occupant i declares its comfort range $[z_i^L, z_I^U]$ truthfully. Then the attained social valuation (i.e. total discomfort plus energy cost) differs from the optimum in (1) by at most $2NB$, where N is the maximum number of occupants at any time.

Proof: Let z^* be the optimal zonal temperature vector corresponding to actual discomfort functions D_i, and $\hat{z}$ be the optimal zonal temperature vector corresponding to deadband penalty discomfort functions $\hat{D}_i$, under truthful declarations of the comfort range $[z_i^L, z_I^U]$ by all occupants. Then, from the optimality condition, the following inequalities hold for all z:

$$\sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j^*) + G(z^*) \leq \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j) + G(z); \tag{19}$$

$$\sum_{j=1}^{m}\sum_{i\in S_j} \hat{D}_i(\hat{z}_j) + G(\hat{z}) \leq \sum_{j=1}^{m}\sum_{i\in S_j} \hat{D}_i(z_j) + G(z). \tag{20}$$

Now, from the definition of B, we have,

$$\Big|\hat{D}_i(z_j) - D_i(z_j)\Big| \leq B. \tag{21}$$

Summing over all N occupants distributed over m zones we obtain,

$$\sum_{j=1}^{m}\sum_{i\in S_j} \hat{D}_i(z_j) - \sum_{j=1}^{m}\sum_{i\in S_j} D_i(z_j) \leq NB. \tag{22}$$

Now, using equations (20), (21), (22) we can establish that

$$\begin{aligned}\sum_{j=1}^{m}\sum_{i\in S_j} D_i(\hat{z}_j)+G(\hat{z}) &\leq NB+\sum_{j=1}^{m}\sum_{i\in S_j}\hat{D}_i(\hat{z}_j)+G(\hat{z})\\ &\leq NB+\sum_{j=1}^{m}\sum_{i\in S_j}\hat{D}_i(z_j^*)+G(z^*)\\ &\leq 2NB+\sum_{j=1}^{m}\sum_{i\in S_j}D_i(z_j^*)+G(z^*)\end{aligned} \tag{23}$$

Note, that the first inequality in (23) follows from (21). The second inequality holds since $\hat{z}$ is the optimal solution for deadband penalty discomfort functions, and the third inequality follows from (22). Therefore, we have

$$\sum_{j=1}^{m}\sum_{i\in S_j} D_i(\hat{z}_j)+G(\hat{z})-\sum_{j=1}^{m}\sum_{i\in S_j}D_i(z_j^*)-G(z^*)\leq 2NB. \tag{24}$$

Hence, we have an upper bound for the social optimality when using deadband penalty discomfort function.

Proposition 5: The maximum utility gained by any occupant k through untruthful declaration of its comfort range $[z_i^L, z_I^U]$ is upper bounded by $2B$.

Proof: Let $\hat{z}$ be the temperature vector resulting from the use of the deadband penalty discomfort functions, under the assumption that occupant i located in zone j declares its comfort range $[z_i^L, z_I^U]$ truthfully, while the other occupants may or may not declare their comfort ranges truthfully. Let $\bar{D}_{i'}(\cdot)$ be the deadband penalty discomfort function corresponding to the comfort range declarations by other occupants $i' \in S \setminus \{i\}$ (which may or may not be truthful). With the comfort range declarations (and hence $\bar{D}_{i'}$) remaining the same, let $\bar{z}$ be the temperature vector under a setting when occupant i does not declare its comfort range truthfully; let $\bar{D}_i$ be the corresponding (untruthful) deadband penalty discomfort function of occupant i. Total discomfort and price (dis-utility) incurred by occupant i when it bids truthfully can be written as,

$$\pi_i(\hat{z}_j) = D_i(\hat{z}_j)+\hat{p}_i. \tag{25}$$

Similarly, the dis-utility incurred by occupant i when it bids untruthfully can be written as:

$$\pi_i(\bar{z}_j) = D_i(\bar{z}_j)+\bar{p}_i. \tag{26}$$

The total utility gained through an untruthful bid by occupant i can thus be represented as $\Delta\pi_i = \pi_i(\hat{z}_j) - \pi_i(\bar{z}_j)$, which can be simplified as:

$$\begin{aligned}\Delta\pi_i &= \big(D_i(\hat{z}_j)+\hat{p}_i\big)-\big(D_i(\bar{z}_j)+\bar{p}_i\big),\\ &= \big(D_i(\hat{z}_j)+\hat{D}_i(\hat{z}_j)-\hat{D}_i(\hat{z}_j)+\hat{p}_i\big)\\ &\quad -\big(D_i(\bar{z}_j)+\hat{D}_i(\bar{z}_j)-\hat{D}_i(\bar{z}_j)+\bar{p}_i\big)\\ &= \big(D_i(\hat{z}_j)-\hat{D}_i(\hat{z}_j)\big)-\big(D_i(\bar{z}_j)-\hat{D}_i(\bar{z}_j)\big)\\ &\quad +\big(\hat{D}_i(\hat{z}_j)+\hat{p}_i\big)-\big(\hat{D}_i(\bar{z}_j)+\bar{p}_i\big).\end{aligned} \tag{27}$$

Now, using the expression for p_i from (5) and following the the same line of analysis as in Proposition 3 it follows that $\big(\hat{D}_i(\hat{z}_j)+\hat{p}_i\big)-\big(\hat{D}_i(\bar{z}_j)+\bar{p}_i\big)\leq 0$. Further, we have $\big|\hat{D}_i(\hat{z}_j)-D_i(\hat{z}_j)\big|\leq B$, and $\big|\hat{D}_i(\bar{z}_j)-D_i(\bar{z}_j)\big|\leq B$. Combining these in a way similar to the proof of Proposition 3, we obtain $\Delta\pi_i \leq 2B$. This concludes the proof.

IV. Simulation Studies

We first use a simple four room model to elaborate on the pricing structure. Note that we use the deadband penalty function approximation for the discomfort function with $\alpha = 1$, as described in Section III-B, in all of our simulation study. Using this model we further establish through simulations that occupants cannot gain by declaring their discomfort function untruthfully. We also make the same observations using parameters based on our Watervliet test facility.

A. Pricing model

The mechanism design algorithm suggests a pricing feedback to the occupants that elicits true temperature preference from the occupants. This comfort price as expressed in (5), has two components to it: the energy cost (e_i) and the discomfort cost (d_i). We consider a four room model from [36] and [42] with two occupants in a room and the rest of the area being unoccupied. The four room model considered for the purpose of simulation is depicted in Figure 4.

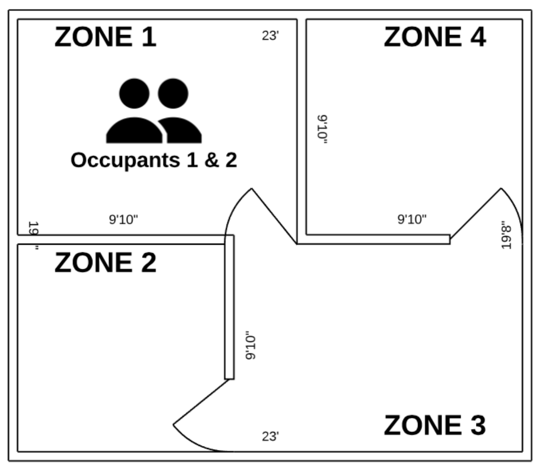

Fig. 4. Four Room Model with two occupants used for the simulation study to demonstrate the occupant comfort pricing model.

The results are presented in Table I. The pricing (p_i) consists of e_i and d_i. We use a deadband penalty function from (18) for occupant discomfort model. An ambient temperature (T_∞) of $18°C$ is considered. For the first set of results in Table I consider the temperature preference of occupant 1 as $y_1^L = 17°C$ and $y_1^U = 19°C$ and that of occupant 2 as $y_2^L = 18°C$ and $y_2^U = 20°C$. Minimizing (4) using CVX tool [43], the optimal zonal temperature comes out to be $18°C$, which is the ambient temperature and hence there is no energy cost associated with maintaining the zone at this optimal temperature. Also, the optimal temperature $18°C$ falls within the comfort range of both the occupants and hence there is no discomfort cost either.

TABLE I
PRICING BASED SIMULATION RESULTS CONDUCTED ON THE FOUR ROOM MODEL.

Occupants (both in Zone 1)	**Ambient Temperature (T_∞)**	**Occupant Temperature Preference (y_i^L, y_i^U)**	**Optimal Zone 1 Temperature (z_j^*)**	**Occupant Pricing ($p_i = e_i + d_i$)**	**Building Energy Cost $G(z^*)$**
			No Cost		
Occupant 1	$18°C$	$17°C$, $19°C$	$18°C$	0 + 0	0
Occupant 2		$18°C$, $20°C$		0 + 0	
			Discomfort Cost Only		
Occupant 1	$18°C$	$16.5°C$, $17.5°C$	$18°C$	0 + 1.5	0
Occupant 2		$18.5°C$, $19.5°C$		0 + 1.5	
			Energy Cost Only		
Occupant 1	$18°C$	$19°C$, $20°C$	$19°C$	0.55 + 0	0.55
Occupant 2		$19°C$, $20°C$		0.55 + 0	
			Both Discomfort & Energy Cost		
Occupant 1	$18°C$	$16°C$, $18°C$	$18.4°C$	-0.45 + 1.72	0.3
Occupant 2		$19°C$, $21°C$		0.3 + 1.35	

Next, we change the preferred temperature range of the occupants slightly ($y_1^L = 16.5°C$ and $y_1^U = 17.5°C$, $y_2^L = 18.5°C$ and $y_2^U = 19.5°C$) to ensure that the ambient temperature ($18°C$) is now out of their comfort range. The comfort price of each occupant is then just the discomfort that they cause to each other. Using (5) we obtain the pricing value of each occupant as shown in Table I. Note that since the optimal temperature is the ambient there is no energy cost associated. However, the occupants still incur a discomfort cost to each other, and this in turn ensures that the occupants declare their truthful comfort ranges.

Next, we set $y_1^L = 19°C$, $y_1^U = 20°C$ and $y_2^L = 19°C$, $y_2^U = 20°C$. Since, both of them have the same preference there is no discomfort cost associated. The optimal temperature turns out to be $19°C$, and the pricing per occupant indicates the energy cost they incur to the building thermal system. In this scenario the comfort price of each occupant just consists of the energy cost.

Finally, we consider $y_1^L = 16°C$, $y_1^U = 18°C$ and $y_1^L = 19°C$, $y_1^U = 21°C$. In this case the comfort price for each occupant would have both the energy and the discomfort cost associated with it. The optimal temperature for zone 1 is obtained as $18.4°C$ that deviates from both the ambient and the comfort range of each occupant, and hence incurring both energy and discomfort cost to each of the occupants. Occupant 1 has a negative energy cost as its presence brings down the zone 1 temperature to $18.4°C$, which in its absence would have been $19°C$ (due to the occupant 2's comfort range) incurring a higher energy cost to the building operator. Note that for all the scenarios we have $\sum_{j=1}^{m}\sum_{i\in S_j} p_i >= G(z^*)$, implying that the sum total credits paid by the occupants is always greater than the total energy cost. Hence, the building management does not loose money (credits) by implementing this mechanism.

B. Untruthful Declaration by Occupants

We conduct further simulation studies for the deadband penalty function based mechanism to establish that an occupant cannot benefit by declaring its comfort range untruthfully, irrespective of whether other occupants decide to be truthful or untruthful. First we consider the *No Cost* (Case 1) scenario from Table I. The results are presented in Table II.

We consider all possible combinations of two occupants in the same zone declaring their comfort ranges untruthfully by either increasing or decreasing both the limit temperatures (y_i^L) and (y_i^U) by a value of δ. We simulated over a range of values for δ, however results presented in II correspond to the value of $\delta = 1°C$. Each cell shows the optimal zonal temperature along with the pricing value for both the occupants relative to the case when both the occupants declare their discomfort functions truthfully. An up arrow $\uparrow$ would indicate an increase of temperature or price relative to the truthful case. Similarly, a down arrow $\downarrow$ would indicate an increase of temperature or price relative to the truthful case. A flat arrow $\leftrightarrow$ on the other hand indicates no change relative to the truthful case. Note that the first set of arrows correspond to occupant 1 and second set for occupant 2. A checkmark ✓corresponding to the temperature indicates that the zonal temperature falls within the comfort preference range of that particular occupant.

Note that a decrease in the pricing value for an occupant combined with a more comfortable zonal temperature would be an indicator of the occupant gaining by declaring the discomfort function untruthfully. However, as can be observed from the simulation results there are no instances of an occupant enjoying a more comfortable temperature at a lower price (such a scenario would be indicated by a combination of down arrows $\downarrow$ for both temperature and price for a particular occupant) relative to the truthful case. We ran simulations for multiple values of δ, to ascertain that similar results holds in those cases as well. i.e., that the occupants did not gain by untruthful declaration of their comfort ranges.

Next we present in Table III the results from scenario corresponding to *Both Discomfort & Energy Cost* case of Table I. In this case too, we make a similar observation, i.e., neither of the occupants can gain by declaring their comfort ranges untruthfully.

Finally, we run simulations to establish the truthfulness property in a more complex layout corresponding to our Watervliet test facility, with the occupancy as depicted in Figure 5. The results are presented in Table IV. Note, that each row corresponds to all possible combinations of untruthful

TABLE II
OPTIMAL ZONE 1 TEMPERATURE, COMFORT EXPERIENCED AND PRICES PAID BY EACH OF THE TWO OCCUPANTS UNDER UNTRUTHFUL COMFORT RANGE DECLARATION, CORRESPONDING TO THE *Both Discomfort & Energy Cost* CASE OF TABLE I.

Optimal Zone Temperature ($^{\circ}C$) and occupant pricing (p_1, p_2)	(y_1^L+0, y_1^U+0)	(y_1^L-1, y_1^U+0)	(y_1^L+1, y_1^U+0)	(y_1^L+0, y_1^U-1)	(y_1^L-1, y_1^U-1)	(y_1^L+1, y_1^U-1)	(y_1^L+0, y_1^U+1)	(y_1^L-1, y_1^U+1)	(y_1^L+1, y_1^U+1)
(y_2^L+0, y_2^U+0)	18.0 * *	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i * *	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L-1, y_2^U+0)	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L+1, y_2^U+0)	18.9 ✓✓	18.9 ✓✓	18.9 ✓✓	18.4 ✓✓	18.2 ✓✓	18.5 ✓✓	18.9 ✓✓	18.9 ✓✓	18.9 ✓✓
	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↔ ↑
(y_2^L+0, y_2^U-1)	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L-1, y_2^U-1)	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L+1, y_2^U-1)	18.6 ✓✓	18.6 ✓✓	18.6 ✓✓	18.2 ✓✓	18.0 ✓✓	18.4 ✓✓	18.6 ✓✓	18.6 ✓✓	18.6 ✓✓
	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↔ ↑
(y_2^L+0, y_2^U+1)	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L-1, y_2^U+1)	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓	18.0 ✓✓
	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔	p_i ↔ ↔
(y_2^L+1, y_2^U+1)	19.0 ✓✓	19.0 ✓✓	19.0 ✓✓	18.5✓✓	18.4✓✓	18.7✓✓	19.0 ✓✓	19.0 ✓✓	19.0 ✓✓
	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↔ ↑	p_i ↔ ↑	p_i ↔ ↑

Note that arrows indicate the direction of change in price and discomfort relative to the truthful declaration scenario. Up arrow ↑ refers to increase in discomfort (price), down arrow ↓ indicates decrease in discomfort (price) and a flat arrow ↔ indicates no change. Checkmark ✓(neutral) indicates zonal temperature within the comfort range of the occupant. First row and first column pricing is with respect to the first cell (*) values. For the inner cells, p_1 value is relative to the first cell value in that row and p_2 value is relative to the first cell value in that column.

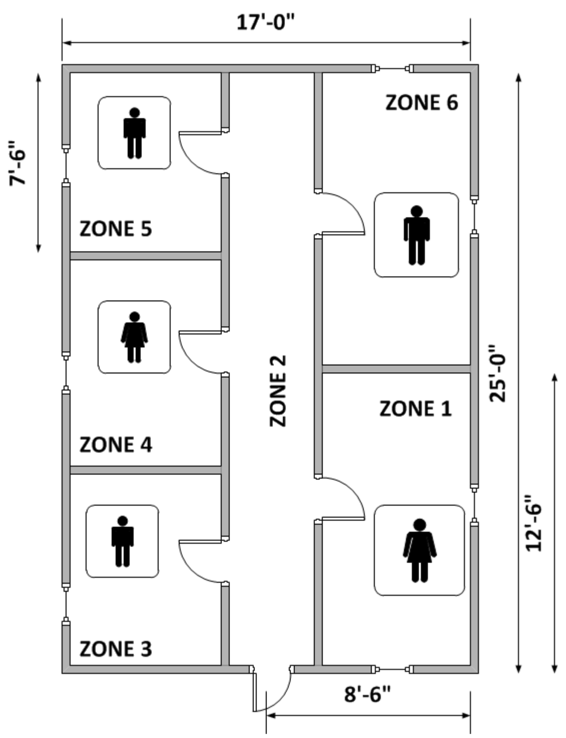

Fig. 5. Schematic layout of the Watervliet test bed with simulated occupancy of each zone depicted by the stick figures used for the truthful simulation study.

declaration of their discomfort function by occupant i. Anytime an occupant can reduce the price and be more comfortable by untruthful declaration, would indicate an incentive for untruthful declaration. Our simulation studies therefore reveal that an occupant does not have an incentive to declare its comfort range untruthfully.

V. CONCLUSION

In this paper, we present an incentive mechanism designed to extract truthful temperature preference (comfort range/function) information from building occupants through the use of "comfort prices". The mechanism utilizes occupant comfort feedback to compute the optimal zonal temperatures, taking into account the thermal correlations between various zones in the building. We further discuss the complexity and practicality issues associated with obtaining the necessary occupant feedback, and propose a couple of approaches towards estimating occupant discomfort functions through simple, low-complexity information input from the occupants. We provide insights to the pricing structure needed for incentive compatibility through a few numerical case studies. Extensive stimulation study based on a 4-zone building model and our 6-zone Watervliet facility parameters establish that occupants cannot benefit by declaring their thermal preferences (comfort ranges) untruthfully. A full scale experimental implementation of our proposed approach, that will utilize occupant feedback in real time, is planned for future investigation.

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TABLE III

OPTIMAL ZONE 1 TEMPERATURE, AND COMFORT EXPERIENCED AND PRICES PAID BY EACH OF THE TWO OCCUPANTS, UNDER UNTRUTHFUL COMFORT RANGE DECLARATION, CORRESPONDING TO THE *No Cost* CASE OF TABLE I

Zone Temp.($^\circ C$) and occupant pricing (p_i)	(y_1^L+0, y_1^U+0)	(y_1^L-1, y_1^U+0)	(y_1^L+1, y_1^U+0)	(y_1^L+0, y_1^U-1)	(y_1^L-1, y_1^U-1)	(y_1^L+1, y_1^U-1)	(y_1^L+0, y_1^U+1)	(y_1^L-1, y_1^U+1)	(y_1^L+1, y_1^U+1)
(y_2^L+0, y_2^U+0)	18.4 * *	18.2 ↓ ↑	18.5 ↑ ↓	18.2 ↓ ↑	18.0 ✓ ↑	18.4 ↔ ↔	19.0 ↑ ✓	19.0 ↑ ✓	19.0 ↑ ✓
	p_i * *	p_i ↑ ↑	p_i ↓ ↓	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↓	p_i ↓ ↓	p_i ↓ ↓
(y_2^L-1, y_2^U+0)	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	17.8 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑
	p_i ↓ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↔ ↓
(y_2^L+1, y_2^U+0)	18.5 ↑ ↓	18.4 ↓ ↓	18.7 ↑ ↓	18.4 ↓ ↓	18.2 ↓ ↓	18.5 ↔ ↓	19.0 ↑ ✓	19.0 ↑ ✓	19.0 ↑ ✓
	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↓ ↑	p_i ↓ ↑
(y_2^L+0, y_2^U-1)	18.2 ↓ ↑	18.0 ✓ ↑	18.4 ↑ ↑	18.0 ✓ ↑	17.8 ✓ ↑	18.2 ↔ ↑	18.9 ↑ ↑	18.9 ↑ ↑	18.9 ↑ ↑
	p_i ↓ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↓ ↓	p_i ↓ ↓	p_i ↓ ↓
(y_2^L-1, y_2^U-1)	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	17.8 ✓ ↑	17.6 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑
	p_i ↓ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↔ ↓
(y_2^L+1, y_2^U-1)	18.4 ↔ ↔	18.2 ↓ ↔	18.5 ↑ ↔	18.2 ↓ ↔	18.0 ✓ ↔	18.4 ↔ ↔	19.0 ↑ ↔	19.0 ↑ ↔	19.0 ↑ ↔
	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↓ ↑	p_i ↓ ↑
(y_2^L+0, y_2^U+1)	18.5 ↑ ↓	18.4 ↓ ↓	18.7 ↑ ↓	18.4 ↓ ↓	18.2 ↓ ↓	18.5 ↔ ↓	19.0 ↑ ✓	19.0 ↑ ✓	19.0 ↑ ✓
	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↔	p_i ↓ ↔	p_i ↓ ↔
(y_2^L-1, y_2^U+1)	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↔	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑	18.0 ✓ ↑
	p_i ↓ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↑ ↓	p_i ↔ ↓	p_i ↔ ↓	p_i ↔ ↓
(y_2^L+1, y_2^U+1)	18.7 ↑ ↓	18.5 ↓ ↓	18.9 ↑ ↓	18.5 ↓ ↓	18.4 ↓ ↓	18.7 ↔ ↓	19.0 ↑ ✓	19.0 ↑ ✓	19.1 ↑ ↑
	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↑ ↑	p_i ↓ ↑	p_i ↓ ↑	p_i ↓ ↑

Note that arrows indicate the direction of change in price and discomfort relative to the truthful declaration scenario corresponding to *Both Discomfort & Energy Cost* case of I. Up arrow ↑ refers to increase in discomfort (price), down arrow ↓ indicates decrease in discomfort (price) and a flat arrow ↔ indicates no change. Checkmark ✓(neutral) indicates zonal temperature within the comfort range of the occupant. First row and first column pricing is with respect to the first cell (*) values. For the inner cells, p_1 value is relative to the first cell value in that row and p_2 value is relative to the first cell value in that column.

TABLE IV

VARIATION FOR WATERVLIET SET UP WITH ONE OCCUPANT BEING UNTRUTHFUL AT A TIME.

Zone Temp.($^\circ C$) and occupant pricing (p_i)	(y_i^L+0, y_i^U+0)	(y_i^L-1, y_i^U+0)	(y_i^L+1, y_i^U+0)	(y_i^L+0, y_i^U-1)	(y_i^L-1, y_i^U-1)	(y_i^L+1, y_i^U-1)	(y_i^L+0, y_i^U+1)	(y_i^L-1, y_i^U+1)	(y_i^L+1, y_i^U+1)
$(i=1)$	18.6 *	18.6 ✓	18.6 ✓	18.0 ✓	18.0 ✓	18.1 ✓	18.6 ✓	18.6 ✓	18.6 ✓
	p_i *	p_i ↔	p_i ↔	p_i ↑	p_i ↑	p_i ↑	p_i ↔	p_i ↔	p_i ↔
$(i=2)$	18.3 *	18.1 ✓	19.0 ✓	18.3 ✓	18.1 ✓	18.9 ✓	18.3 ✓	18.1 ✓	19.0 ✓
	p_i *	p_i ↔	p_i ↑	p_i ↑	p_i ↔	p_i ↑	p_i ↔	p_i ↔	p_i ↑
$(i=3)$	19.0 *	18.2 ↑	19.7 ✓	19.0 ✓	18.2 ↑	19.4 ✓	19.0 ✓	18.2 ↑	19.0 ✓
	p_i *	p_i ↓	p_i ↑	p_i ↔	p_i ↓	p_i ↑	p_i ↑	p_i ↓	p_i ↑
$(i=4)$	20.0 *	19.0 ↑	20.5 ✓	19.8 ↑	19.0 ↑	20.1 ✓	20.0 ✓	19.0 ↑	20.8 ✓
	p_i *	p_i ↓	p_i ↑	p_i ↓	p_i ↓	p_i ↑	p_i ↑	p_i ↓	p_i ↑
$(i=5)$	19.9 *	19.7 ↑	20.2 ↓	19.7 ↑	19.4 ↑	19.9 ↔	20.2 ↓	19.9 ↔	20.4 ↓
	p_i *	p_i ↓	p_i ↑	p_i ↓	p_i ↓	p_i ↑	p_i ↑	p_i ↓	p_i ↑

Note that arrows indicate the direction of change in price and discomfort relative to the truthful declaration scenario. Up arrow ↑ refers to increase in discomfort (price), down arrow ↓ indicates decrease in discomfort (price) and a flat arrow ↔ indicates no change. Checkmark ✓(neutral) indicates zonal temperature within the comfort range of the occupant. Each row corresponds to all possible combinations of untruthful declaration by occupant i.

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