Port Capacity Leasing Games at Internet Exchange Points

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Abstract. Internet Service Providers (ISPs) lease ports at a public switch in an Internet Exchange Point (IXP) to exchange traffic efficiently with other ISPs present at the IXP. The price paid to lease a port depends on the port capacity, which also impacts the Quality of Service (QoS) experienced by the ISP's traffic exchanged through the IXP switch. In this paper, we analyze the leasing of port capacities at an IXP as a non-cooperative game between the ISPs, and analyze the efficiency at equilibrium as compared to the social optimum. We show that when the IXP switch capacity is not changed in response to the port capacities purchased, there is dominant strategy for each ISP that attains a Price of Anarchy (PoA) of at most 2. If the IXP switch capacity is varied to "match" the aggregate port capacity leased by the ISPs, then bad equilibria can exist. However, under certain reasonable assumptions, the PoA is still guaranteed to be within 2. Simulation studies demonstrate the effect of the per-unit leasing price and switch delay functions on the equilibrium performance; in all scenarios simulated, the social cost at equilibrium was found to be very close to the optimum.

Keywords: Internet Exchange Point · Internet Service Provider · Public Peering · Port Capacity Leasing.

1 Introduction

Internet eXchange Points (IXPs) are data centers with network switches through which Internet Service Providers (ISPs) exchange traffic, mostly through peering relationships [1, 8]. In spite of falling transit costs, peering between ISPs has been increasing, resulting in flattening of the Internet [1, 7, 12]. It is estimated that almost 80% of the IP addresses in the world can be reached via public peering, and 20% of all the traffic go through IXPs [4]. Exchange at IXPs typically improves traffic Quality of Service (QoS) due to lower delays and losses associated with shorter Internet paths, and offers reachability to a large number of other ISPs that are present at that IXP.

For peering at the public switch at the IXP, ISPs typically pay a price that depends on the port size (i.e., capacity of the port leased by the ISP) [13]. The larger the port capacity leased, the larger the price paid; however, a larger port capacity also allows faster transfer of traffic. The QoS experienced by the traffic sent by an ISP, however, also depends on the port capacities purchased by other ISPs. This results in the port-capacity leasing decisions of the ISPs being dependent on each other, and whether or not the IXP switch speed (capacity) is varied based on the port capacities purchased by the ISPs.

In this paper, we analyze the port capacity leasing problem for ISPs (at a public switch in an IXP) as a non-cooperative game, where each ISP is making the leasing decision selfishly so as to minimize its own cost in sending the traffic through the IXP. This cost comprises of the port capacity leasing price to be paid to the IXP, as well as the congestion costs at each of the queuing points inside the IXP. We analyze two scenarios: (a) the switch capacity is fixed, i.e., remains unaffected by the port capacities leased by the ISPs; (b) the switch capacity is upgraded/downgraded so as to "match" the aggregate port capacity leased by the ISPs. For each scenario, we evaluate the social cost at equilibrium as compared to the optimum social cost, represented as the Price of Anarchy (PoA) or the Price of Stability (PoS). For scenario (a), we show that there exists a dominant strategy for each ISP, which also attains a PoA of 2. For scenario (b), bad equilibria exist; however, under the reasonable assumption that the switch capacity is varied so that the switch is not the congestion bottleneck in the system, there exist good equilibria and PoA is at most 2. Simulations over a range of parameters, using traffic data for US IXPs as estimated based on their actual locations, show the existence of at least one very good equilibria (PoS very close to 1). Alongside, the values of PoS showed a increasing trend with traffic symmetry and the number of ISPs peering at the IXP.

Our work is related to some of the prior work on the effect of IXP pricing on peering relationships [3, 2]; however the problem investigated in this paper and its game theoretic model is fundamentally different from that analyzed in these earlier work. In [3, 2] the strategy of each ISP involves determining *how much traffic to send through the IXP versus sending it outside of the IXP* (through transit providers). Further, the price charged to each ISP by the IXP depends on the amount of traffic sent by the ISP through the IXP. In contrast, the model in our paper considers determining *how much port capacity an ISP should lease, given a certain amount of traffic to be sent through the IXP*. Given that prices paid by an ISP at an IXP primarily depend on the port capacity [9, 15], this model reflects current practice. The tools used in the game theoretic analysis, and the nature of the results derived, also differ quite significantly. In [3, 2] it is observed that the choice of the traffic-dependent pricing policy has a significant effect on how much of the IXP traffic flows through the IXP at equilibrium, thereby impacting the PoA values significantly. In this paper we observe that when the traffic (to be exchanged via IXP) is already determined, the worst case social cost values at equilibrium is relatively unimpacted by the port leasing costs, i.e., good PoA and PoS values (close to 1) exist irrespective of the port leasing costs. Finally, while [3, 2] draws upon a long line of work on network formation games (e.g., [11, 16]) and utilizes the notion of pairwise equilibrium [10, 6], our model is not a network formation game, and studies Nash equilibria in the context of the port capacity leasing problem.

2 System Model and Properties

In this section, we detail our game theoretic model of the port capacity leasing problem, and argue the existence of the Nash equilibrium of the game.

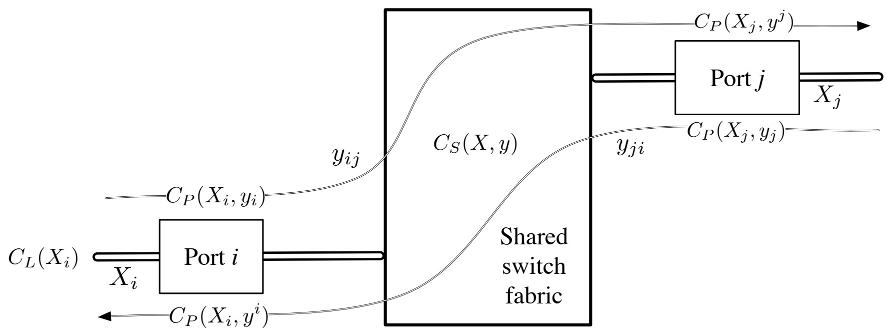

Fig. 1. Schematic of the System Model showing different costs faced by agent (ISP) i.

Consider a public (shared) switch at an IXP, which is utilized by N ISPs to exchange traffic between themselves. The amount of traffic that any ISP (say i) has for the other ISP (say j) is directional and given (fixed), and denoted by y_{ij}. We use $y_i = \sum_j y_{ij}$ ($y^i = \sum_j y_{ji}$) to denote the total traffic sent from (to) ISP i to (from) all other ISPs. Let X_i denote the port capacity leased by any ISP i (ISP i's strategy), which provides a connection bandwidth of X_i (in both directions) between ISP i's server equipment and the shared switch fabric. Note that for stability, $X_i \geq \max(y_i, y^i), \forall i$. As illustrated in Figure 1, the traffic from ISP i to ISP j, y_{ij}, can face congestion at three potential bottlenecks (queues on its path in the IXP) that are impacted by the port capacity choices of the ISPs (i, j and potentially other ISPs): (i) A congestion cost of $C_P(X_i, y_i)$ at the port i which has an aggregate traffic of y_i and served by a link of capacity X_i; (ii) A congestion cost of $C_P(X_j, y^j)$ at port j which has an aggregate traffic of y^j and served by a link of capacity X_j; (iii) A congestion cost of $C_S(X_i, \cdots, y_i, y^i, \cdots, \forall i)$ at the shared switch. Note that $(y_i, y^i, \forall i)$ represents the entire traffic vector that shares the public switch, whose fabric capacity may or may not depend on the vector of port capacities leased, $X = (X_i, \forall i)$. Accordingly, in our analysis in Section 3 we consider both scenarios: when $C_S(\cdot)$ is only a function of the total traffic vector $y = (y_i, y^i, \forall i)$, and when it is a function of the leased capacity vector X as well. Finally, ISP i also faces a port leasing cost given by $C_L(X_i)$.

Let X_{-i} denote the set of strategies of all ISPs except i. Let $[0, M]$ represent the range over which any port capacity can be chosen. Then in the port capacity leasing game, given X_{-i}, each ISP i will choose X_i in the range $[\max(y_i, y^i), M]$ so as to selfishly minimize its cost function $C_i(X_i; X_{-i})$ (continuous in X_i), given below.

$$
\begin{aligned}
C_i(X_i; X_{-i}) = & \sum_j \left[y_{ij} \left(C_P(X_i, y_i) + C_P(X_j, y^j) \right) + y_{ji} \left(C_P(X_j, y_j) + C_P(X_i, y^i) \right) \right] \\
& + C_L(X_i) + \sum_j \left(y_{ij} + y_{ji} \right) C_S(X, y) \\
= & \; y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) + \sum_j \left(y_{ij} C_P(X_j, y^j) + y_{ji} C_P(X_j, y_j) \right) \\
& + C_L(X_i) + \left(y_i + y^i \right) C_S(X, y). \qquad (1)
\end{aligned}
$$

The notion of equilibrium for this game will be of Nash Equilibrium, where no single player (ISP) can improve its cost function (as defined by Eq. 1) by unilaterally changing its own port capacity. We make the following reasonable assumption on the congestion cost functions, which will be used in our analyses throughout the paper.

Assumption 1. *$C_i(X_i; X_{-i})$ has a unique minimum in X_i for any given X_{-i}.*

The functions $C_P(X_i, y_i)$ and $C_P(X_i, y^i)$ are related to the congestion delay at the ports, and can be expected to be decreasing and strictly convex in X_i; for example, the M/M/1 delay function is decreasing and strictly convex in the server capacity. In practice, $C_L(X_i)$ is usually defined for a few discrete port capacity choices [9, 15], but can be reasonably approximated as an increasing and convex function of X_i. The function $C_S(X, y)$ is related to the congestion delay at the switch fabric, and its exact nature will depend on how the switch fabric capacity is varied as a function of the port capacity choices. However, as it is a congestion delay function, and the switch fabric capacity is not likely to decrease with increase in port capacity, it is reasonable to assume that it will be convex function of X. Therefore, $C_i(X_i, X_{-i})$ can be expected to be strictly convex in X_i, thereby satisfying Assumption 1. We now establish the existence of equilibria for this game, denoted by X_{eq}.

Proposition 1. *Under Assumption 1, an equilibrium for the port capacity leasing game always exists.*

Proof. The Nash equilibrium of the port capacity leasing game is characterized by the following fixed point equations: $X_i = f_i(X_{-i}),\ \forall i$, where $f_i(X_{-i}) = \arg\min_{X_i \in [0,M]} C_i(X_i; X_{-i})$. Note that Assumption 1 implies a unique best response strategy, i.e., $\arg\min_{X_i \in [0,M]} C_i(X_i; X_{-i})$ produces a unique value. This implies the function f_i is well defined as well as continuous. Therefore, the conditions of the Brouwer's fixed point theorem are satisfied, implying that there exists a fixed point of the set of equations $X_i = f_i(X_{-i}),\ \forall i$. □

Finally, we define the social optimum (OPT), denoted by X^*, as a port capacity leasing solution that minimizes total cost of all the ISPs, expressed as

$$\begin{aligned}
C(X) &= \sum_i C_i(X_i; X_{-i}) \\
&= \sum_i \left[y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) + \sum_j \left(y_{ij} C_P(X_j, y^j) + y_{ji} C_P(X_j, y_j) \right) \right] \\
&\quad + \sum_i \left[C_L(X_i) + \left(y_i + y^i \right) C_S(X, y) \right] \\
&= \sum_i \left[2 \left(y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) \right) + C_L(X_i) + (y_i + y^i) C_S(X, y) \right]. \quad (2)
\end{aligned}$$

Multiple Equilibria: In many cases of practical interest, the port leasing game has unique equilibrium (as will be discussed in Section 3.1). However there can be cases where multiple equilibria exist; one such example involving 2-ISPs are as follows. Let us assume that the 2 ISPs (say i and j) have $y_{ij} = y_{ji} = 1$ amount of traffic to exchange. Also, $C_P(X_i) = k$ (constant), $C_S(X) = \max(10 - (\sum_i X_i - y), 0)$, and $C_L(X_i) = \log(X_i)$. Then with $C_i(X_i, X_{-i}) = 4k + 2(12 - \sum_i X_i) + \log(X_i), \forall i$, all the combinations of $X_i + X_j = 12$, with $X_i \geq 1$ and $X_j \geq 1$ is an equilibrium solution. For example, $X_i = 8, X_j = 4$ is one such equilibrium, where none of the ISPs can improve their cost by unilaterally changing their leased port capacity.

3 Price of Anarchy Analysis

The Price-of-Anarchy (PoA) (Price-of-Stability (PoS)) is defined as the ratio of the cost at worst (best) equilibrium to the cost at OPT. We analyze two scenarios, based on whether the switch congestion function C_S remains fixed or varies as a function of the leased port capacity vector, X. The first scenario, in which C_S is independent of X, the switch capacity is given and not varied based on leased port capacity. The second scenario models the case where the switch fabric capacity is provisioned based on leased port capacity. The two scenarios require different analytical treatments, and therefore discussed separately.

3.1 Fixed switch capacity

When $C_S(X, y)$ is independent of the strategy vector X (but may depend on the traffic vector y), the port capacity leasing game becomes a potential game, and we obtain the following properties of the equilibrium solution.

Theorem 1. *Under Assumption 1, if $C_S(X, y)$ is independent of X, then:*

(i) Each ISP has a dominant strategy, and the port capacity leasing game has a unique equilibrium.
(ii) $PoA = PoS \leq 2$.

Proof. i) When $C_S(X, y)$ is independent of X, any ISP (say i) can only change $y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) + C_L(X_i)$ part of its cost (Eq. 1) by changing X_i. Hence from Assumption 1, any ISP will always have a dominant strategy (choosing the X_i resulting in minimum cost), and all the ISPs choosing that specific port capacities will result in a unique equilibrium.

ii) Let us define a function $\Phi(X)$ as follows:

$$\Phi(X) = \sum_i \left[\left(y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) \right) + C_L(X_i) + (y_i + y^i) \cdot C_S \right], \quad (3)$$

where X denotes the set of strategies (port capacity choices) of all ISPs. Also, let $\Phi(X_i^{'}, X_{-i})$ express the function for the case where ISP i chooses the strategy $X_i^{'}$, while the other ISPs choose X_{-i}. Then it is straightforward to show the following:

$$\Phi(X_i, X_{-i}) - \Phi(X_i^{'}, X_{-i}) = C_i(X_i, X_{-i}) - C_i(X_i^{'}, X_{-i}). \quad (4)$$

Hence, according to the definition of potential game [16], we have a potential game with a potential function $\Phi(X)$. Also, from potential game analysis [16], if the total cost $C(X) \leq a \cdot \Phi(X)$, for some scalar a, then that game has a $PoS \leq a$. For our current case, $C(X) \leq 2\Phi(X)$, and hence we get $PoS \leq 2$ for this game. Lastly, from part *i)* of this proof we know that there is a unique equilibrium and hence we have $PoA = PoS \leq 2$. □

3.2 Variable switch capacity

The port capacity leasing game in general does not have a potential function when C_S is a function of the port capacities purchased by the ISPs. As shown later in this section, under certain reasonable additional assumptions the PoA can be shown to be small. However, for the general problem (without any additional restrictions) there can be examples which results in very bad PoA; one such example is discussed next.

Bad PoA example: For this example we assume that C_P is zero; $C_S = 2N - \sum_i X_i$, where N is the total number of ISPs and X_i the port capacities of those ISPs; and $C_L(X_i) = (2 + \epsilon)X_i$, where ϵ is a small positive value. Also, let's assume that $y_i = y^i = 1, \forall i$. If all the ISPs except i decide to buy unit port capacity, i.e., $X_j = 1, \forall j \neq i$, then from Eq. 1 we get:

$$C_i(X) = (2 + \epsilon)X_i + 2(2N - (N - 1) - X_i) = 2N + 2 + \epsilon X_i. \quad (5)$$

So, from the perspective of ISP i, it would want to buy the least possible port capacity, which is $X_i = \max(y_i, y^i)$. Hence, ISP i will end up choosing $X_i = 1$. Since, all the other ISPs have already bought port of capacity 1, they are not going to change their purchased port capacities as well (they will face the same cost as Eq. 5), resulting in equilibrium. Hence, $X = \{1, 1, ..., 1\}$ is an equilibrium solution, which results in a total cost of $C = N(2N + 2 + \epsilon)$.

On the other hand, if all of the ISPs were to buy a port capacity of 2, then the total cost would have been $N\left(2(2+\epsilon)+2(2N-2N)\right) = N(4+2\epsilon)$, which is also the OPT solution. Then from the definition of PoA we have, $PoA \geq \frac{N(2N+2+\epsilon)}{N(4+2\epsilon)} = \frac{2N+2+\epsilon}{4+2\epsilon}$, which for large value of N approaches $N/2$.

Bounding *PoA* with Smoothness: For the scenario when C_S is dependent on X_i values, we use the smoothness property [14] to bound the PoA. According to the smoothness analysis if we can show that,

$$\sum_i \left[\lambda \cdot C_i(X^*) + \mu \cdot C_i(X) - C_i(X_i^*, X_{-i})\right] \geq 0, \tag{6}$$

then $PoA \leq \frac{\lambda}{1-\mu}$ holds for all X when X^* is the OPT.

Theorem 2. *The port capacity leasing game has a $PoA \leq 2$ if $y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) \geq (y_i + y^i) C_S(X, y)$ holds for any value of $X, \forall i$.*

Proof. We prove the smoothness property (Eq. 6) for $\lambda = 1$ and $\mu = \frac{1}{2}$. Hence, the left hand side of Eq. 6 becomes,

$$\begin{aligned}
&\sum_i \left[C_i(X^*) + \frac{1}{2} \cdot C_i(X) - C_i(X^*, X_{-i})\right] \\
= &\sum_i y_i \left[C_P(X_i^*, y_i) + C_S(X^*, y) + \frac{1}{2} C_S(X, y) - C_S(X_i^*, X_{-i}, y)\right] \\
+ &\sum_i y^i \left[C_P(X_i^*, y^i) + C_S(X^*, y) + \frac{1}{2} C_S(X, y) - C_S(X_i^*, X_{-i}, y)\right] \\
+ &\frac{1}{2} \sum_i C_L(X_i).
\end{aligned} \tag{7}$$

Now, when the assumption of Theorem 2 holds, we have $y_i C_P(X_i^*, y_i) + y^i C_P(X_i^*, y^i) \geq (y_i + y^i) C_S(X_i^*, X_{-i}, y)$ $\forall i$. Then from Eq. 7 we always get a non-negative value and from smoothness argument we get, $PoA \leq \frac{1}{1-\frac{1}{2}} = 2$. $\square$

Corollary 1. *If both C_P and C_S represent M/M/1 delay functions, and the switch has a capacity of $\sum_i X_i$, then $PoA \leq 2$.*

Proof. This is actually a special case of Theorem 2. Since C_P represent M/M/1 delay function, $C_P(X_i, y_i) = \frac{1}{X_i - y_i}$, and $C_P(X_i, y^i) = \frac{1}{X_i - y^i}, \forall i$. Also, since the switch capacity is $\sum_i X_i$, $C_S(X, y) = \frac{1}{\sum_i \left[X_i - \frac{1}{2}(y_i + y^i)\right]}$. The multiplier of $\frac{1}{2}$ in front of $y_i + y^i$ is to avoid double counting, since y_{ij} is included in both y_i and

y^j. To prove the condition of Theorem 2 we observe that,

$$
\begin{aligned}
& y_i \left[C_P(X_i^*, y_i) - C_S(X_i^*, X_{-i}, y)\right] + y^i \left[C_P(X_i^*, y^i) - C_S(X_i^*, X_{-i}, y)\right] \\
&= \left[\left(\frac{y_i}{X_i^* - y_i} + \frac{y^i}{X_i^* - y^i}\right) - \frac{y_i + y^i}{\sum_i (X_i^* + X_j + X_k + ...) - \frac{1}{2}(y_i + y^i)}\right] \\
&\geq \left[\left(\frac{y_i}{X_i^* - y_i} + \frac{y^i}{X_i^* - y^i}\right) - \frac{y_i + y^i}{\left(X_i^* - \frac{1}{2}(y_i + y^i)\right)}\right] \geq 0; \qquad (8)
\end{aligned}
$$

where the first inequality is true because $X_i \geq \max(y_i, y^i), \forall i$; and $\frac{1}{a+b} < \frac{1}{a}$, for any $a, b > 0$. Then the last inequality can be proved by simple algebraic manipulation. Hence, the condition of Theorem 2 is met and we get a $PoA \leq 2$. □

Finally, we argue why the assumption $y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i) \geq (y_i + y^i) C_S(X, y)$ is expected to hold even for fairly general C_P and C_S functions (beyond M/M/1 delay functions), as long as the switch is not the bottleneck in the traffic path through the IXP. Let $\bar{X} = \sum_i X_i$ denote the aggregate port capacity, and $\bar{y} = \frac{1}{2}\sum_i (y_i + y^i)$ denote the aggregate traffic through the IXP. Let us model the switch as a server with aggregate capacity $\bar{X}$ and aggregate traffic $\bar{y}$, and with slight abuse in notation we denote the switch congestion (delay) cost as $C_S(\bar{X}, \bar{y})$. For any i, we have $\bar{X} - \bar{y} = X_i + \sum_{j \neq i} X_j - y_i - \sum_{j \neq i} y_j \geq X_i - y_i$, since $X_j \geq y_j \; \forall j$. As in typical queuing systems, the average delay is inversely proportional to the idle time $\bar{X} - \bar{y}$. Therefore, for any i, we can write, $C_S(\bar{X}, \bar{y}) \leq C_S(X_i, y_i)$. We further assume that the switch is not the bottleneck in the traffic path in terms of congestion cost, i.e., for any scalar values $\tilde{X}, \tilde{y}$, with $\tilde{X} \geq \tilde{y}$, we have $C_P(\tilde{X}, \tilde{y}) \geq C_S(\tilde{X}, \tilde{y})$. Taking $\tilde{X} = X_i, \tilde{y} = y_i$, we get $C_S(X_i, y_i) \leq C_P(X_i, y_i)$. Therefore, we have, $C_S(\bar{X}, \bar{y}) \leq C_S(X_i, y_i) \leq C_P(X_i, y_i)$. Similarly, we can argue, $C_S(\bar{X}, \bar{y}) \leq C_S(X_i, y^i) \leq C_P(X_i, y^i)$. Combining, we get $(y_i + y^i) C_S(\bar{X}, \bar{y}) \leq y_i C_P(X_i, y_i) + y^i C_P(X_i, y^i)$.

4 Simulations

4.1 Simulation Setup

Simulation of the port capacity leasing game requires: a) determining a viable traffic matrix (values of $y_{ij}, \forall (i, j)$ pair), and b) deciding the cost functions C_P, C_S, C_L such that they reflect real world behavior.

Traffic Matrix: To determine the traffic matrix, we individually calculated the y_{ij} values using a modified gravity model similar to the one used in [17]. We utilized PeeringDB to obtain information about IXP locations, the number of ISPs publicly peering at an IXP, and the port capacity purchased by the ISPs. On the other hand, utilized the CAIDA database [5] for router information (i.e., location and frequency at that location) of the ISPs. Using these information we calculated the value of $y_i = \sum_{A,B} y_{i_A j_B}$, where $y_{i_A j_B}$ is the traffic that ISP

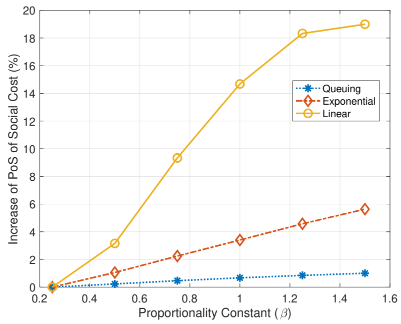

Fig. 2. Average PoS vs β for different switch congestion cost (delay) functions (C_S).

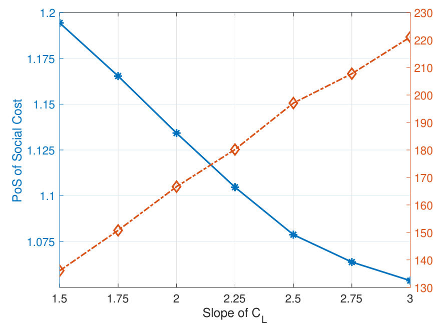

Fig. 3. Average PoS and SC vs per-unit Leasing Cost.

i needs to send to ISP j from location A to location B. Using the gravitational law we have, $y_{i_A j_B} = k\frac{R_{i_A} \cdot R_{j_B}}{d^2_{AB}}$, where R_{i_A} is the number of Routers that ISP i has in area A, R_{j_B} is the number of Routers that ISP j has in area B, d_{AB} is the distance between area A and B, and K is a constant. The generated traffic matrix showed non-homogeneous traffic with exponentially decaying distribution, which supports the asymmetry usually observed in internet traffic [12].

4.2 Results and observation

All of the simulation results discussed in this section are expressed in terms of PoS, calculated as the ratio of cost at any equilibrium to the OPT. Also, all the PoS results are for 4-ISP cases unless otherwise stated.

Shared Switch Delay: We model the switch cost C_S as the average packet delay through the switch and consider the following delay functions (C_S as a function of $\bar{X} = \sum_i X_i$ and $\bar{y} = \frac{1}{2}\sum_i (y_i + y^i)$): i) M/M/1 queuing delay function ($C_S = \frac{\beta}{\bar{X} - \bar{y}}$), ii) exponentially decaying delay function ($C_S = \beta \cdot \exp(-(\bar{X} - \bar{y}))$), and iii) linearly decaying delay function ($C_S = \beta \cdot (k - (\bar{X} - \bar{y}))$), where β is a proportionality factor and k is some constant. The effect on PoS for these three functions with the change of β is shown in Figure 2. We observe that if we scale up C_S (increase β), then PoS generally becomes larger. The outcome is in agreement to the main idea of Theorem 2, where we argued that if C_S is small compared to the C_P, then PoA will be small.

Port Leasing Cost: The effect of increasing the per unit leasing cost (C_L) is depicted in Figure 3. We have assumed C_L to be linear in X_i and have increased the slope ($\frac{C_L(X_i)}{X_i}$) to characterize the increase of per unit port leasing cost. As we can see, although increasing C_L decreases the PoS value of the system, it results in an increase in Social Cost (as defined in Eq. 2). Hence, although increasing the value of C_L will proportionally increase the revenue of IXP, it will also increase the social cost, eventually forcing the ISPs to look for alternate

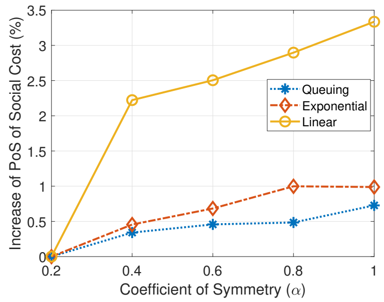

Fig. 4. Average PoS vs α for different switch congestion cost (delay) functions (C_S).

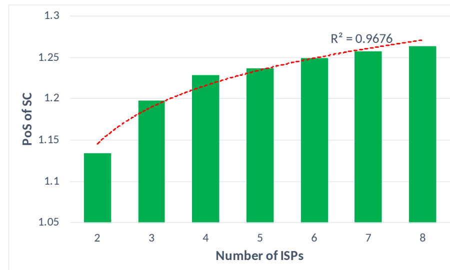

Fig. 5. Average PoS vs Number of ISPs.

ways to exchange their traffic. The decrease of PoS with the increase of C_L can be explained in the following way: with the increase of C_L both the equilibrium and OPT solution results in very small values of X_i (because ISPs will try to buy as small port capacity as possible), thus making cost at equilibrium very close to cost at OPT.

Traffic Symmetry: The effect of traffic symmetry on PoS is measured by a factor $\alpha \in [0, 1]$ (coefficient of symmetry). The traffic values are generated using a uniform distribution with bounds $[\alpha \cdot y_{max}, y_{max}]$, where $y_{max} = \max_{i,j}(y_{ij}), \forall(i, j)$. Hence, the lower the value of α the higher the asymmetry. As shown by the simulation results in Figure 4, the more symmetric the traffic ($\alpha = 1$ is homogeneous traffic) the higher the PoS. Also, this effect is more prominent when C_S is linear. Since internet traffic is usualy not homogeneous [12], we can expect smaller PoS values in real world scenario.

Number of ISPs: The change of PoS with the increase of ISPs doing public peering at some IXP is shown in Figure 5. Since the computing time for finding the OPT solution is in the order of $O(k^N)$, where N is the number of ISPs, we limited our simulation to find the PoS values only for $N \leq 8$. From the results for $N = 2$ to 8, we observe that PoS increases with increasing N. Also, the variation of PoS against N seems to follow a logarithmic trend.

5 Conclusion

Port capacity leasing by ISPs at an IXP is modeled as a non-cooperative game, where all ISPs try to unilaterally minimize their individual costs. Theoretical analysis of this game indicates that while there can be equilibria with very bad PoA, under reasonable practical assumptions PoA is bounded by 2. Simulation results also corroborate the findings of the theoretical results, and provide insight about the effect of congestion and port leasing cost functions on the PoS values. Traffic models generated using IXP data indicates high asymmetry of Internet traffic, for which the PoS values are observed to be very close to 1.

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