

Extended Second Price Auctions with Elastic Supply for PEV Charging in the Smart Grid

Saptarshi Bhattacharya, *Student Member, IEEE*, Koushik Kar, *Member, IEEE*, Joe H. Chow, *Fellow, IEEE*,
and Aparna Gupta, *Member, IEEE*

Abstract—In this paper, we explore the question of efficient allocation of energy, while buying the same from generation companies, to PEVs by aggregator (electricity utility or load serving entities) through auction mechanisms. Recognizing the practical limitations of the Vickrey-Clarke-Groves (VCG) mechanism which would be natural to apply in this context, we investigate two practical mechanisms that can be viewed as extensions of second price auction mechanisms, and have limited message (bid) complexity. In the first mechanism, the *elastic-supply Multi-level Second Price (es-MSP)*, each PEV agent submits a number of price bids, one for each of a given set of energy levels (energy quantities). In the second mechanism, the *elastic-supply Progressive Second Price (es-PSP)*, the PEV agents submit a two-dimensional bid indicating the price as well as the desired energy quantity. Taking into account differences across PEV-owners in terms of their willingness-to-pay values and charging time constraints, we analyze the *social optimality* and *incentive compatibility* properties of the two auction mechanisms. We also complement our theoretical findings with numerical simulations.

Index Terms—PEV charging, smart grid, auctions, incentive compatibility, social optimality.

I. INTRODUCTION

EFFECTIVE management of the electricity demand from Plug-in Electric Vehicles (PEVs) will be crucial for maintaining the stability and operational efficiency of the power grid in the near future [1]-[3]. Fortunately, PEVs provide significant flexibility in terms of their energy consumption rates and schedules, which can be utilized towards reducing the variability of the aggregate demand over time. Additionally, it can also help to partially absorb the variability associated with the supply side, particularly when a significant fraction of the energy is being supplied from intermittent renewable energy sources. Coordinated charging of PEVs is necessary for optimizing electricity dispatch over a temporal scale, hence ensuring that undesirable demand peaks are not created in the power grid [4]. Coordination of PEV charging under varying levels of PEV penetration through controlling the price dynamics has also recently been studied in [7]. A congestion pricing based distributed framework for controlling PEV charging has also been studied in [6]. A price driven charging mechanism that results from non-linear pricing of PEV demand and results in load variance minimization is reported in [14]. Several other approaches to solve the problem of PEV charging has

S. Bhattacharya, K. Kar and J. H. Chow are with the Department of Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY - 12180, USA.

A. Gupta is with the Lally School of Management and Technology, Rensselaer Polytechnic Institute, Troy, NY - 12180, USA.

been taken including game theory [5], [8], [15], gradient optimization [9], sequential quadratic optimization [10], [11], dynamic programming [12] and other heuristic methods [13].

In this paper, we study the use of *auction mechanisms* for solving the *efficient* charging control (scheduling) problem for PEVs in the smart grid. Such auctions can be run in an automated manner where information flow between the aggregator and PEV owners are done through interactive smart meters/chargers. The smart meters/chargers can collect some simple information from the PEV owners (such as amount of energy required, maximum per-unit energy cost it is willing to pay, and time by which it needs the PEV to be charged), and compute and place the bids accordingly. The auction mechanisms studied in this paper are most suited for residential charging of PEVs. However, they can also be applied in commercial charging facilities (such as parking lots offering charging service) by having secured, personalized user profiles. Users can log-in to these profiles and set their preferences, based on which the smart meters at the charging facility can conduct the auctions discussed in the paper.

We realize that the cost of procuring energy from a generation company (by an electricity aggregator) can be modeled as a convex function of the energy. We also realize that PEV agents (PEV owners) can differ in terms of their charging constraints and their willing-to-pay values (for the energy given to them). Since the valuation and charging constraints are private information to the agents, the auction mechanism must induce the agents to be *truthful* in declaring that information to the aggregator, or in making its bids as required by the auction process. A natural candidate for this auction is a *Vickrey-Clark-Groves (VCG)* mechanism. In the VCG mechanism, users are expected to submit their valuation functions (of the resource) to the auctioneer. Users payments are based on the *social opportunity cost*, which in turn ensures that the mechanism is *dominant strategy incentive compatible*. This ensures that users do not have any incentive to bid their valuation functions untruthfully. Based on these valuation functions, the auctioneer (aggregator in our case) can compute and assign a resource allocation (charging solution in our case) that is *socially optimal*, i.e., maximizes the *economic surplus* in the grid. The economic surplus (*or social value*) of a charging solution (schedule) corresponds to the aggregate valuation of the PEV agents for the energy supplied to them, minus the total cost of supplying the energy, given their charging constraints as well as the energy availability constraints of the distribution network.

The VCG mechanism is associated with some practical

limitations however, posing difficulties in implementing it directly in our problem context. Firstly, direct implementation of the VCG mechanism requires the agents to declare their entire valuation function, which is often not exactly known to (or may be hard to estimate by) the agents (PEV owners) themselves. Secondly, even if the valuation functions are known explicitly, declaring them to the aggregator (exactly, or to a close degree of approximation) requires very high message complexity, as the function is defined over a continuous space of real numbers. The discretized or low-complexity approximate VCG frameworks that we consider in this paper are motivated by these limitations of the VCG mechanism. In this paper we apply and study two extensions of this auction mechanism to the PEV charging context, that require the agent to declare only a small number of price and/or quantity values to the auctioneer (aggregator). Despite this, we show that the proposed auction mechanisms retain the desirable incentive compatibility and social optimality properties, at least to a reasonable/desired degree of accuracy, or when equilibrium is attained. Since the two auction mechanisms can be viewed as extensions of VCG, or more generally extensions of the second price auctions of a resource having an elastic supply, we will refer them to as *elastic supply-Extended Second Price (es-ESP)* auction mechanisms. Note that the implementation of the auction mechanisms studied in this paper relies on information provided by the user such as willingness to pay values at certain energy levels (quantities). Truthfulness of the auction mechanisms is important, as it ensures that the users will declare such information truthfully, without which the implemented auction may not result in socially optimal resource sharing.

In the first mechanism, which we call the *elastic supply-Multi-level Second Price (es-MSP)* auction, each agent is required to declare a set of prices that it is willing to pay for certain (given) levels of energy (quantities of charge). This mechanism can rightly be seen as a discretized approximation of the VCG mechanism. In the second mechanism, which we call the *elastic supply-Progressive Second Price (es-PSP)* auction, each agent is required to declare a two-dimensional bid comprising of a price per unit energy (willingness to pay) and a requested quantity (of energy).

Analogue of the mechanisms that we study in this paper have been considered in prior work, mostly in the context of bandwidth allocation in the Internet [16]-[23]. The *es-MSP* mechanism is related to the notion of multi-bid auctions investigated in [16], [17]. The application context, and the network model is significantly different however; in particular our system reduces to a bipartite network graph with elastic supply limits whereas [16], [17] studies a single node (single block of divisible resource of fixed quantity) or inelastic resources connected in a tree network topology. While the broad nature of our approximation results (for incentive compatibility and social optimality) are similar to those in these prior work, we consider convex supply costs and also provide a pricing mechanism that is not only simple to express and compute, but also allows much simpler proofs of the results. The PSP mechanism was proposed in [18], and further analyzed in [19], all in the context of bandwidth auctions. We not only provide

parallels of the full suite of results in [18] and a key result in [19] in the PEV charging context (i.e., bipartite network model and convex supply costs), but also obtain stronger results by taking into account the elastic nature of the supply. In particular, we show that the energy allocation at all Nash equilibria of the auction mechanism is efficient (Proposition 3), a result that does not hold for the fixed resource model, as shown in [19]. Recently, Zou *et al.*, in [24]-[25] have studied the PSP auction mechanism for a model and application that is closely related to ours. However, our model is more general in that it considers heterogeneous charging (time) constraints across PEVs, and we prove two additional, important results (Propositions 3 and 5), which have not been shown in [25]. Furthermore, we also provide a different and general analysis method that combines modeling via ramp functions and using subgradient optimality conditions, and is likely to be useful in analyzing the PSP auction in broader classes of network models and convex environments.

The rest of the paper is structured as follows. Section II describes the system model. Sections III and IV describe and analyze the *es-MSP* and *es-PSP* auction mechanisms, respectively. We evaluate these mechanisms through simulations in Section V and conclude in Section VI.

II. SYSTEM MODEL

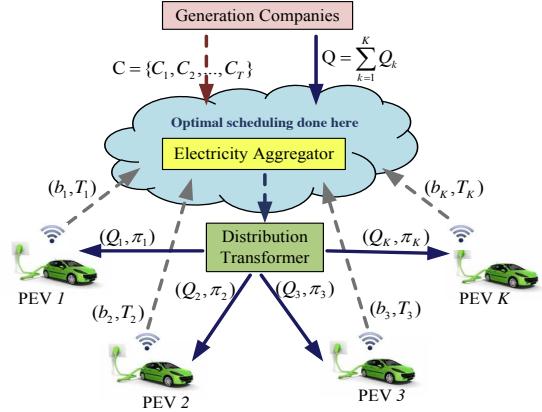


Fig. 1. Schematic diagram of the elastic supply-Extended Second Price (*es-ESP*) auction mechanisms.

Consider an auction window comprising of T time slots, denoted by $\mathcal{T} = \{1, 2, \dots, T\}$. Let $\mathcal{K} = \{1, 2, \dots, K\}$ be the set of all PEVs in the distribution network under consideration. The set of charging constraints (preferences) can differ across PEVs; for PEV k it is given by a set $T_k \subseteq \mathcal{T}$ at which it can charge (i.e., it is connected to the grid). Also, let each PEV $k \in \mathcal{K}$ have a remaining battery capacity of α_k at the start of the auction window i.e. the amount of energy which can still be injected into the battery. We assume that the non-PEV based inelastic demand is given by D_t for $t = 1, \dots, T$. The cost of supplying electricity in any time slot t , denoted by C_t , is assumed to be an *increasing, strictly convex* function of the total load (sum of PEV load and non-PEV load) in that time slot. The supply cost at time t is thus given by $C_t(D_t + \sum_{k=1}^K q_k^t)$,

where q_k^t represents the energy allotted to PEV agent k in time slot t . Let the vector $\mathbf{q}_k = (q_k^1, q_k^2, \dots, q_k^T)$ represent the charging schedule for PEV k . We call $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_K)$ the *schedule vector* for all PEVs in a feasible solution. Let Q_k represent the total energy received by the PEV k over all accessible time slots. We call $\mathbf{Q} = (Q_1, Q_2, \dots, Q_K)$ the *allocation vector* for all PEVs. Note that $Q_k = \sum_{t \in T_k} q_k^t$, and therefore \mathbf{Q} can be expressed as $\mathbf{Q} = \mathbf{M}\mathbf{q}$, where \mathbf{M} is an appropriately defined matrix of dimension $K \times KT$ containing either 0 or 1 as elements. The total energy allocated for PEV charging at time t , denoted by Q^t , is then expressed as $Q^t = D_t + \sum_{k \in \mathcal{K}} q_k^t$. Every PEV (agent) k is associated with

a (privately known) *valuation function* $v_k(\cdot)$ for the energy obtained. We assume $v_k(0) = 0$, and $v_k(Q_k)$ is *increasing, twice differentiable and strictly concave* in Q_k in the range $[0, \alpha_k]$. Figure 1 provides a schematic diagram of the system model and the studied auction mechanisms.

A. VCG mechanism for PEV charging with elastic supply

The socially optimal goal of PEV charging is that of maximizing the “economic surplus”, defined as the total valuation of the energy allocated minus the total cost of supply, subject to the the charging constraints. This is expressed as,

$$\max S(\mathbf{q}) = \sum_{k \in \mathcal{K}} v_k \left(\sum_{t \in T_k} q_k^t \right) - \sum_{t \in \mathcal{T}} C_t (D_t + \sum_{k \in \mathcal{K}} q_k^t), \quad (1)$$

$$\text{s.t. } \sum_{t \in T_k} q_k^t \leq \alpha_k, \quad k \in \mathcal{K}, \quad (2)$$

$$q_k^t = 0, t \notin T_k; \quad q_k^t \geq 0, t \in T_k; \quad k \in \mathcal{K}. \quad (3)$$

Let $\mathcal{D}_k = \{q_k^t, t = 1, \dots, T, \text{subject to (2) and (3)}\}$ be the feasibility constraint set for the charging of PEV k . Then all feasible schedule vectors (for all PEVs) that satisfy (2) and (3) must be contained in $\mathcal{D} = \mathcal{D}_1 \times \mathcal{D}_2 \times \dots \times \mathcal{D}_K$. Therefore, (2) and (3) can be replaced by a single constraint $\mathbf{q} \in \mathcal{D}$. Specifically, the VCG mechanism requires the PEV agent (user) k to make a payment expressed as,

$$\pi_k = \underbrace{\sum_{j \in \mathcal{K} \setminus \{k\}} v_j(Q_{j,-k}^*)}_{\pi_k^o} - \underbrace{v_j(Q_j^*)}_{\pi_k^g} + \sum_{t \in \mathcal{T}} \left(C_t(Q^{*,t}) - C_t(Q_{-k}^*) \right) \quad (4)$$

Here Q_j^* ($Q^{*,t}$) represents the energy allocation to agent j (total load at time t , resp.) under socially optimal energy allocation (one that solves (1)-(3)) when all agents (including agent k) are present. Also $Q_{j,-k}^*$ (Q_{-k}^*) represents the energy allocation to agent j (total load at time t , resp.) under socially optimal energy allocation when agent k is absent from the auction. The part π_k^o denotes the opportunity cost of the resource incurred by agent k (the amount of loss it causes to others through its inclusion in the auction process) and the part π_k^g denotes the additional generation cost that the aggregator has to incur due to the inclusion of agent k in the auction process. This VCG payment policy ensures that rational agents (acting in self-interest) do not have any incentive to declare their valuation functions untruthfully.

There are practical difficulties however in implementing a VCG mechanism as has been mentioned in Section I. These factors motivate the need to look at VCG-like mechanisms that require PEV agents to submit their bids in some simple form that is both convenient to them and requires low message complexity. When the bid space is restricted, however, the challenges are: (i) How can the auction mechanism be designed so that rational agents do not have any incentive to declare the bids untruthfully? ii) How can the socially optimal allocation be attained based on the submitted bids?

III. THE ELASTIC SUPPLY MULTI-LEVEL SECOND PRICE (*es*-MSP) AUCTION MECHANISM

In this mechanism, the aggregator specifies a finite number of energy levels (assume that is n) at the start of the auction window. PEV agents are expected to submit their charging preferences b_k which is of the form $b_k = \{P_{1,k}, P_{2,k}, \dots, P_{n,k}\}$. Here $P_{j,k}$ represents the valuation of j^{th} energy level by PEV agent k . Based on the vector b_k and the timing constraints T_k (also communicated through smart metering equipment), the aggregator constructs a piecewise linear approximate valuation function. Let this be denoted as $\bar{v}_k(\cdot)$ for any $k \in \mathcal{K}$. Based on this piecewise linear valuation function, the aggregator attempts to solve the following optimization problem for optimally buying the electricity and scheduling it for dispatch to the individual PEVs:

$$\max \underline{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \bar{v}_k \left(\sum_{t \in T_k} q_k^t \right) - \sum_{t \in \mathcal{T}} C_t (D_t + \sum_{k \in \mathcal{K}} q_k^t), \quad (5)$$

$$\text{s.t. } \mathbf{q} \in \mathcal{D} \quad (6)$$

Assuming that $\mathbf{q} = \tilde{\mathbf{q}}$ optimizes the above problem (resulting in allocation vector of $\tilde{\mathbf{Q}}$), the price of the electricity which is to be paid by any PEV agent k is computed as $\pi_k = \sum_{j \in \mathcal{K} \setminus \{k\}} \bar{v}_j(\tilde{Q}_{j,-k}) - \bar{v}_j(\tilde{Q}_j) + \sum_{t \in \mathcal{T}} (C_t(\tilde{Q}^t) - C_t(\tilde{Q}_{-k}^t))$. Note that the payment structure is similar to the VCG mechanism with the $v_k(\cdot)$ in the VCG mechanism being replaced by the $\bar{v}_k(\cdot)$, $\forall k \in \mathcal{K}$ in the *es*-MSP mechanism. Also note that the *es*-MSP mechanism is a single-shot mechanism meaning that the allocations and prices as computed above are the final values.

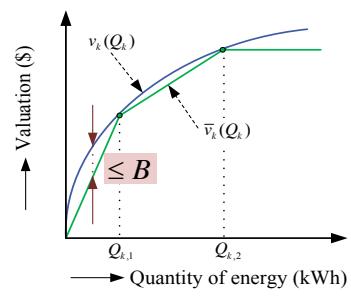


Fig. 2. Piecewise linear approximation of valuation function $v_k(Q_k)$.

A. Analysis of the *es*-MSP auction mechanism

Assume that the actual valuation function $v_k(\cdot)$ is non-decreasing, strictly concave with $v_k(0) = 0$, and $\bar{v}_k(0) = 0$.

Clearly, $\bar{v}_k(x) \leq v_k(x)$, $\forall x$, as seen from Figure 2. Let $B = \max_k \max_{x \leq \alpha_k} (v_k(x) - \bar{v}_k(x))$ represent the maximum deviation of the approximate valuation function \bar{v}_k from the actual valuation v_k over the energy range of interest. (Note that α_k is upper bounded by battery capacity.) Then we have the following results.

Proposition 1. *Assuming truthful bidding, the social valuation of the energy being auctioned off differs from the maximum possible social valuation by KB .*

Proof. Consider $V(\mathbf{Q}) \triangleq \sum_{k \in \mathcal{K}} v_k(Q_k)$ and $\bar{V}(\mathbf{Q}) \triangleq \sum_{k \in \mathcal{K}} \bar{v}_k(Q_k)$ where $\mathbf{Q} = (Q_1, Q_2, \dots, Q_K)$ is the complete

allocation vector of energy to all PEVs. \mathbf{Q}^* and $\tilde{\mathbf{Q}}$ are taken to be the optimal allocations to the VCG mechanism and the *es*-MSP mechanism respectively. We need to show that, $V(\mathbf{Q}^*) - V(\tilde{\mathbf{Q}}) \leq KB$ where B is constant as defined earlier. From optimality of VCG mechanism, $V(\mathbf{Q}^*) \geq V(\mathbf{Q}) \forall \mathbf{Q}$. Similarly, from optimality of the *es*-MSP mechanism, $\bar{V}(\tilde{\mathbf{Q}}) \geq \bar{V}(\mathbf{Q}) \forall \mathbf{Q}$. We know that $v_k(Q_k) - \bar{v}_k(Q_k) \leq B$, $\forall Q_k$ and for all $k \in \mathcal{K}$. Therefore, we can write that,

$$\sum_{k \in \mathcal{K}} (v_k(Q_k) - \bar{v}_k(Q_k)) \leq KB, \quad (7)$$

$$\Rightarrow V(\mathbf{Q}) - \bar{V}(\mathbf{Q}) \leq KB. \quad (8)$$

Identifying the fact that $V(\mathbf{Q}) \geq \bar{V}(\mathbf{Q}) \forall \mathbf{Q}$, and noting that $\tilde{\mathbf{Q}}$ is the optimal value for the *es*-MSP mechanism (auction with discretized functions $\bar{v}_k(\cdot)$), it can be written that,

$$V(\tilde{\mathbf{Q}}) \geq \bar{V}(\tilde{\mathbf{Q}}) \geq \bar{V}(\mathbf{Q}^*) \geq V(\mathbf{Q}^*) - KB. \quad (9)$$

The first inequality in equation (9) comes from the fact that $V(\mathbf{Q}) \geq \bar{V}(\mathbf{Q}) \forall \mathbf{Q}$ (refer to Figure 2 for a clearer understanding); the second inequality follows from the fact that $\tilde{\mathbf{Q}}$ is optimal for the *es*-MSP mechanism (using $\bar{V}(\cdot)$) and the third inequality follows from equation (8). Therefore, we have $V(\mathbf{Q}^*) - V(\tilde{\mathbf{Q}}) \leq KB$. \square

Define $u_k(x) = v_k(x) - \pi_k(x)$ to be the utility obtained by agent k after allocation of x kWh of energy.

Proposition 2. *The maximum utility gained by any agent k through untruthful declaration of its valuation function is upper bounded by B .*

Proof. Let $\mathbf{Q}^* = (Q_1^*, Q_2^*, \dots, Q_K^*)$ be the vector of post *es*-MSP mechanism allocation to all PEV agents under assumption that agent i is bidding its discretized valuation function $\bar{v}_i(\cdot)$ truthfully. Assume that the other agents $j \in \mathcal{K} \setminus \{i\}$ bid $\bar{w}_j(\cdot)$ as their discretized valuation function; \bar{w}_j may or may not be truthful. Considering the $\bar{w}_j(\cdot)$, $j \in \mathcal{K} \setminus \{i\}$ remaining the same, let $\hat{\mathbf{Q}} = (\hat{Q}_1, \hat{Q}_2, \dots, \hat{Q}_K)$ be the vector of post *es*-MSP allocation under a setting when agent i does not bid truthfully; $\bar{y}_i(\cdot)$ being the untruthful discretized valuation function of agent i . Let $\mathbf{Q}_{-i} = (Q_{1,-i}, Q_{2,-i}, \dots, Q_{i-1,-i}, Q_{i+1,-i}, \dots, Q_{K,-i})$ be the allocation vector under a setting when agent i is not present in the auction. For notational simplicity, we will consider

$u_i(Q_i) = u_i(\mathbf{Q})$ for the rest of the proof. The utility obtained by the agent i when it bids truthfully is given as,

$$\begin{aligned} u_i(\mathbf{Q}^*) &= v_i(\mathbf{Q}^*) - \pi_i(\mathbf{Q}^*), \\ &= v_i(\mathbf{Q}^*) + \underbrace{\sum_{j \in \mathcal{K} \setminus \{i\}} \bar{w}_j(Q_j^*) - \sum_{t \in \mathcal{T}} C_t(Q^{t,*})}_{\Psi(\mathbf{Q}^*)} \\ &- \underbrace{\left(\sum_{j \in \mathcal{K} \setminus \{i\}} \bar{w}_j(Q_{j,-i}) - \sum_{t \in \mathcal{T}} C_t(Q_{-i}^t) \right)}_{\Psi(\mathbf{Q})}. \end{aligned} \quad (10)$$

♦

Similarly, the utility obtained by agent i when it bids untruthfully can be written in the same way as (10):

$$\begin{aligned} u_i(\hat{\mathbf{Q}}) &= v_i(\hat{\mathbf{Q}}) - \pi_i(\hat{\mathbf{Q}}), \\ &= v_i(\hat{\mathbf{Q}}) + \underbrace{\sum_{j \in \mathcal{K} \setminus \{i\}} \bar{w}_j(\hat{Q}_j) - \sum_{t \in \mathcal{T}} C_t(\hat{Q}^t)}_{\Psi(\hat{\mathbf{Q}})} - \clubsuit. \end{aligned} \quad (11)$$

The amount of unilateral gain through an untruthful bid by i can thus be represented as $\Delta u_i = u_i(\hat{\mathbf{Q}}) - u_i(\mathbf{Q}^*)$, which can be simplified as,

$$\begin{aligned} \Delta u_i &= \left(v_i(\hat{\mathbf{Q}}) - \bar{v}_i(\hat{\mathbf{Q}}) \right) - \left(v_i(\mathbf{Q}^*) - \bar{v}_i(\mathbf{Q}^*) \right) \\ &+ \underbrace{\left(\bar{v}_i(\hat{\mathbf{Q}}) + \sum_{j \in \mathcal{K} \setminus \{i\}} \bar{w}_j(\hat{Q}_j) - \sum_{t \in \mathcal{T}} C_t(\hat{Q}^t) \right)}_{\Psi(\hat{\mathbf{Q}})} \\ &- \underbrace{\left(\bar{v}_i(\mathbf{Q}^*) + \sum_{j \in \mathcal{K} \setminus \{i\}} \bar{w}_j(Q_j^*) - \sum_{t \in \mathcal{T}} C_t(Q^{t,*}) \right)}_{\Psi(\mathbf{Q}^*)}. \end{aligned} \quad (12)$$

Note that \mathbf{Q}^* optimizes $\Psi(\mathbf{Q})$ over all \mathbf{Q} so $\Psi(\hat{\mathbf{Q}}) - \Psi(\mathbf{Q}^*) \leq 0$. Also note that $0 \leq v_i(Q_i) - \bar{v}_i(Q_i) \leq B \forall Q_i, \forall i \in \mathcal{K}$, as seen from Figure 2. Combining these with (12), we can write $\Delta u_i \leq B$. \square

Note that B can be bounded in terms of the “granularity” at which the agents are required to submit their bids, as follows. Let $\hat{\delta} = \max_k \max_{n=1}^{N_k} (Q_{k,n} - Q_{k,n-1})$, where $Q_{k,0}$ is assumed to be zero for all k . Also, assume that the valuation function $v_k(\cdot)$ is differentiable and *strictly concave*, and $0 < -v_k''(x) \leq \nu$ for all $x \leq \alpha_k$. Then, it can be shown through the following lemma that the degree of approximation in Proposition 1 (social optimality) and Proposition 2 can be made as small as desired by reducing $\hat{\delta}$.

Lemma 1. *Assume that the valuation function v_k is differentiable and strictly concave, and $0 < -v_k''(x) \leq \nu$ for all $x \leq \alpha_k$. Then $B \leq \frac{\nu \hat{\delta}^2}{2}$.*

Proof. Refer to Figure 3 for this proof. Assume that $\Delta(x) \triangleq (v_k(x) - \bar{v}_k(x))$. From the definition of ν and noting that $\bar{v}_k''(x) = 0$, we can write that $\max |\Delta''(x)| = \max |v_k''(x) - \bar{v}_k''(x)| = \nu$. Assume $x_1 \leq x_0 \leq x_2$. Using Mean Value Theorem, we can write,

$$\Delta'(x_2) = \Delta'(x_1) + (x_2 - x_1)\Delta''(x_0), \quad (13)$$

$$\geq \Delta'(x_1) - \nu(x_2 - x_1). \quad (14)$$

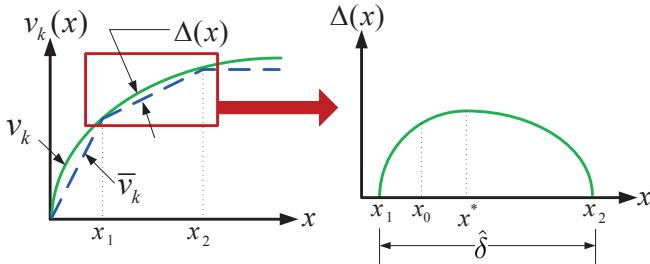


Fig. 3. Variation of the difference between resource valuation under VCG mechanism and MSP mechanism.

Therefore, it can be written that,

$$\Delta'(x_1) - \Delta'(x_2) \leq \nu(x_2 - x_1), \quad (15)$$

$$\Rightarrow \Delta'(x_1) - \Delta'(x_2) \leq \nu\hat{\delta}. \quad (16)$$

Observing that $\Delta'(x_1) \geq 0$, $\Delta'(x_2) \leq 0$ (in Figure 3) and using equation (16), the following inequalities can be written,

$$\Delta'(x_1) \leq \nu\hat{\delta}, \quad (17)$$

$$-\Delta'(x_2) \leq \nu\hat{\delta}. \quad (18)$$

Assume x^* to be the point where $\Delta(x)$ is maximized. Assume $x_1 \leq x_{10} \leq x^*$ and $x^* \leq x_{20} \leq x_2$. Using Mean Value Theorem, we can write,

$$\Delta(x^*) = \Delta(x_1) + \Delta'(x_{10})(x^* - x_1), \quad (19)$$

$$\leq \Delta'(x_1)(x^* - x_1), \quad (20)$$

$$\leq \nu\hat{\delta}(x^* - x_1). \quad (21)$$

The above set of inequalities are written combining (17), (19) and the fact that $\Delta(x_1) = \Delta(x_2) = 0$. Again, expanding $\Delta(x)$ about x_2 and using a similar argument as in (19)-(21), we can write that,

$$\Delta(x^*) \leq \nu\hat{\delta}(x_2 - x^*). \quad (22)$$

Adding inequalities (21) and (22), we get,

$$2\Delta(x^*) \leq \nu\hat{\delta}(x_2 - x_1), \quad (23)$$

$$\Rightarrow 2\Delta(x^*) \leq \nu\hat{\delta}^2, \quad (24)$$

$$\Rightarrow \Delta(x^*) \leq \frac{\nu\hat{\delta}^2}{2}. \quad (25)$$

This clearly shows that $B \leq \frac{\nu\hat{\delta}^2}{2}$. □

IV. THE ELASTIC-SUPPLY PROGRESSIVE SECOND PRICE (es-PSP) AUCTION MECHANISM

In this mechanism, the bid of any PEV agent $k \in \mathcal{K}$ is given by $b_k = (a_k, p_k)$ where a_k is the amount of energy demanded by agent k , and p_k is the price per unit of electricity it is willing to pay. Let $\mathcal{B}_k \subseteq \mathbb{R}_+^2$ be the set of all possible bids by agent k . Let $\mathcal{B}_k^T \subset \mathcal{B}_k$ denote the set of all *truthful* bids where the bid price reflects the *marginal valuation* of the bid quantity, i.e. the 2-d bid is of the form $(a_k, v'_k(a_k))$. A general bid vector (truthful or untruthful) \mathbf{b} is then defined as $\mathbf{b} = (b_1, b_2, \dots, b_K)$. Also let \mathbf{b}_{-k} denote the set of bids of all agents other than k , i.e. $\mathbf{b}_{-k} = (b_1, b_2, \dots, b_{k-1}, b_{k+1}, \dots, b_K)$.

In our *es-PSP* mechanism, once the bid b_k and time constraints T_k are reported to the auctioneer, it solves the following optimization problem for optimal allocation of the energy for charging the PEVs,

$$\max \bar{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \left(\underbrace{\sum_{t \in T_k} q_k^t}_{l_k} \right) p_k - \sum_{t \in \mathcal{T}} C_t (D_t + \sum_{k \in \mathcal{K}} q_k^t) \quad (26)$$

$$s.t. \quad \sum_{t \in T_k} q_k^t \leq a_k, \quad k \in \mathcal{K}, \quad (27)$$

$$\mathbf{q} \in \mathcal{D}. \quad (28)$$

Note that the constraint (28) just captures the feasibility constraints (2)-(3)¹. Similar in principle to the VCG and the *es-MSP* mechanisms, the payment that needs to be made by any agent k is expressed as, $\pi_k = \sum_{j \in \mathcal{K} \setminus \{k\}} (\tilde{Q}_{j,-k} - \tilde{Q}_j) p_j + \sum_{t \in \mathcal{T}} (C_t(\tilde{Q}^t) - C_t(\tilde{Q}_{-k}^t))$ where \tilde{Q}_j (\tilde{Q}^t) represents the energy allocation to agent j (total load at time t , resp.) in the *es-PSP* mechanism. For a given schedule (and corresponding payments) as computed by the *es-PSP* auctioneer, the utility of any agent k is a function of its own bid as well as the bids of others, and is expressed as (similar to the *es-MSP* mechanism),

$$u_k(b_k, \mathbf{b}_{-k}) = v_k(b_k, \mathbf{b}_{-k}) - \pi_k(b_k, \mathbf{b}_{-k}) \quad (29)$$

$$= \overbrace{v_k(\tilde{Q}_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \tilde{Q}_j p_j}^{U_k(b_k, \mathbf{b}_{-k})} - \sum_{t \in \mathcal{T}} C_t(\tilde{Q}^t) - h_k(\mathbf{b}_{-k}). \quad (30)$$

Note that in (30), the term $h_k(\mathbf{b}_{-k}) = \sum_{j \in \mathcal{K} \setminus \{k\}} \tilde{Q}_{j,-k} p_j + \sum_{t \in \mathcal{T}} C_t(\tilde{Q}_{-k}^t)$ depends only on the bids of the other agents.

Therefore, given the bids of others \mathbf{b}_{-k} , a rational agent k would look towards choosing its bid b_k so as to maximize the term $U_k(b_k, \mathbf{b}_{-k})$ subject to (27) and (28), and the given tie-breaking rule (as discussed later). Since this term depends on the allocation of the *es-PSP* mechanism (when all agents including k is present), with slight abuse of notation we refer to this term later in this paper simply as a function of the corresponding schedule ($\tilde{\mathbf{q}}$) or allocation vector ($\tilde{\mathbf{Q}}$), as $U_k(\tilde{\mathbf{q}})$ or $U_k(\tilde{\mathbf{Q}})$. In this paper, we look at the properties of the game in which each agent k , who is assumed to be aware of the *es-PSP* payment and allocation policy, and the bids and constraints of the other agents, attempts to choose its 2-d bid b_k so as to maximize $U_k(b_k, \mathbf{b}_{-k})$. Note that we have analyzed the *es-PSP* mechanism as a complete information game, as optimizing the users individual objective in (30) requires knowledge of bids of the other users. Analysis of an incomplete information equivalent of the game would require assumptions on probabilistic beliefs of each PEV agent about others bids, and remains open for future work

¹Note that (2) is subsumed by (27): it is easy to argue that an agent would not have any incentive to demand an energy quantity a_k larger than α_k . Similarly, it can be argued that an agent does not have any incentive to declare its constraint set T_k untruthfully to the auctioneer.

A. Preliminaries

Definition 1. For a two-dimensional bid $b_k = (a_k, p_k) \in \mathcal{B}_k$, an equivalent ramp function $\hat{w}_{k,b_k} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined as $\hat{w}_{k,b_k}(x) = p_k \min(x, a_k)$.

Note: For any “truthful” bid $b_k \in \mathcal{B}_k^T$ with bid quantity a_k , the ramp function can be represented as \hat{v}_{k,a_k} which is same as earlier, but with $p_k = v'_k(a_k)$, i.e. $\hat{v}_{k,a_k}(x) = v'_k(a_k) \min(x, a_k)$. This is illustrated in Figure 4.

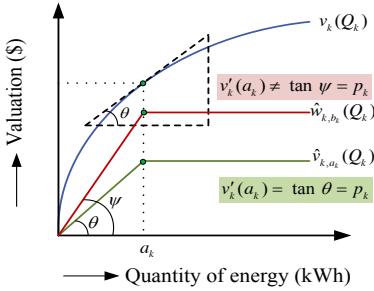


Fig. 4. Ramp function representation of 2-d bid b_k : here $\hat{w}_{k,b_k}(Q_k)$ represents a (possibly *untruthful*) bid and $\hat{v}_{k,a_k}(Q_k)$ represents a *truthful* bid corresponding to bid quantity a_k .

Note that replacing the terms l_k in the *es-PSP* allocation objective $\bar{S}(\mathbf{q})$ in (26) by their corresponding ramp functions, allows us to ignore (27) and optimize only with respect to (28), i.e. the feasibility constraints (2)-(3). It is easy to see that any schedule vector that optimizes \bar{S} (with l_k ’s replaced by their ramp functions) under $\mathbf{q} \in \mathcal{D}$, will not realize an allocation vector $\tilde{\mathbf{Q}}$ such that $\tilde{Q}_k > a_k$ for any k . Since the supply cost functions are (strictly) increasing in the load, if $\tilde{Q}_k > a_k$ for any k , then the objective $\bar{S}(\mathbf{q})$ could be increased by adjusting the corresponding schedule vector $\tilde{\mathbf{q}}$ so as to reduce \tilde{Q}_k to a_k . Due to the nature of the ramp functions, such adjustment would not change $\sum_{k \in \mathcal{K}} l_k$ but reduces $\sum_{t \in \mathcal{T}} g_t$ in (26), thus improving \bar{S} . Hence $\tilde{\mathbf{q}}$ cannot be optimal. Therefore, the energy allocation problem in the *es-PSP* mechanism can be equivalently expressed as

$$\max \bar{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \hat{w}_{k,b_k} \left(\sum_{t \in T_k} q_k^t \right) - \sum_{t \in \mathcal{T}} C_t (D_t + \sum_{k \in \mathcal{K}} q_k^t) \quad (31)$$

$$s.t. \quad \mathbf{q} \in \mathcal{D}. \quad (32)$$

Note that constraints (27) are implied by the definition of the ramp functions, and therefore only constraint (28), i.e. $\mathbf{q} \in \mathcal{D}$ need to be accounted for in the optimization. Recall from the analysis in (29)-(30) that each rational agent seeks to maximize $U_k(b_k, \mathbf{b}_{-k})$ subject to (27) and (28) and the tie-breaking rule (which we will describe shortly). Expressed in terms of ramp functions, this becomes equivalent to maximizing $U_k(\tilde{\mathbf{q}})$ (where $\tilde{\mathbf{q}}$ is a schedule vector resulting from the *es-PSP* auction for the bid vector \mathbf{b}_k), given by

$$U_k(\tilde{\mathbf{q}}) = v_k(\tilde{Q}_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,b_j}(\tilde{Q}_j) - \sum_{t \in \mathcal{T}} C_t(\tilde{Q}^t), \quad (33)$$

where $\tilde{\mathbf{q}}$ satisfies $\mathbf{q} \in \mathcal{D}$.

Finally, note that the allocation resulting from the *es-PSP* mechanism may not be unique, when two agents bid the same

price. To resolve this, we assume that the auctioneer utilizes a *tie-breaking* rule which is known to every agent participating in the auction. This tie-breaking rule allows the auctioneer to determine a unique allocation for the agents even if some of the price bids are equal. Any fixed tie-breaking rule works for our purpose; for definiteness, we assume that the agents submitting the same price bids are prioritized in increasing order of their indices: a higher PEV index gets that PEV agent a higher priority in allocation.

Lemma 2. Assuming a fixed tie-breaking rule, the allocation vector \mathbf{Q} in any solution of the *es-PSP* mechanism is unique.

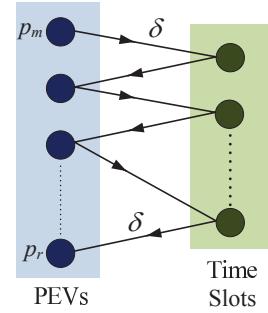


Fig. 5. Flow redirection that i) either increases the objective value \bar{S} , or ii) directs flow from a low priority user to a higher priority user as defined by the tie-breaking rule.

Proof. Consider a given bid vector \mathbf{b} , for which $\hat{\mathbf{q}}$ is an optimal schedule vector (possibly non-unique) resulting from (26)-(28) (or equivalently, (31)-(32)) and the fixed tie-breaking rule. We want to show that the allocation vector $\hat{\mathbf{Q}} = \mathbf{M}\hat{\mathbf{q}}$ is unique, even though the optimal schedule vector $\hat{\mathbf{q}}$ may be non-unique.

It is easy to see from the strict convexity of $C_t(Q^t)$ in Q^t , that $\hat{Q}^t \forall t \in \mathcal{T}$ is unique for all optimal schedule vectors $\hat{\mathbf{q}}$. Therefore, it follows that the total flow (of energy) given by $f(\hat{\mathbf{q}}) = \sum_{t \in \mathcal{T}} \hat{Q}^t$ is a constant under any optimal schedule.

Now, for the sake of contradiction let us assume that $\hat{\mathbf{Q}}$ is not unique i.e. there exists some $\check{\mathbf{Q}}$ which is realized by a scheduling vector $\check{\mathbf{q}}$, such that $\hat{\mathbf{Q}} \neq \check{\mathbf{Q}}$. Let us order the users (from top to bottom in the bipartite graph representation) in increasing order of their price bids; users with the same price bids are ordered in the increasing order of their indices. Let us renumber the indices of the users (PEVs) now according to this new order. Let m be the smallest index user (in this new order just defined) in which the two allocations differ. Without loss of generality, let us assume $\hat{Q}_m > \check{Q}_m$. Since $f(\hat{\mathbf{q}}) = f(\check{\mathbf{q}})$ (as argued before, the total energy allocation is the same in any optimal schedule), there must exist an index $r > m$, with $\check{Q}_r > \hat{Q}_r$, such that we can direct some positive flow $\delta > 0$ flow from m to r in the solution $\hat{\mathbf{q}}$ (see Figure 5). (Note that the flow $\check{\mathbf{q}} - \hat{\mathbf{q}}$ can be resolved into a set of *path flows*, each of which start and end at a PEV node, such as the one shown in Figure 5.) Note that r is such that either (i) $p_r > p_m$, or (ii) $p_r = p_m$ and user r has a higher priority than m according to the tie-breaking rule. In case (i) the flow

redirection from m to r improves the objective $\bar{S}(\hat{\mathbf{q}})$; in case (ii) the solution $\hat{\mathbf{q}}$ could not have satisfied the tie-breaking rule. In either case, we arrive at a contradiction to our assumption that $\hat{\mathbf{q}}$ optimizes \bar{S} subject to the tie-breaking rule, thereby proving the result. \square

B. Relation between Nash equilibrium and Social optimality

In this section we provide the main results stating the relationship between the Nash equilibrium and social optimality of the *es*-PSP mechanism. Note that if a bid vector results in an allocation such that no PEV agent has any incentive of improving their utility by unilaterally changing their bid, then this is a point of Nash equilibrium. Let $\mathbf{q}^* = (\mathbf{q}_1^*, \mathbf{q}_2^*, \dots, \mathbf{q}_K^*)$ be any socially optimal schedule vector that realizes the socially optimal allocation vector $\mathbf{Q}^* = (Q_1^*, Q_2^*, \dots, Q_K^*)$. Note that the optimal schedule \mathbf{q}^* can be non-unique, but due to the strict concavity of $v_k(Q_k)$ in Q_k , it follows that the optimal allocation vector \mathbf{Q}^* is unique.

Proposition 3. *The allocation at any Nash equilibrium of the *es*-PSP mechanism is socially efficient.*

Proof. Consider a bid vector $\check{\mathbf{b}}$ that is at Nash equilibrium, and let $\hat{w}_{k,\check{b}_k}(\cdot)$, $\forall k \in \mathcal{K}$ be the corresponding ramp functions. (We will see from Proposition 4 that a Nash equilibrium to the *es*-PSP mechanism exists.) Let $\check{\mathbf{q}}$ be a schedule vector resulting from the *es*-PSP mechanism for this bidding strategy, and the corresponding allocation vector be $\check{\mathbf{Q}}$.

Now from (33), given the bids of other agents $\check{\mathbf{b}}_{-k}$, agent k seeks to maximize $\check{U}_k(\mathbf{q})$, given by

$$\check{U}_k(\mathbf{q}) = v_k(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t). \quad (34)$$

We first argue that $\check{U}_k(\mathbf{q})$ is maximized by $\check{\mathbf{q}}$, $\forall k \in \mathcal{K}$ subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule. To see this, for sake of contradiction, suppose that for any $k \in \mathcal{K}$, $\check{U}_k(\mathbf{q})$ is not maximized at $\check{\mathbf{q}}$. In other words, there exists some schedule vector $\bar{\mathbf{q}} \neq \check{\mathbf{q}}$ that maximizes \check{U}_k subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule; let $\bar{\mathbf{Q}} (\neq \check{\mathbf{Q}})$ be the corresponding allocation vector. Let $\partial \check{U}_k(\mathbf{q})$ be the set of sub-gradients of the function $\check{U}_k(\mathbf{q})$ at \mathbf{q} , and $\Gamma^+(\mathbf{q})$ be the conjugate to the cone of feasible directions in \mathcal{D} at \mathbf{q} . Then since $\mathbf{q} = \check{\mathbf{q}}$ optimizes $\check{U}_k(\mathbf{q})$, from [26] we can write (sub-gradient constrained optimality condition):

$$\partial \check{U}_k(\check{\mathbf{q}}) \cap \Gamma^+(\check{\mathbf{q}}) \neq \emptyset. \quad (35)$$

Now let agent k unilaterally change its bid from \check{b}_k to $\bar{b}_k = (\bar{Q}_k, v'_k(\bar{Q}_k))$, and let $\hat{v}_{k,\bar{Q}_k}(\cdot)$ be the corresponding (truthful) ramp function. We will show that agent k gains for this deviation. Since the bids of the other agents are kept fixed at $\check{\mathbf{b}}_{-k}$, allocation will be determined by the auctioneer by maximizing $\bar{S}(\mathbf{q})$ given by

$$\bar{S}(\mathbf{q}) = \hat{v}_{k,\bar{Q}_k}(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t), \quad (36)$$

subject to (32) and the tie-breaking rule. Note that the only difference in $\bar{S}(\mathbf{q})$ and $\check{U}_k(\mathbf{q})$ is the replacement of $v_k(Q_k)$ in $\check{U}_k(\mathbf{q})$ by $\hat{v}_{k,\bar{Q}_k}(Q_k)$ in $\bar{S}(\mathbf{q})$. Also, note that $\hat{v}_{k,\bar{Q}_k}(Q_k)$ is non-differentiable with respect to Q_k at \bar{Q}_k . Further, the

component corresponding to q_k^t (for any $t \in T_k$) in any sub-gradient of $\bar{S}(\mathbf{q})$ at $\mathbf{q} = \check{\mathbf{q}}$ is given by $\lambda v'_k(\bar{Q}_k)$ for $0 \leq \lambda \leq 1$, which contains $v'_k(\bar{Q}_k)$ ($\lambda = 1$) case. Thus, we can write $\partial \check{U}_k(\check{\mathbf{q}}) \subset \partial \bar{S}(\check{\mathbf{q}})$. This fact and (35) gives

$$\partial \bar{S}(\check{\mathbf{q}}) \cap \Gamma^+(\check{\mathbf{q}}) \neq \emptyset. \quad (37)$$

This implies that $\check{\mathbf{q}}$ also optimizes $\bar{S}(\mathbf{q})$. From Lemma 2, we know that the corresponding allocation $\bar{\mathbf{Q}}$ is unique. This implies that when the other agents' bids are remain fixed at $\check{\mathbf{b}}_{-k}$ and agent k changes its bid to $\bar{b}_k = (\bar{Q}_k, v'_k(\bar{Q}_k)) \neq \check{b}_k$, the allocation resulting from the *es*-PSP auction must be $\bar{\mathbf{Q}}$, which improves $\check{U}_k(\mathbf{q})$ beyond its value at Nash equilibrium $\check{U}_k(\check{\mathbf{q}})$. This provides incentive for agent k to change its bid from \check{b}_k , contradicting the fact that the bid vector $\check{\mathbf{b}}$ is at Nash equilibrium. Therefore, our supposition was wrong, implying that $\check{U}_k(\mathbf{q})$ is indeed maximized at $\check{\mathbf{q}}$ for all $k \in \mathcal{K}$. Define $C(\mathbf{q}) = \sum_{t \in \mathcal{T}} C_t(Q^t)$. From the first order (necessary) conditions for optimality of $\check{U}_k(\mathbf{q})$ at $\mathbf{q} = \check{\mathbf{q}}$ along the direction of \mathbf{q}_k , and identifying the fact that $\sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j)$ in (34) is independent of \mathbf{q}_k , we can write

$$[\nabla_{\mathbf{q}_k} v_k(\check{Q}_k) - \nabla_{\mathbf{q}_k} C(\check{\mathbf{q}})]_{\mathcal{D}_k} = 0. \quad (38)$$

Note that (38) holds for all $k \in \mathcal{K}$. Now consider $S(\mathbf{q})$ in (1) for computing the social optimum subject to (32). The corresponding first order conditions for optimality (which are both necessary and sufficient in this case, due to the convexity of $S(\mathbf{q})$ in \mathbf{q}) of $S(\mathbf{q})$ subject to (32) are given as,

$$[\nabla_{\mathbf{q}_k} v_k(Q_k) - \nabla_{\mathbf{q}_k} C(\mathbf{q})]_{\mathcal{D}_k} = 0, \quad \forall k \in \mathcal{K}. \quad (39)$$

Note that (38) when considered for all $k \in \mathcal{K}$, is the same as the conditions in (39). Therefore the schedule vector $\check{\mathbf{q}}$ (which realizes an allocation vector of $\check{\mathbf{Q}}$) also maximizes $S(\mathbf{q})$, and is therefore socially optimal. This also implies that the allocation vector $\check{\mathbf{Q}}$ at any Nash equilibrium of the *es*-PSP mechanism equals the unique socially optimal allocation \mathbf{Q}^* . \square

Proposition 3 should not be interpreted as the uniqueness of the Nash equilibrium bids. Proposition 3 only implies that the allocation vector at all Nash equilibria is the same, and is socially optimal. Note however that this (socially optimal) allocation vector is in general realizable by multiple schedule vectors.

Proposition 4. *Truthful bidding at the socially optimal allocations is a Nash equilibrium of the *es*-PSP mechanism, i.e., the bids $b_k^* = (Q_k^*, v'_k(Q_k^*))$ for $k \in \mathcal{K}$, constitute a Nash equilibrium of the *es*-PSP mechanism.*

Proof. Consider the bid vector $\mathbf{b}^* = (b_1^*, b_2^*, \dots, b_K^*)$ where $b_k^* = (Q_k^*, v'_k(Q_k^*)) \forall k \in \mathcal{K}$. The allocation problem solved by the auctioneer in this case is:

$$\max \hat{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k) - \sum_{t \in \mathcal{T}} C_t(Q^t), \quad (40)$$

subject to $\mathbf{q} \in \mathcal{D}$ (and the tie-breaking rule), where $\hat{v}_{k,Q_k^*}(Q_k) = v'_k(Q_k^*) \min(Q_k, Q_k^*)$ is the truthful ramp function corresponding to the socially optimal allocation for user

k . Since \mathbf{q}^* optimizes $S(\mathbf{q})$ in (1) subject to $\mathbf{q} \in \mathcal{D}$, from [26] we can say,

$$\partial S(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \emptyset. \quad (41)$$

Here, $\partial S(\mathbf{q}^*)$ is the set of sub-gradients of the function $S(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$; in our case, since $S(\mathbf{q})$ is differentiable for all \mathbf{q} , $\partial S(\mathbf{q}^*)$ will just consist of the gradient of $S(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$. Also, $\Gamma^+(\mathbf{q}^*)$ represents the conjugate to the cone of feasible directions in \mathcal{D} , at the point $\mathbf{q} = \mathbf{q}^*$. We can see that the only difference in (1) and (40) is that $v_k(Q_k)$ has been replaced by $\hat{v}_{k,Q_k^*}(Q_k)$. Also, note that $\hat{v}_{k,Q_k^*}(Q_k)$ is non-differentiable with respect to Q_k at Q_k^* . Further, the component corresponding to q_k^t (for any $t \in T$) in any sub-gradient of $\hat{S}(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$ is given by $\lambda v'_k(Q_k^*)$ for $0 \leq \lambda \leq 1$, which contains $v'_k(Q_k^*)$ ($\lambda = 1$) case. Thus, we can argue that,

$$\partial S(\mathbf{q}^*) \subset \partial \hat{S}(\mathbf{q}^*). \quad (42)$$

From (41) and (42), we can write,

$$\partial \hat{S}(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \emptyset, \quad (43)$$

which implies that \mathbf{q}^* also optimizes $\hat{S}(\mathbf{q})$ subject to $\mathbf{q} \in \mathcal{D}$. Now for any $\mathbf{q} = \bar{\mathbf{q}}$ that optimizes $\hat{S}(\mathbf{q})$ subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule, the corresponding allocation vector $\bar{\mathbf{Q}}$ is unique (from Lemma 2). We claim that $\bar{\mathbf{Q}} = \mathbf{Q}^*$. To see this, let us assume for the sake of contradiction, $\bar{\mathbf{Q}} \neq \mathbf{Q}^*$. Note that any optimal allocation $\bar{\mathbf{Q}}$ must satisfy $\bar{Q}_k \leq Q_k^*$, $\forall k \in \mathcal{K}$. Also note that $\hat{v}_{k,Q_k^*}(\bar{Q}_k) = v'_k(Q_k^*)\bar{Q}_k$ for all such allocations. Since $\hat{S}(\mathbf{q}^*) = \hat{S}(\bar{\mathbf{q}})$, we have

$$\sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k^*) - \sum_{t \in T} C_t(Q^{*,t}) = \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(\bar{Q}_k) - \sum_{t \in T} C_t(\bar{Q}^t). \quad (44)$$

From the strict convexity of $C_t(\cdot)$, it follows that Q^t , $\forall t \in T$ must be unique in any optimal solution. Therefore, $Q^{*,t} = \bar{Q}^t$, $\forall t \in T$. Thus, $\sum_{t \in T} C_t(Q^{*,t}) = \sum_{t \in T} C_t(\bar{Q}^t)$. Thus,

$$\begin{aligned} \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k^*) &= \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(\bar{Q}_k). \\ \Rightarrow \sum_{k \in \mathcal{K}} v'_k(Q_k^*)(Q_k^* - \bar{Q}_k) &= 0. \end{aligned} \quad (45)$$

Since $v'_k(\cdot) > 0$ at all points (we have assumed the valuation functions to be (strictly) increasing), it follows that $Q_k^* = \bar{Q}_k$, $\forall k \in \mathcal{K}$. Thus, $\mathbf{Q}^* = \bar{\mathbf{Q}}$. This shows that given the bid vector \mathbf{b}_{-k}^* , i.e. when every other agent $j \in \mathcal{K} \setminus \{k\}$ bids the ramp function $\hat{v}_{j,Q_j^*}(\cdot)$, the allocation is \mathbf{Q}^* provided agent k bids $\hat{v}_{k,Q_k^*}(Q_k)$. Now from (33), given \mathbf{b}_{-k}^* , a rational (selfish) user k 's objective is to maximize $U_k^*(\mathbf{q})$, given by

$$U_k^*(\mathbf{q}) = v_k(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{v}_{j,Q_j^*}(Q_j) - \sum_{t \in T} C_t(Q^t). \quad (46)$$

Comparing $U_k^*(\mathbf{q})$ and $S(\mathbf{q})$ (defined in (1)), we see that the only differences are the replacement of $v_j(Q_j)$ by $\hat{v}_{j,Q_j^*}(Q_j)$, $\forall j \in \mathcal{K} \setminus \{k\}$. Using similar arguments as provided earlier in this proof (when comparing the sub-gradients of $S(\mathbf{q})$ and $\hat{S}(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$), it follows that

$$\partial S(\mathbf{q}^*) \subset \partial U_k^*(\mathbf{q}^*). \quad (47)$$

From (41) and (47), we can write,

$$\partial U_k^*(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \emptyset. \quad (48)$$

This shows that $U_k^*(\mathbf{q})$ is maximized at $\mathbf{q} = \mathbf{q}^*$ subject to (32) and the tie-breaking rule, provided $\mathbf{b}_{-k} = \mathbf{b}_{-k}^*$. Now suppose that given the bids of other users remains fixed at \mathbf{b}_{-k}^* , user k deviates by bidding b'_k , which realizes in a schedule vector \mathbf{q}' (and corresponding allocation vector \mathbf{Q}'), as a result of the *es*-PSP auction. Since \mathbf{q}^* optimizes $U_k^*(\mathbf{q})$ subject to (32) and the tie-breaking rule, $U_k^*(\mathbf{q}') \leq U_k^*(\mathbf{q}^*)$. Thus we see that given $\mathbf{b}_{-k} = \mathbf{b}_{-k}^*$, agent k has no incentive to change its bid from \hat{v}_{k,Q_k^*} or equivalently $b_k^* = (Q_k^*, v'_k(Q_k^*))$. Therefore the bid vector $\mathbf{b}^* = (b_k^*, \mathbf{b}_{-k}^*)$ is a Nash equilibrium of the *es*-PSP mechanism. \square

Loosely speaking, Proposition 4 can be viewed as a converse of Proposition 3. The result is an extension of Proposition 1 in [19] which considers a generalized network model but fixed resource supply. From the proof of Proposition 4, it can also be seen that for this bidding strategy, each agent k gets the quantity Q_k^* that it asks for, i.e., the optimal energy allocation resulting from the auction when the bid vector is \mathbf{b}^* is \mathbf{Q}^* .

C. Truthful price-bid declaration

Recall that Proposition 4 shows the existence of a *truthful* bid that is a Nash equilibrium. In the proof of Proposition 3, we have also used truthful bidding in constructing potentially better bids for a user, given the bids of others. These are not accidental, as can be seen from Proposition 5 as stated below. This result shows that given the bids of other agents (which need not be at Nash equilibrium or result in socially optimal allocation), any agent cannot gain by lying about its price bid for the quantity bid it declares (the quantity bid that optimizes its individual utility, given other agents' bids).

Proposition 5. *Given the bids of other agents, \mathbf{b}_{-k} , there exists a truthful best bid for agent k , $\tilde{b}_k(\mathbf{b}_{-k}) \in \mathcal{B}_k^T$.*

Proof. From (33), given the bids of other agents \mathbf{b}_{-k} , agent k seeks to maximize

$$U_k(\mathbf{q}) = v_k(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,b_j}(Q_j) - \sum_{t \in T} C_t(Q^t), \quad (49)$$

subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule. Let an optimal schedule vector (that maximizes $U_k(\mathbf{q})$) be $\mathbf{q} = \tilde{\mathbf{q}}$ which realizes an allocation vector $\tilde{\mathbf{Q}} = (\tilde{Q}_1, Q_2, \dots, \tilde{Q}_K)$. While $\tilde{\mathbf{q}}$ can be non-unique, owing to strict concavity of $v_k(Q_k)$ with respect to Q_k , it follows that \tilde{Q}_k is unique. From an argument similar to that in the proof of Lemma 2, we can show that \tilde{Q}_j , $\forall j \in \mathcal{K} \setminus \{k\}$ are unique as well, when the tie-breaking rule is taken into account. Thus, $\tilde{\mathbf{Q}}$ is unique. Note that for optimality of (49) at $\tilde{\mathbf{q}}$, it follows from [26] that,

$$\partial U_k(\tilde{\mathbf{q}}) \cap \Gamma^+(\tilde{\mathbf{q}}) \neq \emptyset. \quad (50)$$

Now let us define the following “truthful” bid for agent k : $\tilde{b}_k(\mathbf{b}_{-k}) = (\tilde{Q}_k, v'_k(\tilde{Q}_k))$. Then \tilde{b}_k can be represented as the ramp function $\hat{v}_{k,\tilde{Q}_k}(\cdot)$. Now when agent k bids \tilde{b}_k in the *es*-PSP mechanism, while the bids of other agents remain

fixed at \mathbf{b}_{-k} , the auctioneer computes the energy allocation by maximizing $\tilde{S}(\mathbf{q})$ given by

$$\tilde{S}(\mathbf{q}) = \hat{v}_{k,\tilde{Q}_k}(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,b_j}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t), \quad (51)$$

subject to (32) and the tie-breaking rule. Compare $U_k(\mathbf{q})$ and $\tilde{S}(\mathbf{q})$. The only differences are replacement of $v_k(Q_k)$ in $U_k(\mathbf{q})$ by $\hat{v}_{k,\tilde{Q}_k}(Q_k)$ in $\tilde{S}(\mathbf{q})$. Hence, arguing as before (see proofs of Propositions 4 and 3), $\partial U_k(\tilde{\mathbf{q}}) \subset \partial \tilde{S}(\tilde{\mathbf{q}})$. Combining this with (50) we get $\partial \tilde{S}(\tilde{\mathbf{q}}) \cap \Gamma^+(\tilde{\mathbf{q}}) \neq \emptyset$, which implies that $\tilde{\mathbf{q}}$ also optimizes $\tilde{S}(\mathbf{q})$. Since the allocation vector corresponding to any optimum solution of $\tilde{S}(\mathbf{q})$ subject to the tie-breaking rule is unique (Lemma 2), it follows that if agent k submits a bid of \tilde{b}_k when the other bids are kept at \mathbf{b}_{-k} , the resulting allocation is $\tilde{\mathbf{Q}}$. Since $\tilde{\mathbf{Q}}$ maximizes $U_k(\mathbf{q})$, therefore the truthful bid $\tilde{b}_k = (\tilde{Q}_k, v'_k(\tilde{Q}_k))$, which depends on \mathbf{b}_{-k} , represents the agent's best bid given \mathbf{b}_{-k} . The result follows. \square

The question of convergence to equilibrium for the PSP mechanism has been addressed in [22], in context of a single fixed resource and quantized user bids. The extension of such results to our network model - which has elastic supply and a bipartite graph structure - however remains open for future investigation.

V. NUMERICAL STUDY

In order to validate our theoretical findings with numerical evidence, we consider an urban residential power distribution network with a baseline demand profile as given in [28]. We consider 200 PEVs in the residential network: each having a valuation function of the nature $v_k = \kappa(1 - e^{-ax})$ where κ and a are concavity parameters of the valuation function. Ideally all PEVs would have unique parametric values for κ and a but for simplicity, we assume that PEVs are of two types: each type represented by a unique set of κ values. For our study, we chose $\kappa_1 = 15$; $\kappa_2 = 12$. For all PEVs, $a = 0.1$. We assume that all PEVs are available to charge in all time slots. Based on the nature of market clearing prices observed in New York City [27], the aggregator's cost of buying x kWh of energy for PEV charging in any time slot t is assumed to be determined by the function $C_t(D_t + x) = \frac{1}{2}c(D_t + x)^2$ where $c = 0.0006932$. We see from Figure 6 that the VCG mechanism results in *valley filling* of the load curve: it directs the PEVs to charge from time slots that have lesser inelastic demand in them (12am - 6am i.e. overnight charging).

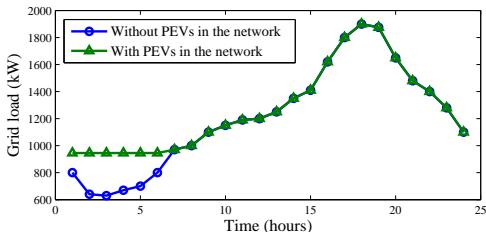


Fig. 6. Grid load profile before and after energy allocation to PEVs through the VCG mechanism.

The socially optimal allocations of the VCG mechanism to PEV agents of type 1 and 2 are found to be 8.2798 kWh and 6.0483 kWh respectively. Note that this is sufficient for driving 20 – 30 miles approximately (depending upon PEV manufacturer type) which we believe, is suitable for daily commuting requirements in a typical urban (or semi-urban) setting. However, one must note that PEVs can avail greater energy by raising their valuation of the resource. The following table shows how PEV agent 1 can increase its allocation by raising its valuation. Here, we increase the parameter κ_1 and hold κ_2 constant and observe the change in the socially optimal allocation. For our subsequent simulations, we use $\kappa_1 = 15$

TABLE I
EFFECT OF INCREASING VALUATION ON SOCIAL OPTIMUM ALLOCATION
FOR TYPE 1 PEV AGENTS

κ_1	15	20	25	30	35
Allocation (kWh)	8.2798	10.7993	12.7859	14.4287	15.8197

and $\kappa_2 = 12$ and $a = 0.1$. We now consider the effect of increasing the number of PEVs in the network on the social optimum allocation. In Figure 7, we report the net allocation to PEV agents of type 1 and type 2 under different levels of PEV penetration in the network. We use an equal proportion of type 1 and type 2 agents for this study. We observe that

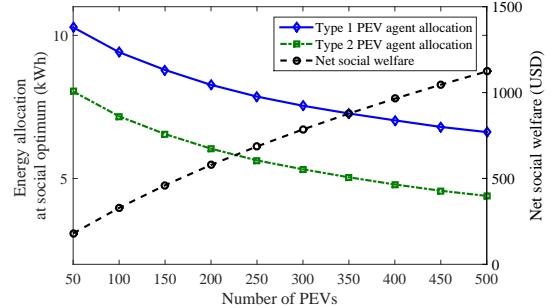


Fig. 7. Effect of increasing the number of PEVs in the distribution network on the social optimum values. Note that $\kappa_1 = 15$, $\kappa_2 = 12$ for this study.

with the same valuation functions, the individual allocations to each PEV agent decreases as the total number of PEV agents increase in the network. This indicates that a higher penetration of PEVs in the network increases competition among rational PEV agents to avail the energy. We also note that the overall social valuation of the resource monotonically increases as we increase the PEV penetration in the network. This proves that a higher PEV penetration allows the aggregator to increase the overall social valuation of the energy resource being auctioned.

Having studied the variation of the social optimum with respect to PEV user preferences and overall levels of PEV penetration, we concentrate on studying the proposed approximations to the pure VCG mechanism. We first consider the *es*-MSP mechanism. Define $\gamma = \frac{E_{VCG} - E_{es-MSP}}{E_{VCG}}$, where E is the economic surplus (aggregate valuation of the PEV agents for the energy supplied to them, minus the total cost of supplying that energy, given their charging constraints) as defined in Section I. The parameter γ is the measure of the

relative degree of inefficiency resulting from the discretization of the valuation functions. We observe from Figure 8 that the relative inefficiency approximately decreases with increase in the number of bids.

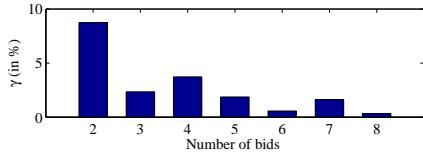


Fig. 8. Effect of discretization in *es*-MSP mechanism on social welfare.

Next we consider the *es*-PSP auction, and demonstrate that a social optimum solution is a Nash equilibrium. The socially optimal allocations resulting from the VCG mechanisms are 8.2798 kWh for Type 1 PEVs, and 6.0483 kWh for Type 2 PEVs. Now, we study the effect of increasing the quantity of bid for each agent k in an entire range of interest of charging, while the bids of all other 199 agents (of both type 1 and type 2) are held constant at their socially optimal values. We assume truthful bidding, so determining the quantity also determines the per-unit price p_k . We observe from Figure 9 that, agents

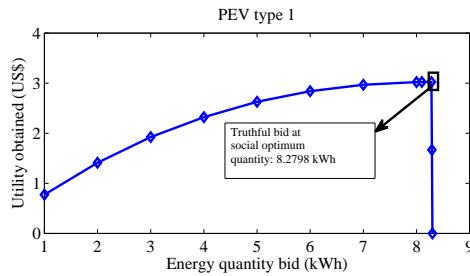


Fig. 9. Utility received by agents of type 1 by varying their bids unilaterally, while the other agent's are kept at their socially optimal values.

of type 1 gain maximum utility by bidding at the socially optimal value of 8.2798 kWh, at which point there is no incentive to unilaterally deviate and gain a greater utility. This validates that bidding socially optimal quantities is indeed a Nash equilibrium, under truthful bidding.

We also observe that given all the other 199 agents bid at their socially optimal values (and truthfully), any agent of type 1 drastically loses utility if it overbids (truthfully) its energy quantity even marginally above its social optimum. This can be explained as follows. At its socially optimum bid of 8.2798 kWh, the corresponding truthful price bid of the singular type 1 PEV agent is 0.6554 \$/kWh, at which point it obtains its desired energy quantity (and maximum utility). A marginal unilateral increase of bid quantity by the agent from the socially optimal value to 8.29 kWh (say) causes its price bid to be 0.6547 \$/kWh and results in the agent receiving a non-zero but smaller allocation (also lesser than its requested bid). A further increase of bid to 8.3 kWh corresponds to a price bid of 0.6541 \$/kWh, at which point its allocation (and corresponding utility) is zero (owing to the price bid becoming lesser than marginal cost of energy i.e. 0.6544 \$/kWh).

We also study the effect of charging time constraints on the social optimal allocation and payments. For this case, we

assume that type 1 PEV agents are unavailable to charge from 12am - 3am and type 2 PEV agents are unavailable to charge from 2am - 3am. Under such setting, we observe that the social optimum allocation becomes 7.8438 kWh and 5.6124 kWh for type 1 and 2 agents respectively. This is clearly lesser than the allocations when no charging time constraints are present. We observe that when agents of type 1 increase their valuation from $\kappa_1 = 15$ to $\kappa_1 = 20$ ($\kappa_2 = 12$ and $a = 0.1$ are unchanged), with similar time constraints as assumed, the social optimum allocation changes to 10.2677 kWh and 5.5251 kWh for type 1 and 2 agents respectively. Thus agents who are (severely) time constrained can get greater energy allocated through raising their true valuations. The corresponding *es*-PSP mechanism payments (assuming that it has converged to the point of social optimum) for type 1 agents are observed to be 5.3664\$ and 7.3473\$ respectively for the cases where $\kappa_1 = 15$ and $\kappa_2 = 20$. This shows that a higher payment needs to be made by the time constrained type 1 PEV agents to obtain greater resource.

We now compare the payments to be made to the aggregator by the PEV agents under the VCG mechanism, the 8-bid *es*-MSP mechanism and the *es*-PSP mechanism (assuming it has converged to social optimum) under the setting where there are no charging time restrictions for any agents. We compare these payments with a specific rule based payment strategy in which the socially optimum allocations in any time slot are charged at a flat per unit price equal to the marginal price of energy in that time slot (after PEV load inclusion). Refer to Figure 10 for this study.

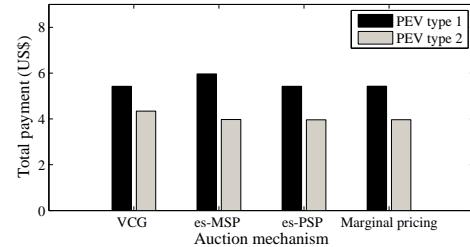


Fig. 10. Comparison of total payments to be made by a PEV agent of type 1 or type 2 under different mechanisms.

The VCG payments for an agent of type 1 and type 2 are seen to be 5.4236\$ and 4.3414\$ respectively. The payments under the *es*-PSP mechanism are observed to be 5.4226\$ and 3.9619\$ for an agent of type 1 and 2 respectively. In the 8-bid *es*-MSP mechanism, the net payments to be made by PEV agents of type 1 and type 2 are observed to be 5.9638\$ and 3.9731\$ respectively. Assuming that the PEVs have been allocated their socially optimal energy quantities, the rule based marginal pricing policy results in payments of 5.4266\$ and 3.9641\$ for type 1 and type 2 PEV agents respectively. Thus we infer that the studied auction mechanisms yields prices for energy use (PEV charging in our case) that are comparable to the rule based marginal pricing policy often employed for customer billing by load serving entities in retail electricity markets.

Remarks: We conclude this section with a complexity analysis of the studied mechanisms. Assume there are K

agents participating in the auction and the auction window of T time slots. Firstly, note that for the VCG mechanism and the *es*-MSP mechanism (both of which are “single-shot” mechanisms), the allocation to PEV agents is determined by a convex program ((1)-(3) for the VCG and (5)-(6) for the *es*-MSP mechanism) which has KT decision variables. The determination of payment of an agent also requires the solving of one convex program which has $(K-1)T$ decision variables (to determine the allocations to other agents in the absence of the agent whose payment is being determined). So overall we require $K+1$ convex programs to entirely compute the allocation and payments of all PEV agents for the VCG mechanism and the *es*-MSP mechanism.

The *es*-PSP mechanism, could be implemented as an iterative mechanism, where each round has the same computational requirements of the overall VCG (or *es*-MSP mechanism). Typically, the number of rounds and hence the computational requirements will increase with the increase in the number of agents. However, note that in practice, many PEVs will have similar charging preferences in the network. This can also be implicitly achieved by engineering the smart meters in a manner that the PEV agent preferences are constrained to lie within some select choices made available through an interactive user interface. Through such schemes, the central auctioneer (the utility or aggregator) can reduce computational requirements by grouping similar PEV agents (and hence reducing the number of optimization decision variables) while determining the auction allocations and agent payments.

ACKNOWLEDGMENT

This work was supported in part by the National Science Foundation (NSF) under award no. 1408333.

REFERENCES

- [1] G. T. Heydt, The impact of electric vehicle deployment on load management strategies, *IEEE Transactions on Power Application Systems*, vol. PAS-102 , no. 5, pp. 1253-1259, May 1983.
- [2] A. Heider and H. J. Haubrich, Impact of wide-scale EV charging on the power supply network, in *Proc. IEE Colloq. Electric Vehicles - A Technology Roadmap for the Future*, vol. 6, no. 262, pp. 6/1-6/4, 1998.
- [3] R. Liu, L. Dow, and E. Liu, A survey of PEV impacts on electric utilities, in *Proc. IEEE PES Innov. Smart Grid Tech. Conf.*, Jan. 2011.
- [4] M. D. Galus and G. Andersson, Demand management of grid connected plug-in hybrid electric vehicles, in *Proc. IEEE Energy 2030 Conference*, Anaheim, CA, Nov. 2008.
- [5] Z. Ma, D. Callaway, and I. Hiskens, Decentralized charging control for large populations of plug-in electric vehicles, *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67-78, Oct. 2011.
- [6] Z. Fan, A distributed demand response algorithm and its application to PHEV charging in smart grids, *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1280-1290, Mar. 2012.
- [7] I. S. Bayram, A. Tajer, M. Abdallah and K. Qaraqe, Capacity planning frameworks for electric vehicle charging stations with multiclass customers, *IEEE Transactions on Smart Grid*, vol. 6, no. 4, Jul. 2015.
- [8] W. Tushar, W. Saad, V. Poor, and D. Smith, Economics of electric vehicle charging: a game theoretic approach, *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1767-1778, Dec. 2012.
- [9] L. Gan, U. Topcu, and S. Low, Optimal decenteralized protocols for electric vehicle charging, in *Proc. 50th IEEE Conference on Decision and Control (CDC)*, Dec. 2011.
- [10] K. Clement-Nyns, E. Haesen, and J. Driesen, The impact of charging plug-in hybrid electric vehicles on a residential distribution grid, *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 371-380, Jan. 2010.
- [11] A. Hajimiragh, C. A. Canizares, M. Fowler, and A. Elkamel, Optimal transition to plug-in hybrid electric vehicles in Ontario, Canada, considering the electricity-grid limitations, *IEEE Transactions on Industrial Electronics*, vol. 57, no. 2, pp. 690-701, Feb. 2010.
- [12] N. Rotering and M. Ilic, Optimal charge control of plug-in hybrid electric vehicles in deregulated electricity markets, *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1021-1029, Aug. 2012.
- [13] E. Sortomme, M. Hindi, S. D. MacPherson, and S. S. Venkata, Coordinated charging of plug-in hybrid electric vehicles to minimize distribution system losses, *IEEE Transactions on Smart Grid*, vol. 2, no. 1, pp. 198-205, Mar. 2012.
- [14] A. Ghavami, K. Kar, S. Bhattacharya, A. Gupta, Price-driven charging of plug-in electric vehicles: Nash equilibrium and best-response strategy convergence, in *Proc. 47th Annual Conference on Information Sciences and Systems (CISS)*, Baltimore, MD, Mar. 2013.
- [15] O. Beaude, S. Lasaulce and M. Hennebel, Charging games in networks of electrical vehicles, in *Proc. 6th Intl. Conference on Network Games, Control and Optimization (NetGCoop)*, Avignon, France, Nov. 2012.
- [16] P. Maillé and B. Tuffin, Multi-bid auctions for bandwidth allocation in communication networks, in *Proc. IEEE INFOCOM*, Mar. 2004.
- [17] P. Maillé and B. Tuffin, Pricing the internet with multibid auctions, *IEEE/ACM Transactions on Networking*, vol. 14, no. 5, p. 992-1004, Oct. 2006.
- [18] A. A. Lazar and N. Semret, The PSP auction mechanism for network resource sharing, in *Proc. 8th International Symposium on Dynamic Games and Applications*, Maastricht, Netherlands, July 1998.
- [19] R. Jain and J. Walrand, An efficient Nash-implementation mechanism for network resource allocation, *Automatica*, vol. 46, no. 8, pp. 1276-1283, Oct. 2010.
- [20] R. Johari and J. N. Tsitsiklis, Efficiency of scalar-parameterized mechanisms, *Operations Research*, vol. 57, no. 4, pp. 823-839, July-Aug. 2009.
- [21] S. Yang and B. Hajek, VCG-Kelly mechanisms for allocation of divisible goods: adapting VCG mechanisms to one-dimensional signals, *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 6, pp. 1237-1243, Aug. 2007.
- [22] C. W. Qu, P. Jia and P. E. Caines, Analysis of a class of decentralized decision processes: quantized progressive second price auctions, in *Proc. 46th IEEE Conference on Decision and Control (CDC)*, New Orleans, LA, Dec. 2007.
- [23] M. Bitsaki, G. Stamoulis, and C. Courcoubetis, A new strategy for bidding in the network-wide progressive second price auction for bandwidth, in *Proc. CoNEXT '05*, Toulouse, France, Oct. 2005.
- [24] S. Zou, Z. Ma and X. Liu, Efficient coordinations of large-scale elastic loads with MCP and PSP auction mechanisms, in *Proc. 32nd Chinese Control Conference*, Xi'an, China, July 2013.
- [25] S. Zou, Z. Ma and X. Liu, Distributed efficient charging coordinations for electric vehicles under progressive second price auction mechanism, in *Proc. 52nd IEEE Conference on Decision and Control (CDC)*, Florence, Italy, Dec. 2013.
- [26] V. F. Dem'yanov and L. V. Vasil'ev, Nondifferentiable Optimization (Chapter 4), *Springer-Verlag*, 1985.
- [27] Pricing Data. [Online]. Available: http://www.nyiso.com/public/markets_operations/market_data/pricing_data/index.jsp
- [28] S. Shao, T. Zhang, M. Pipattanasomporn and S. Rahman, “Impact of TOU rates on distribution load shapes in a smart grid with PHEV penetration,” in *Proc. 2010 IEEE PES Transmission and Distribution Conf. and Expo.*, New Orleans, LA, Apr. 2010.