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## 2D Image Features

Two dimensional image features are interesting local structures. They include junctions of different types like 'Y', 'T', 'X', and 'L'. Much of the work on 2D features focuses on junction 'L', aks, *corners*.

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## Corner

Corners are the intersections of two edges of sufficiently different orientations.

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## Corner Detection

- Corners are important two dimensional features.
- They can concisely represent object shapes, therefore playing an important role in matching, pattern recognition, robotics, and mensuration.

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## Previous Research

- Corner detection from the underlying gray scale images.
- Corner detection from binary edge images (digital arcs).

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## Corner Detection from Gray Scale Image I

Corners are located in the region with large intensity variations in every direction.

Let $I_c$ and $I_r$ be image gradients in horizontal and vertical directions, we can defined a matrix $C$ as

- in two directions

$$C = \begin{bmatrix} \sum I_c^2 & \sum I_c I_r \\ \sum I_c I_r & \sum I_r^2 \end{bmatrix}$$

where the sums are taken over a small neighborhood

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Compute the eigenvalue of $C$, $\lambda_1$ and $\lambda_2$. If the minimum of the two eigen values is larger than a threshold, the the point is declared as a corner. It is good for detecting corners with orthogonal edges.

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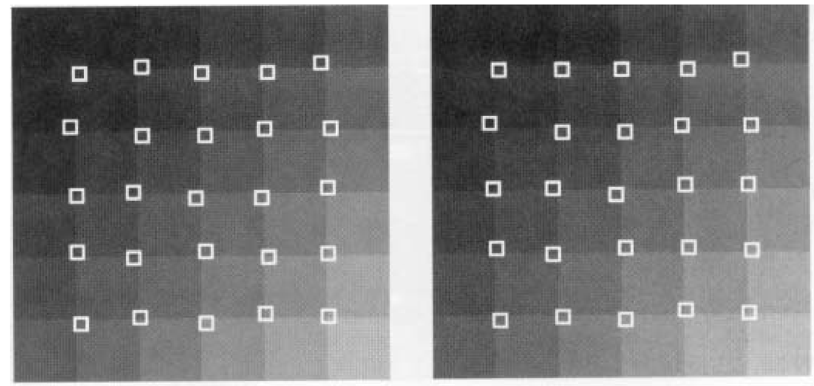

(a) (b)

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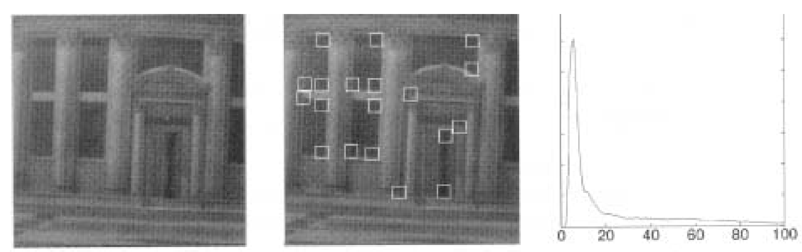


(a) (b) (c)

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- in four directions

  compute local autocorrelation in four directions and take the local lowest result as the measure of interest. Image response at each point is thresholded to determine whether the point is a corner or not.

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**Corner Detection from Gray Scale Image II**

Assume corners are formed by two edges of sufficiently different orientations, in a $N \times N$ neighborhood, compute the direction of each point and then construct a histogram of edgel orientations. A pixel point is declared as a corner if two distinctive peaks are observed in the histogram.

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**Corner Detection from Gray Scale Image II**

Fit the image intensities of a small neighborhood with a local quadratic or cubic facet surface. Look for saddle points by calculating image Gaussian curvature (the product of two principle curvatures, see appendix A 5).

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**Corner Detection from Gray Scale Image II**

Saddle points are points with zero gradient, and a max in one direction but a min in the other. Different methods are used to detect saddle points.

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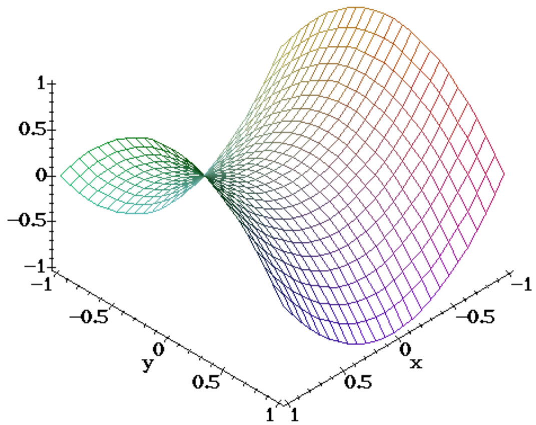


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## Corner Detection from Digital Arcs

- Objective

  Given a list of connected edge points (a digital arc) resulted from an edge detection, identify arc points that partition the digital arc into maximum arc subsequences.

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## Corner Detection from Digital Arcs

- Criteria

  maximum curvature.

  deflection angle.

  maximum deviation.

  total fitting errors.

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## A Statistical Approach

- Problem Statement

  Given an arc sequence $S = \{ \begin{pmatrix} \hat{x}_n \\ \hat{y}_n \end{pmatrix} | n = 1, \ldots N\}$, statistically determine the arc points along $S$ that are most likely corner points.

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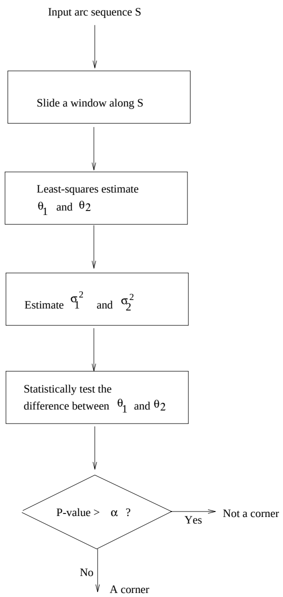
## Approach Overview

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## Approach Overview (cont'd)

- Slide a window along the input arc sequence S.
- Estimate $\hat{\theta}_1$ and $\hat{\theta}_2$, the orientations of the two arc subsequences located to the right and left of the window center, via least-squares line fitting.
- Analytically compute $\sigma_1^2$ and $\sigma_2^2$, the variances of $\hat{\theta}_1$ and $\hat{\theta}_2$.
- Perform a hypothesis test to statistically test the difference between $\hat{\theta}_1$ and $\hat{\theta}_2$.

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## Details of the Proposed Approach

- Noise and Corner Models
- Covariance Propagation
- Hypothesis Testing

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## Noise Model

Given an observed arc sequence $S = \{(\hat{x}_n \ \hat{y}_n)^t | n = 1, \ldots N\}$, it is assumed that $(\hat{x}_n, \hat{y}_n)$ result from random perturbations to the ideal points $(x_n, y_n)$, lying on a line $x_n \cos\theta + y_n \sin\theta - \rho = 0$, through the following noise model:

$$\begin{pmatrix} \hat{x_n} \\ \hat{y_n} \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \end{pmatrix} + \xi_n \begin{pmatrix} \cos\theta \\ sin\theta \end{pmatrix} ; n = 1, \ldots, N$$

where $\xi_n$ are iid as N(0, $\sigma^2$).

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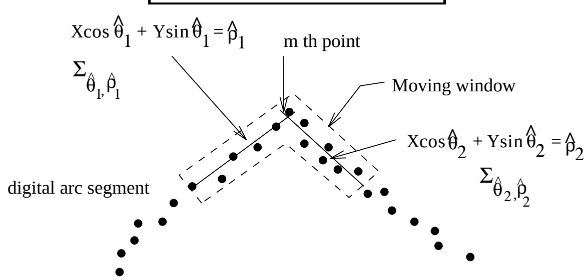
## Corner Model




$$H_0 : \theta_{12} < \theta_0 \qquad H_1 : \theta_{12} \geq \theta_0$$

where $\hat{\theta}_{12} = |\hat{\theta}_1 - \hat{\theta}_2|$, $\theta_{12}$ is the population mean of

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$\hat{\theta}_{12}$, and $\theta_0$ a threshold.

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## Covariance Propagation

- Problem statement

  Analytically estimate $\Sigma_{\Delta\Theta}$, the covariance matrix of least-squares estimate $\hat{\Theta} = (\hat{\theta} \ \hat{\rho})^t$, in terms of the input covariance matrix $\Sigma_{\Delta X}$.

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## Covariance Propagation (cont.)

From Haralick's covariance propagation theory, define

$$F(\hat{\Theta}, \hat{X}) = \sum_{n=1}^{N} (\hat{x}_n cos\hat{\theta} + \hat{y}_n \sin\hat{\theta} - \hat{\rho})^2$$

and

$$g^{2\times 1}(\Theta, X) = \frac{\partial F}{\partial \Theta}$$

then

$$\sum_{\triangle\Theta} = (\frac{\partial g(X,\Theta)}{\partial \Theta})^{-1} (\frac{\partial g(X,\Theta)}{\partial X})^t \sum_{\triangle X}$$

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$$(\frac{\partial g(X,\Theta)}{\partial X})[(\frac{\partial g(X,\Theta)}{\partial \Theta})^{-1}]^t$$

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## Covariance Propagation (cont.)

Define

$$k = \begin{cases} +\sqrt{x^2 + y^2 - \rho^2} & \text{if } y\cos\theta \geq x\sin\theta \\ -\sqrt{x^2 + y^2 - \rho^2} & \text{otherwise} \end{cases}$$

and

$$\mu_k = \frac{1}{N}\sum_{n=1}^{N} k_n$$

$$\sigma_k^2 = \sum_{n=1}^{N} (k_n - \mu_k)^2$$

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## Geometric Interpretation of $k$

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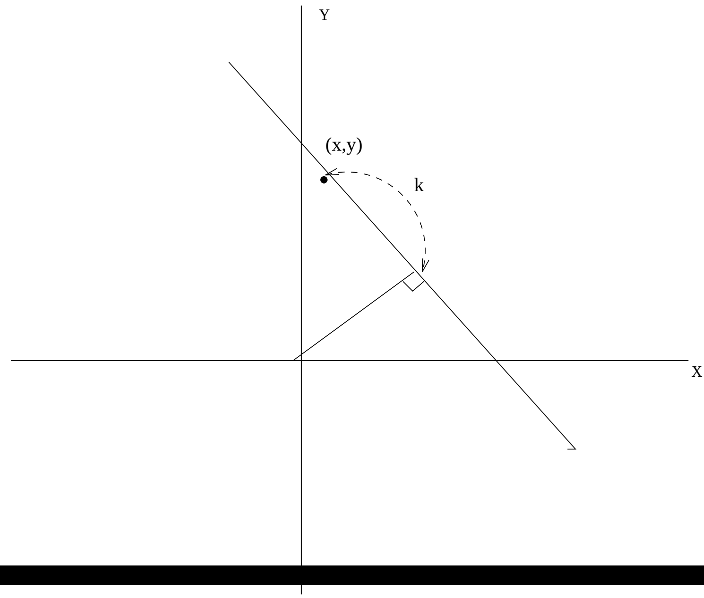


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## Covariance Propagation (cont.)

$$\sum_{\Delta\Theta} = \sigma^2 \begin{pmatrix} \frac{1}{\sigma_k^2} & \frac{\mu_k}{\sigma_k^2} \\ \frac{\mu_k}{\sigma_k^2} & \frac{1}{N} + \frac{\mu_k^2}{\sigma_k^2} \end{pmatrix}$$

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## Hypothesis Testing

$$H_0 : \theta_{12} < \theta_0 \qquad H_1 : \theta_{12} \geq \theta_0$$

where $\theta_0$ is an angular threshold and $\theta_{12}$ is the population mean of RV $\hat{\theta}_{12} = |\hat{\theta}_1 - \hat{\theta}_2|$.
Let

$$T = \frac{\hat{\theta}_{12}^2}{\hat{\sigma}_{\theta_1}^2 + \hat{\sigma}_{\theta_2}^2}$$

Under null hypothesis

$$T \sim \chi_2^2$$

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if $P(T) < \alpha$, then a corner else not a corner.

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## Corner Detection Example

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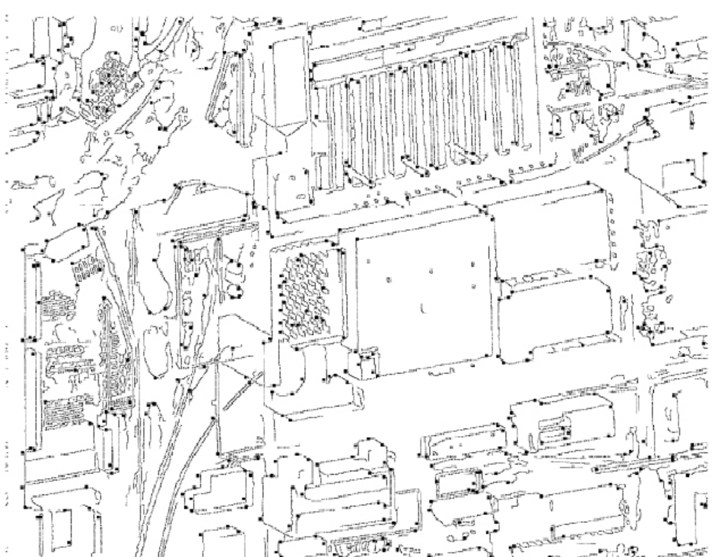

Fig. 2. Extracted edges of building model for a RADIUS image with detected corners (represented by black square dots) overlaid.