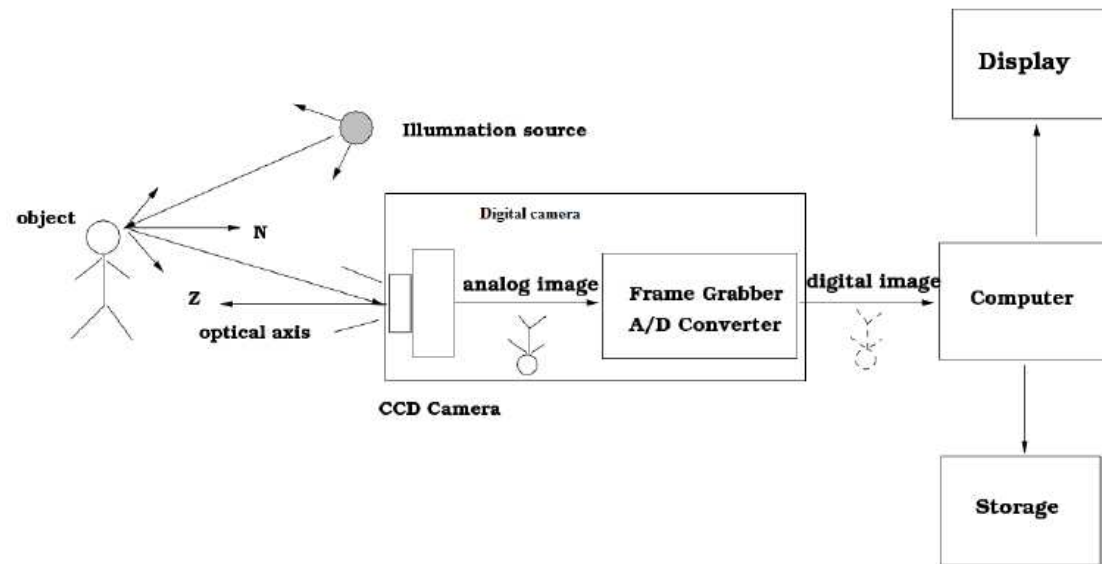


# Image Acquisition and Representation

- how digital images are produced
- how digital images are represented
- photometric models-basic radiometry
- image noises and noise suppression methods

# Image Acquisition Hardware



Note a signal amplifier through an automatic gain control (AGC) is often added before the A/D converter to boost the analog image

## Camera

- First photograph was due to Niepce of France in 1827.
- Employ either the photochemical (film) or photoelectric principles (analog/digital)
- Basic abstraction is the pinhole camera
  - lenses required to ensure image is not too dark
  - , to focus the light, and to increase image size
  - various other abstractions can be applied

# CCD Camera

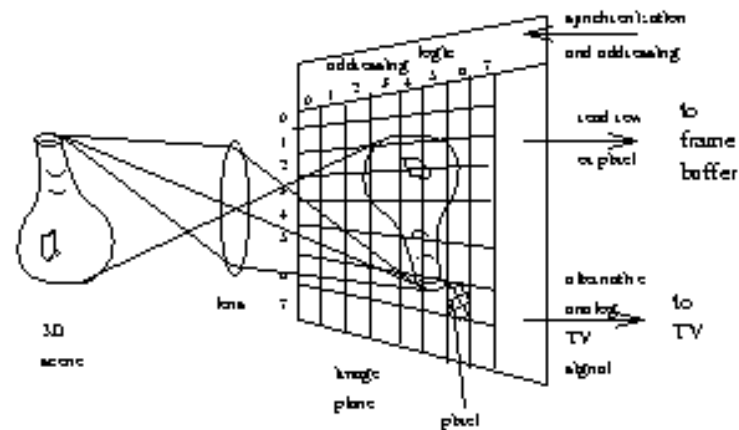


Figure 2.2: A CCD (digital) camera imaging a vase; discrete cells convert light energy into electrical charges, which are represented as small numbers when input to a computer.

CCD (Charged Couple Device) camera consists of a lens and an image plane (chip array) containing tiny photoelectric cells that convert light energy into

electrical charge (electrons) when a cell receives enough photons. The output is analog image. The key camera parameters include

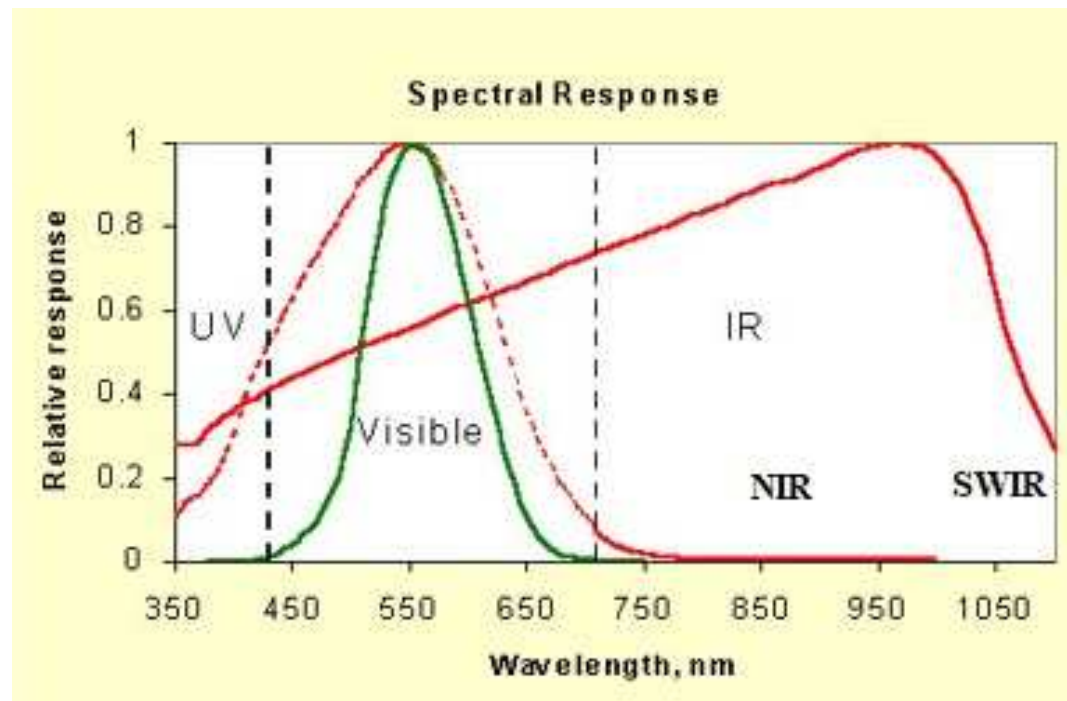
- cell size (e.g.,  $16.6 \times 12.4\mu m$ , aspect ratio=4:3, not square)
- number of cells (e.g.  $512 \times 512$ , also referred to as camera resolution, i.e., the number of cells horizontally and vertically).
- image plane geometries: rectangle, circular, or linear.
- Spectral response (28%(450nm), 45%(550nm),

62%(650nm) )

visible light: 390-750 nm, IR light 750 nm and higher

- Aperture- control the amount of light entering into the camera

# Spectrum Response



IR ranges: Near infrared (NIR) from 780 nm to 1400 nm, shortwave infrared (SWIR) from 1400 nm to 3000 nm, middlewave infrared (MWIR) from 3000 nm to 5000 nm, and longwave infrared (LWIR) from 8000 nm to 14000 nm. They have different applications.

# CCD array geometries

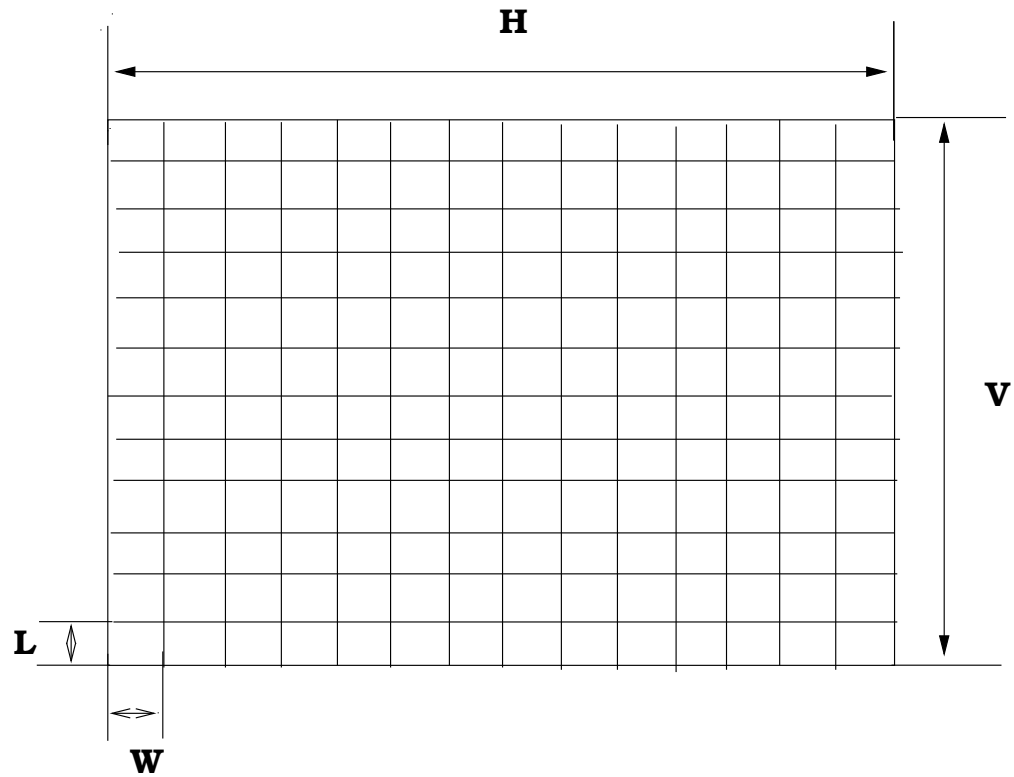
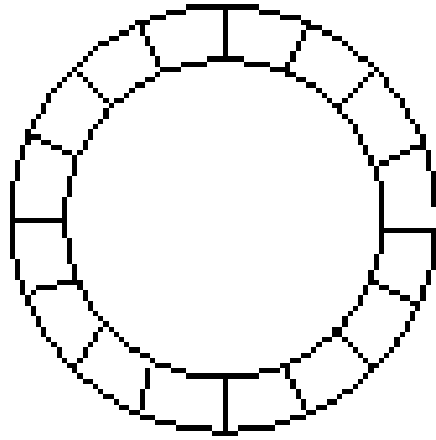


Figure 1: CCD camera image plane layout



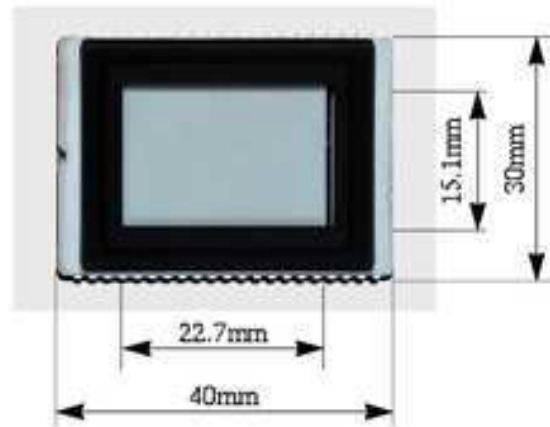
## Other CCD array geometries



Usually,  $H \times W/V \times L=4:3$ . This aspect ratio is more suitable for human viewing. For machine vision, aspect ratio of 1:1 is preferred.

## CMOS Camera

A CMOS (Complementary Metal Oxide Silicon) camera is an alternative image sensor.



It follows the same principle as CCD by converting photons into electrical changes. But it uses different technologies in converting and transporting the electrical charges. Compared to CCD, it is

faster, smaller, cheaper, consumes less power but its light sensitivity is lower and its image is more noisy. Mainly for low-end consumer applications.

## Single Photon Camera

The latest development is the single photon camera, a new camera that is sensitive enough to detect a single photon. It has single-photon sensitivity in the visible and near infrared (400 nm - 850 nm) wavelength range.  
<https://spectrum.ieee.org/single-photon-camera>

## Analog Image

An analog image is a 2D image  $F(x, y)$  which has infinite precision in spatial parameters  $x$  and  $y$  and infinite precision in intensity at each point  $(x, y)$ .

## Frame Grabber

An A/D converter that spatially **samples** the camera image plane and **quantizes** the voltage of into a numerical intensity value.

- Sample frequency (sampling interval) v. image resolution through spatial sampling
- Range of intensity value through amplitude quantization
- On-board memory and processing capabilities

## Spatial sampling process

Let  $(x,y)$  and  $(c,r)$  be the image coordinates before and after sampling. Spatial sampling converts  $(x,y)$  to  $(c,r)$

$$\begin{pmatrix} c \\ r \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (1)$$

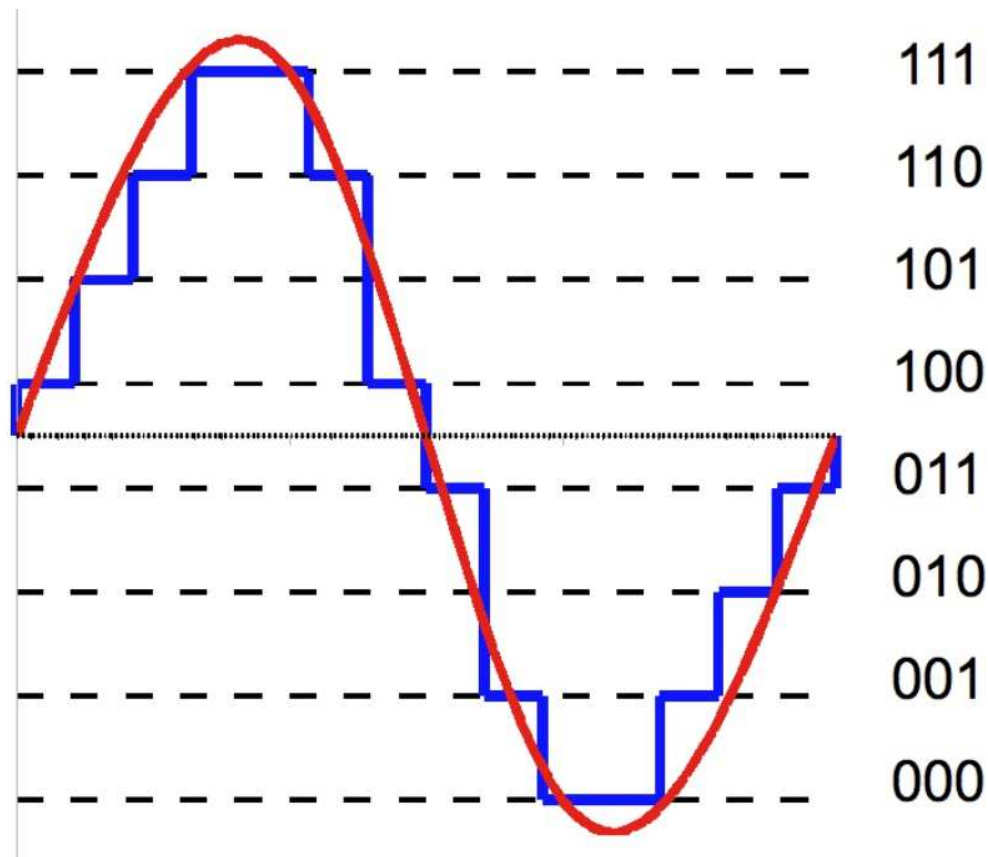
where  $s_x$  and  $s_y$  are sampling frequency (*pixels/mm*) due to spatial quantization. They are referred to as Pixels Per Inch (PPI) or scale factors. A high quality image with good details usually involves 300 PPI. The sampling frequency determines the image resolution. The higher sampling frequency, the higher image

resolution. But the image resolution is limited by camera resolution. Oversampling by the frame grabber requires interpolation and does not necessarily improve image perception.



## Amplitude Quantization

Amplitude quantization converts the magnitude of the signal  $F(x,y)$  to produce pixel intensity  $I(c,r)$ . The  $I(c,r)$  is obtained by dividing the range of  $F(x,y)$  into intervals and representing each interval with an integer number.



The number of intervals to represent  $I(c,r)$  is determined by the number of bits allocated to represent  $F(x,y)$ . For example, if 8-bit is used, then  $F(x,y)$  can be

divided into 256 intervals with the first interval represented by 0 and the last interval represented by 255.  $I(c,r)$  therefore ranges from 0 to 255.

## Digital Camera

Combines analog camera and digitizer into one system.  
It directly outputs digital images.

## Computer

Computer (including CPU and monitor): used to access images stored in the frame grabber, process them, and display the results on a monitor

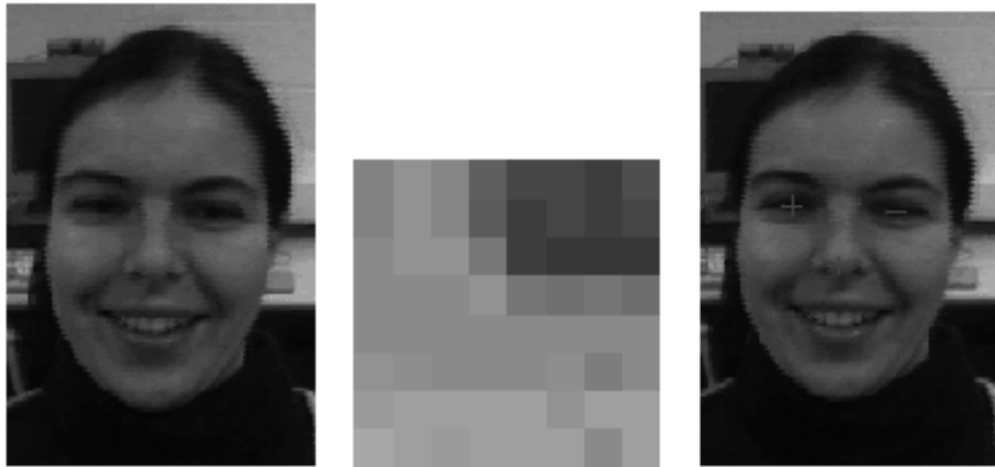
## Digital Image

The result of digitization of an analog image  $F(x,y)$  is a digital image  $I(c,r)$ .  $I(c,r)$  represented by a discrete 2D array of intensity samples, each of which is represented using a limited precision determined by the number of bits for each pixel.

## Digital Image (cont'd)

- Image resolution ( $H \times W$ )
- Intensity range  $[0, 2^N-1]$
- Color image (RGB)

# Digital Representation

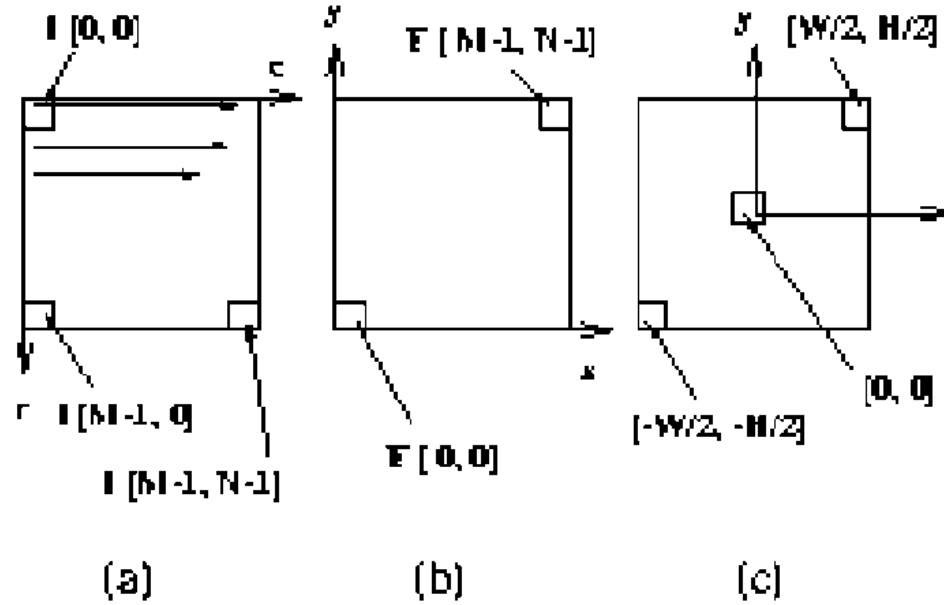


	0	1	2	3	4	5	6	7
0	130	146	133	95	71	71	62	78
1	130	146	133	92	62	71	62	71
2	130	146	146	120	62	55	55	55
3	130	130	130	146	117	112	117	110
4	130	130	130	130	130	130	130	130
5	146	142	130	130	130	143	125	130
6	156	150	150	150	150	146	150	150
7	168	150	156	150	150	150	130	150

Figure 1.1: Image of a face (top left), subimage from the right eye region ( top center ), eye location detected by a computer program ( top right. ), and intensity values from the eye subimage (bottom).

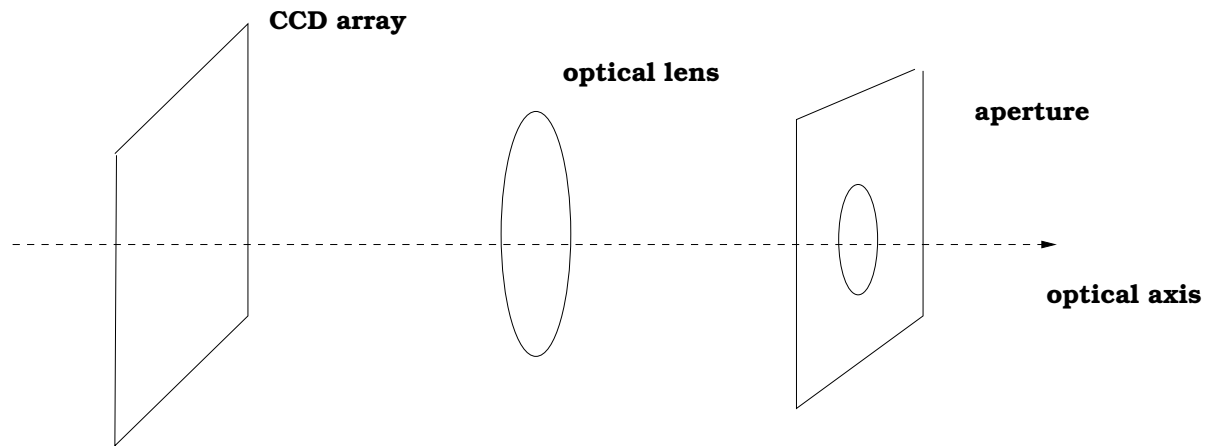


## Different coordinate systems used for images

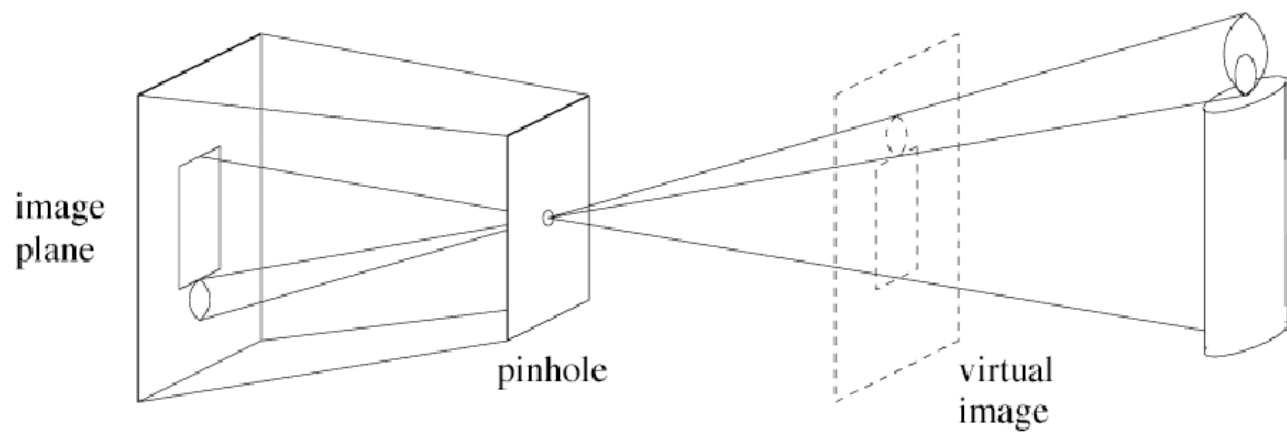


(a) Row-column coordinate system with  $(0,0)$  at the upper-left corner, (b) Cartesian coordinate system with  $(0,0)$  at the lower left corner, and (c) Cartesian coordinate system with  $(0,0)$  at the center.

## Basic Optics: Pinhole model

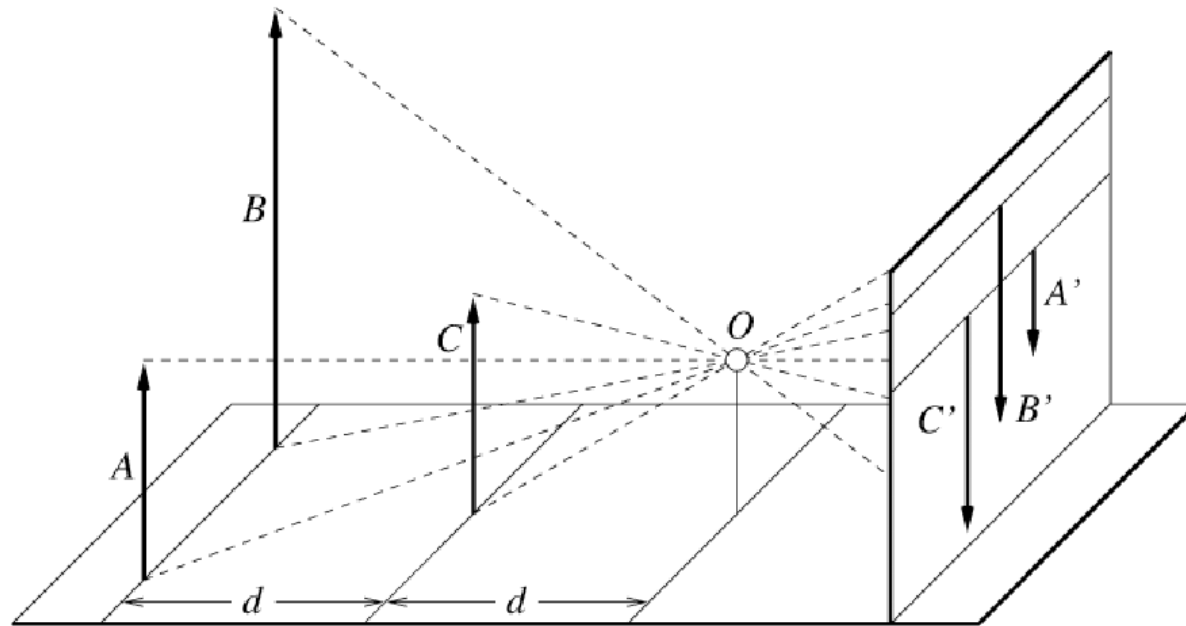


Reducing the camera's aperture to a point so that one ray from any given 3D point can enter the camera and create a one-to-one correspondence between visible 3D points and image points.



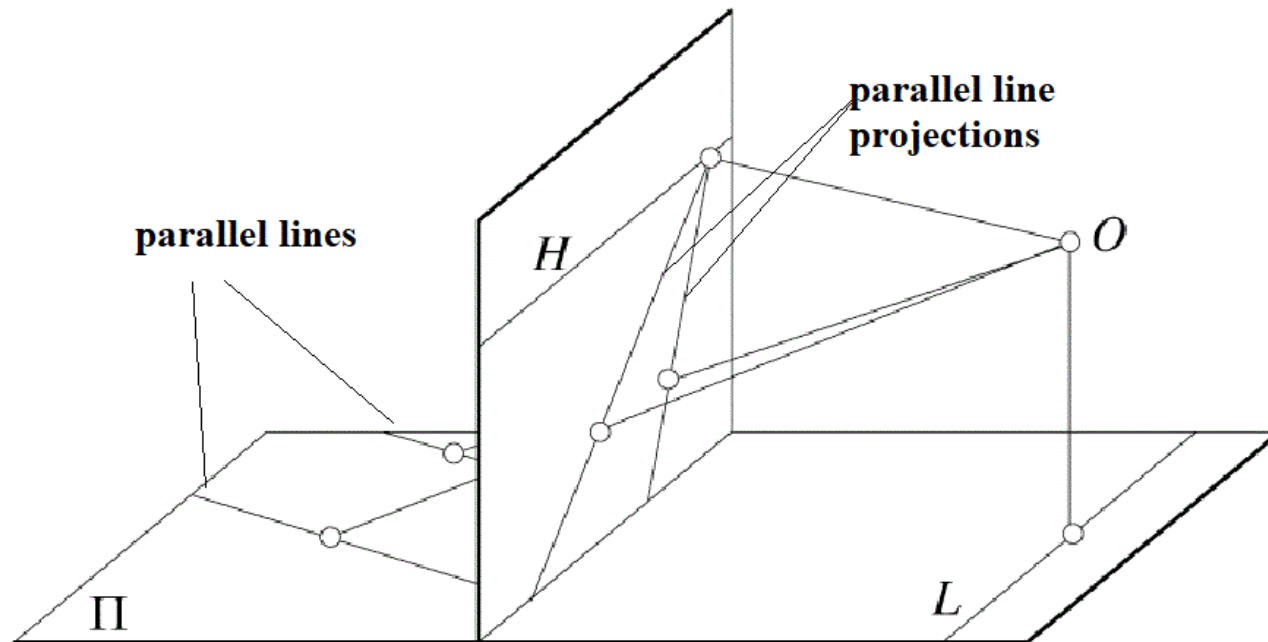
## Pinhole model (cont'd)

Distant objects are smaller due to perspective projection. Larger objects appear larger in the image.



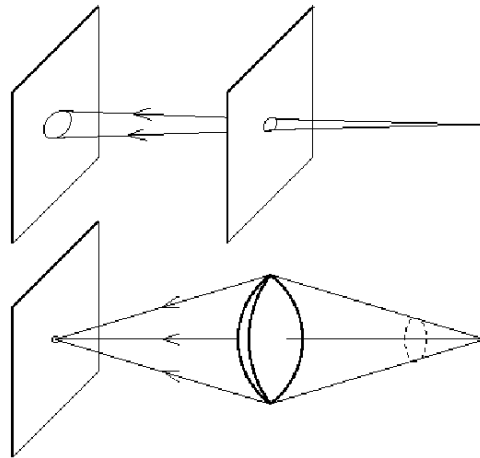
## Pinhole model (cont'd)

Parallel lines meet at horizon, where line  $H$  is formed by the intersection of the plane parallel to the lines and passing through  $V$ , which is referred as vanishing point.



## Camera Lens

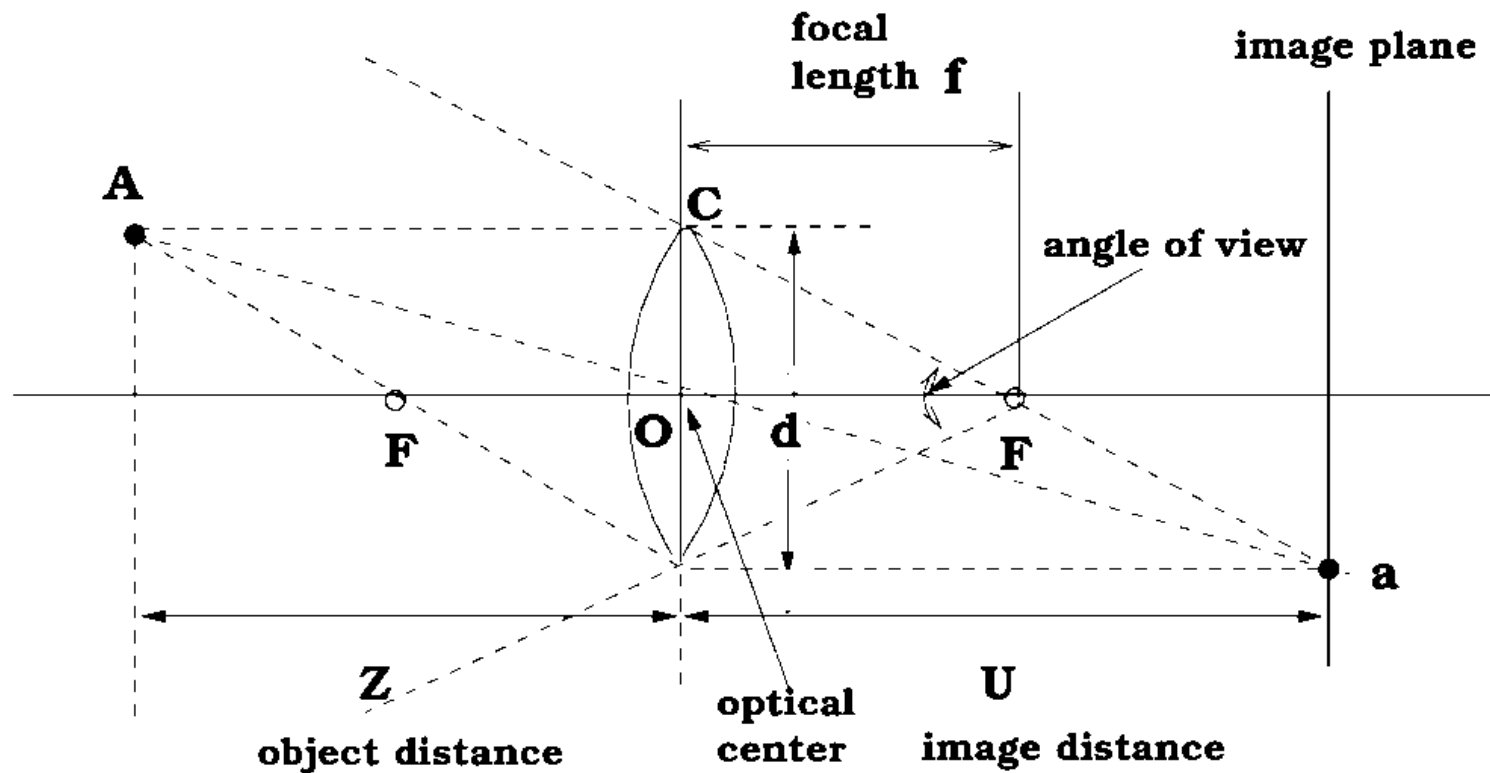
Lens may be used to focus light so that objects may be viewed brighter. Lens can also increase the size of the objects so that objects in the distance can appear larger.



Without lens in the top figure and with lens in the bottom figure

## Basic Optics: Lens Parameters

Lens parameters: focal length ( $f$ ) and effective diameter ( $d$ ), angle of view, light refraction



## Fundamental equation of thin Lens

$$\frac{1}{Z} + \frac{1}{U} = \frac{1}{f}$$

It is clear that increasing the object distance, while keeping the same focus length, reduces image size. Keeping the object distance, while increasing the focus length, increases the image size.

U is the image plane focusing distance, i.e., where image is focused. As the object distance increases to infinity, i.e.,  $Z \rightarrow \infty$ , the image focusing distance  $U=f$ , i.e., objects at infinity stay focused when image plane is



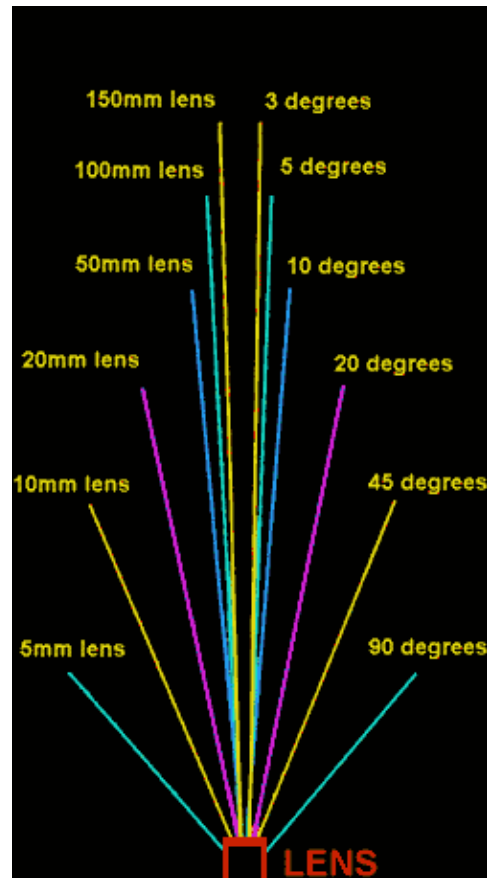
located at focus point.  $U$  is close to  $f$  for object farther away.

## Angle (Field) of View (AOV)

Angular measure of the portion of 3D space actually seen by the camera. It is defined as

$$\omega = 2 \arctan \frac{d}{2f}$$

AOV is inversely proportional to focal length and proportional to lens size. Larger lens or smaller focal length give larger AOV.

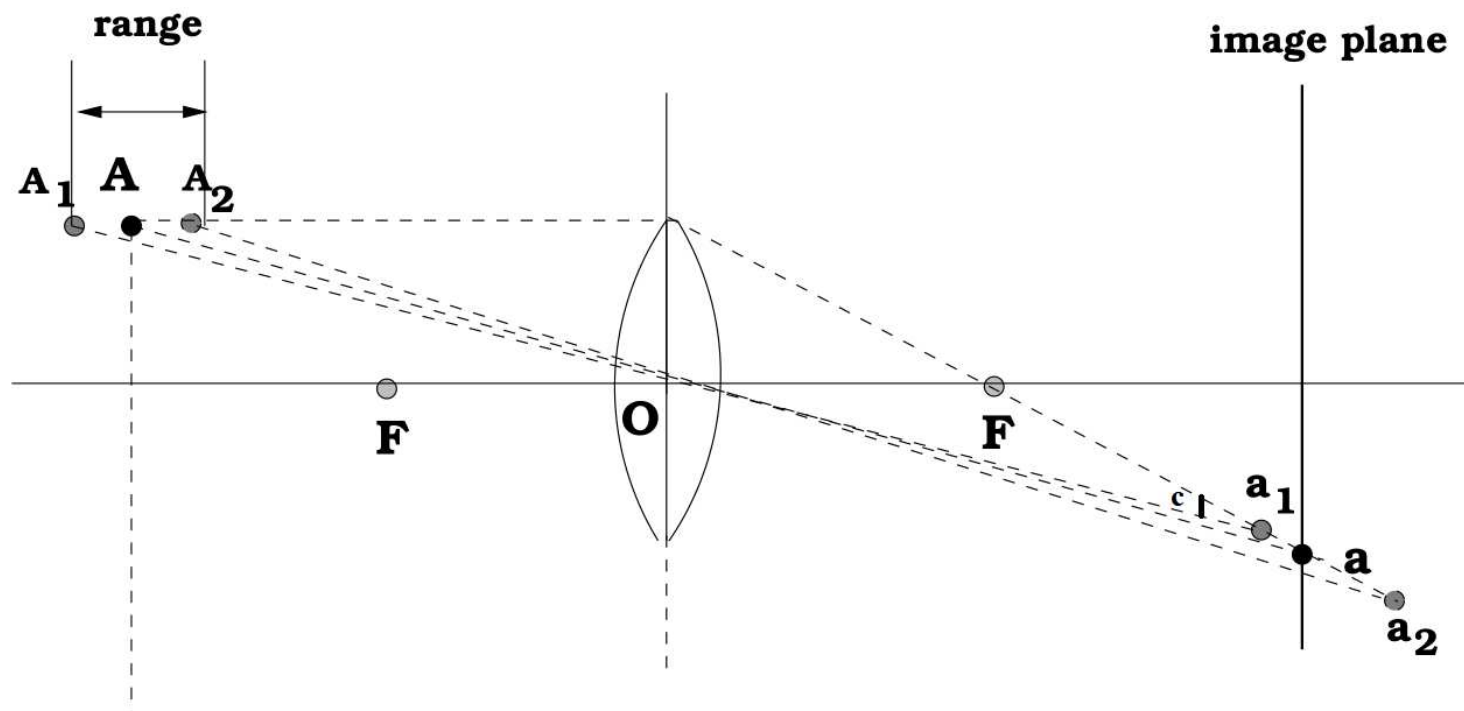


$\frac{f}{d}$  is called F-number. AOV is inversely proportional to F-number.

Similar to AOV, Field of View (FOV) determines the portion of an object that is observable in the image. But different from AOV, which is a camera intrinsic parameter and is a function of only lens of parameters, FOV is a camera extrinsic parameter that depend both on lens parameters and object parameters. In fact, FOV is determined by focus length, lens size, object size, and object distance to the camera.

# Depth of Field

The allowable distance range such that all points within the range are acceptably in focus in the image, which is inversely proportional to the circle of confusion radius  $c$ .



Depth of field is inversely proportional to focus length, proportional to shooting distance, and inversely proportional to the aperture (especially for close-up or with zoom lens).



**Small DoF**



**Large DoF**

See more at

<http://www.azuswebworks.com/photography/dof.html>

Since “acceptably in focus” is subjective, as the focus length increases or shooting distance decreases (both make the picture more clear and larger), the tolerance in picture blurriness also decreases, hence a reduction in depth of field.

## Camera and Lens Parameter Summary

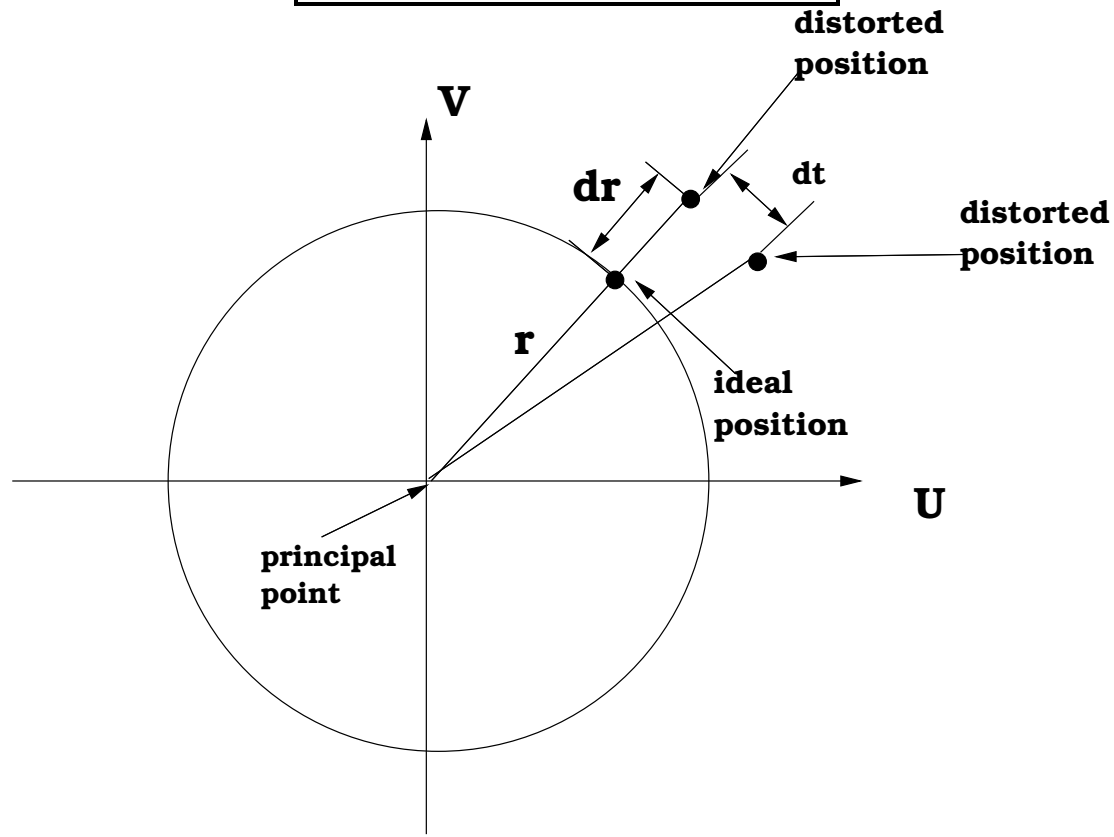
- Camera resolution
- Camera spectral response
- Aperture
- Image resolution
- Lens focus length ( $f$ )
- Lens diameter ( $d$ )
- Angle of view
- Field of view
- Depth of field



## Other Lens Parameters

- fixed focal length v. Zoom lens
- Motorized zoom Lenses—zoom lenses are typically controlled by built-in, variable-speed electric motors. These electric zooms are often referred to as servo-controlled zooms
- Supplementary lens: positive and negative (increase/decrease AOV)
- Digital zoom: a method to digitally change the focus length to focus on certain region of the image typically through interpolation.

# Lens distortion



**dr: radial distortion**  
**dt: tangential distortion**

Tangential distortion is usually small and is often

ignored.

# Effects of Lens Distortion

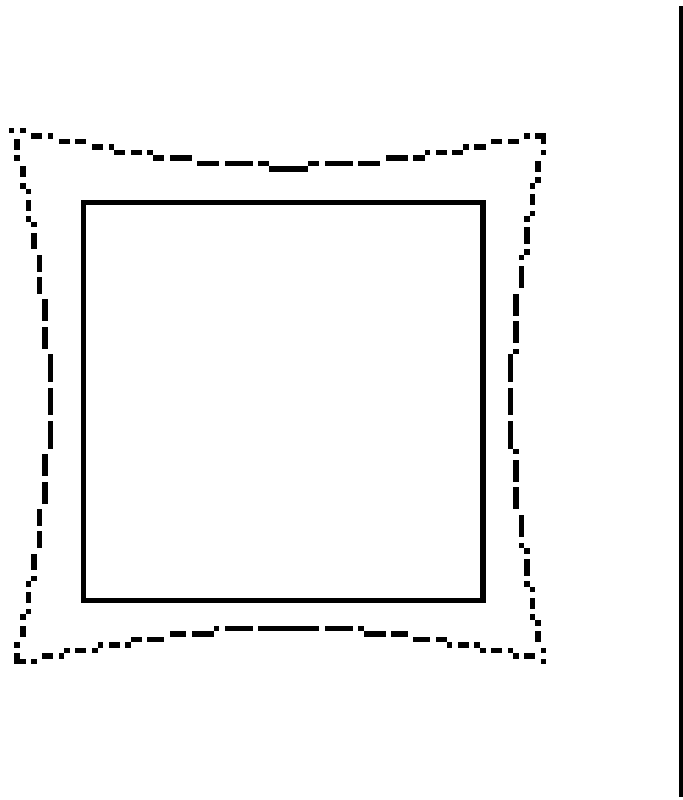
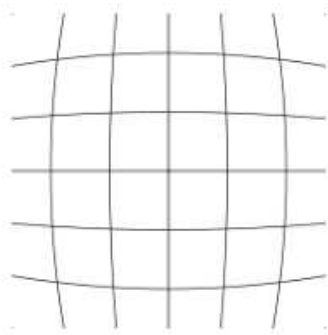


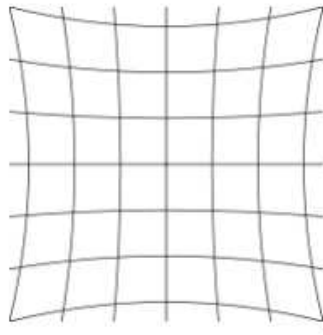
Figure 2: Effect of radial distortion. Solid lines: no distortion; dashed lines with distortion. More distortion far away from the center

## Radial Lens Distortion

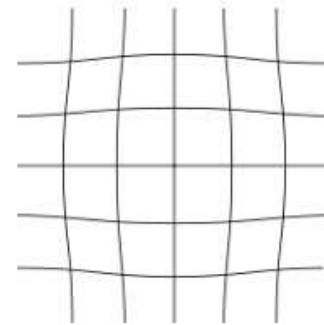
Radial lens distortion can be divided into: barrel distortion ( magnification decreases with distance from image center), pincushion distortion ( magnification increases with distance from the optical axis), and moustache distortion (mixture).



**Barrel distortion**



**Pincushion distortio**



**Mustache distortion**

## Lens Distortion modeling and correction

Radial lens distortion causes image points to be displaced from their proper locations along radial lines from the image center. The distortion can be modeled by

$$u = u_d(1 + k_1 * r^2 + k_2 * r^4)$$

$$v = v_d(1 + k_1 * r^2 + k_2 * r^4)$$

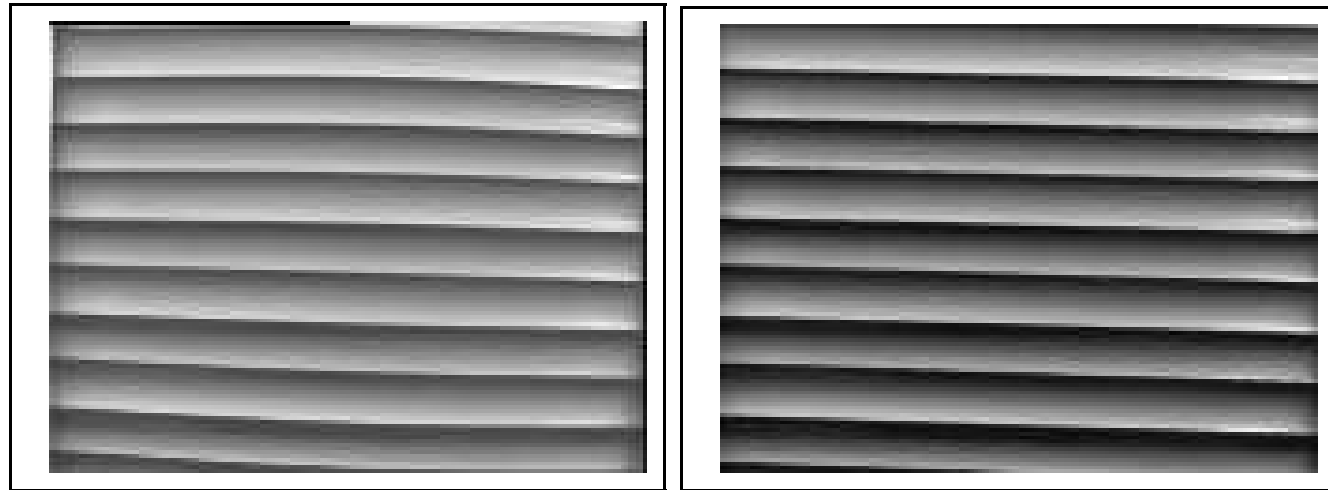
where  $r = \sqrt{(u - u_0)^2 + (v - v_0)^2}$ ,  $(u, v)$  is the ideal and unobserved image coordinates relative to the (U,V) image frame,  $(u_d, v_d)$  is the observed and distorted image coordinates,  $(u_0, v_0)$  is the center of the image,  $k_1$  and  $k_2$

are coefficients.  $k_2$  is often very small and can be ignored.

The geometric knowledge of 3D structure (e.g. collinear or coplanar points, parallel lines, angles, and distances) is often used

to solve for the distortion coefficients. Refer to Wikipedia [https://en.wikipedia.org/wiki/Distortion\\_\(optics\)#Radial\\_distortion](https://en.wikipedia.org/wiki/Distortion_(optics)#Radial_distortion) for lens calibration using parallel lines.





(a)

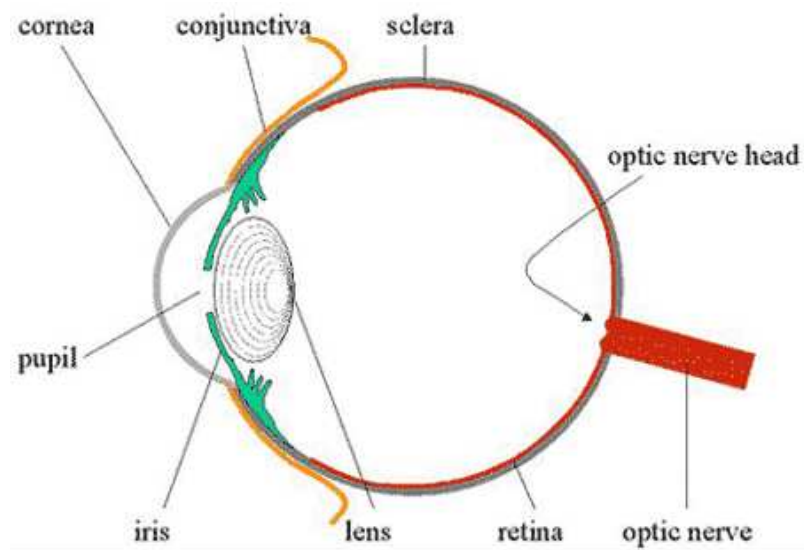
(b)

Figure 3: Radial lens distortion before (a) and after (b) correction

Commercial graphics software such as Photoshop and Corel include functions to correct the lens distortion.

With the modern optics technology and for most computer vision applications, both types of geometric lens distortions are often negligible.

## Structure of Eye



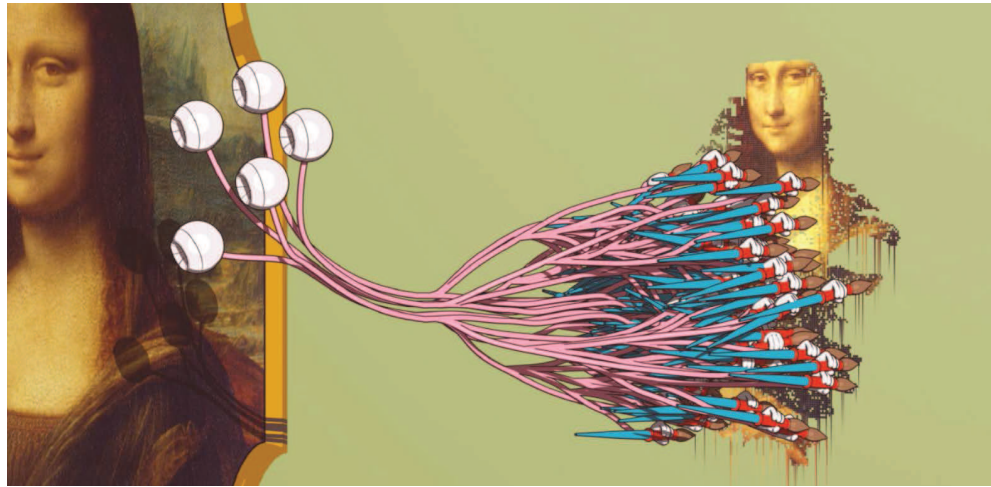
- cornea-the front and the transparent part of the coat of the eyeball that reflects and refracts the incoming light
- pupil-the opening in the center of iris that controls

the amount of light entering into the eyes

- iris-the colored tiny muscles that surround the pupil. It controls the opening and closing of the pupil
- lens-the crystalline lens located just behind the iris. its purpose is to focus the light on retina.
- retina-the sensory photo-electric sensitive tissue at the back of the eye. It captures light and converts it to electrical impulses.
- optic nerve-the optic nerve transmits electrical impulses from the retina to the visual cortex, where image of the world is formed.

## Sparse Connections Between Retina and Visual Cortex

The latest study below shows that retina is only sparsely connected to the visual cortex. This suggests 1) retina selectively passes only important signals to visual cortex, and 2) the visual cortex reconstructs the image of the world primarily from the prior knowledge it already has, coupled with sparse observational signals from the retina.



See the article *A Mathematical Model Unlocks the Secrets of Vision* at

<https://www.quantamagazine.org/a-mathematical-model-unlocks-the-secrets-of-vision-20190821/>

## Bionic Eye

The question is if it is possible to produce (simulate) the electrical impulses by other means (e.g. through hearing or other sensing channels) and send the signals to the brain as if they were from the eyes.

Yes, this is can be done!. Research about bionic eyes is doing this. See the video at

<http://www.youtube.com/watch?v=696dxY6BYBM>

Moreover, Harvard researchers developed a tunable metalens that can change the focus in bionic eye real-time to automatically correct for common aberrations in human vision.

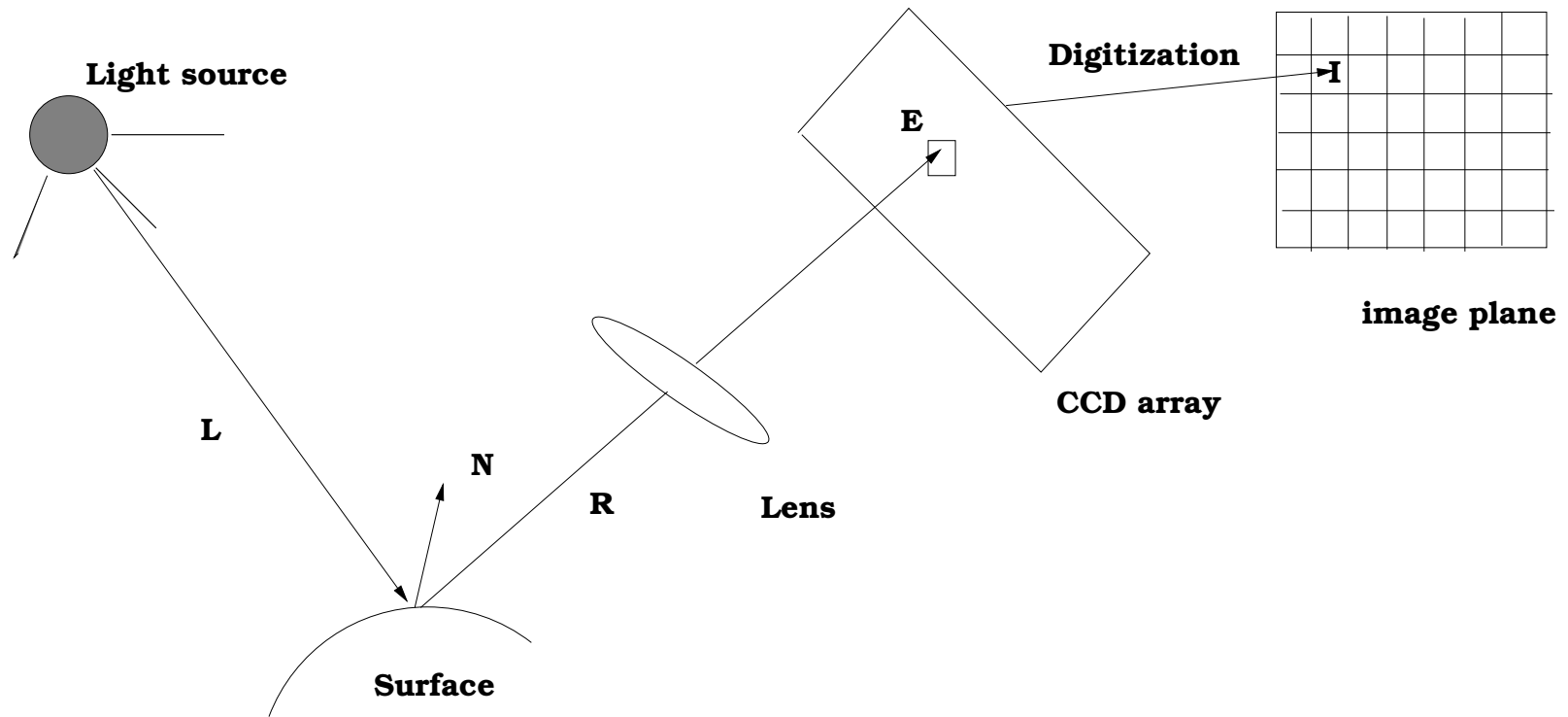
Under the Bionic Eye: Harvard Develops Metalens that  
Can Autofix Common Vision Problems

<https://interestingengineering.com/under-the-bionic-eye-harvard-develops-metalens-that-can-autofix-common-vision-problems>



# Basic Radiometry

We introduce the basic photometric image model.



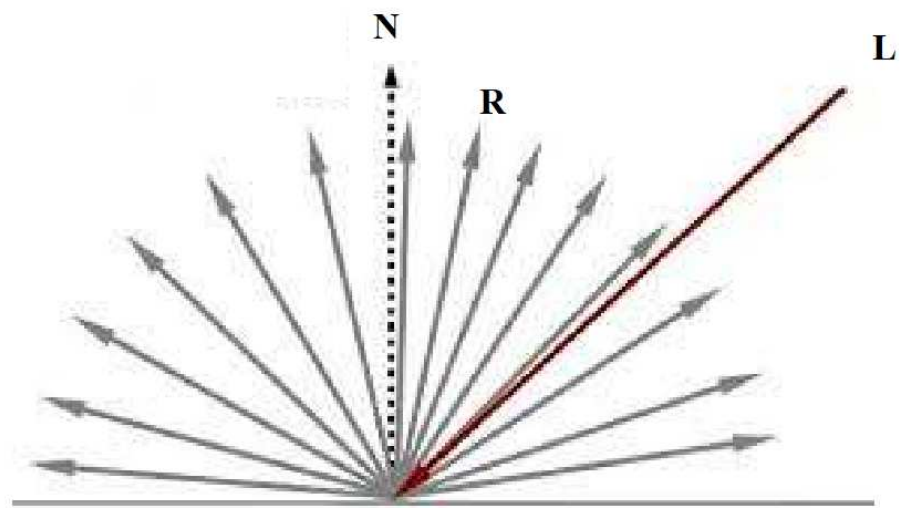
- Illumination vector **L**

- Scene radiance  $\mathbf{R}$ : is the power of the light, per unit area, ideally emitted by a 3D point
- Image irradiance  $\mathbf{E}$ : the power of the light per unit area a CCD array element receives from the 3D point
- Image intensity  $\mathbf{I}$ : the intensity of the corresponding image point

## Lambertian Surface Reflectance Model

$$R = \rho \mathbf{L} \cdot \mathbf{N}$$

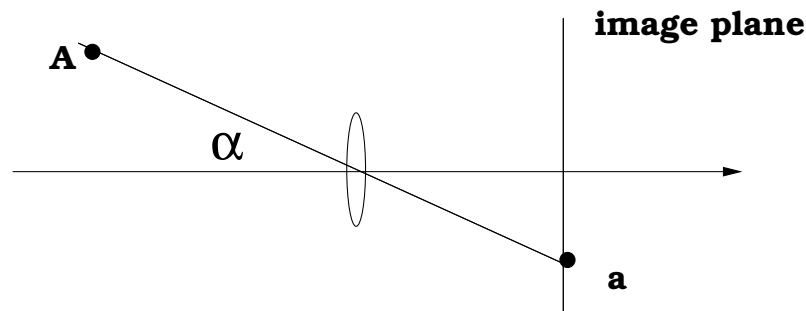
where  $\mathbf{L}$  represents the incident light,  $\mathbf{N}$  surface normal, and  $\rho$  surface albedo. The object looks equally bright from all view directions.



## Surface Radiance and Image Irradiance

The fundamental radiometric equation:

$$E = R \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cos^4 \alpha$$



For small angular aperture (pin-hole) or object far from camera,  $\alpha$  is small, the  $\cos^4 \alpha$  can be ignored. The image irradiance is uniformly proportional to scene

radiance. Large  $d$  or small  $F$  number produces more image irradiance and hence brighter image.

## Image Irradiance and Image Intensity

$$I = \beta E$$

where  $\beta$  is a coefficient dependent on camera and frame grabber settings.

## The Fundamental Image Radiometric Equation

$$I = \beta \rho \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4 \alpha L \cdot N$$



## Image Formats

Images are usually stored in computer in different formats. There two image formats: Raster and Vector.

## Raster Format

A Raster image consists of a grid of colored dots called pixels. The number of bits used to represent the gray levels (or colors) denotes the depth of each pixel. Raster files store the location and color of every pixel in the image in a sequential format.

## Raster Formats

There are many different Raster image formats such as TIFF, PGM , JPEG, GIF, and PNG. They all can be organized as follows:

- image header (in ASCII, image size, depth, date, creator, etc..)
- image data (in binary either compressed or uncompressed) arranged in sequential order.

## PGM

PGM stands for Portable Greyscale Map. Its header consists of

P5

number of columns

number of rows

Max intensity (determine the no of bits)

Raw image data (in binary, pixels are arranged sequentially)

P5

640 480

255

## PGM (cont'd)

Some software may add additional information to the header. For example, the PGM header created by XV looks like

P5

# CREATOR: XV Version 3.10a Rev: 12/29/94

320 240

255

## PPM

PPM (Portable PixMap) format is for color image. Use the same format.

P6

640 480

255

raw image data (each pixel consists of 3 bytes data in binary, respectively representing the R, G, and B components of the color image). Besides RGB, color image can also be represented in other color space such as XYZ. Different color spaces are convertible.

## Vector Format

A Vector image is composed of lines, not pixels. Pixel information is not stored; instead, formulas that describe what the graphic looks like are stored. They're actual vectors of data stored in mathematical formats rather than bits of colored dots. Vector format is good for image cropping, scaling, shrinking, and enlarging but is not good for displaying continuous-tone images and for image processing. Common vector formats include portable document format (pdf), postscript (ps), and encapsulated postscript (eps).

## Image noise

- intensity noise
- positional error

Note image noise is the intrinsic property of the camera or sensor, independent of the scene being observed. It may be used to identify the imaging sensors/cameras.



## Intensity Noise Model

Let  $\hat{I}$  be the observed image intensity at an image point and  $I$  be the ideal image intensity, then

$$\hat{I}(c, r) = I(c, r) + \epsilon(c, r)$$

where  $\epsilon$  is white image noise, following a distribution of  $\epsilon \sim \mathcal{N}(0, \sigma^\epsilon(\cdot, \nabla))$ . Note we do not assume each pixel is identically and independently perturbed.

## Estimate $\sigma$ from Multiple Images

Given  $N$  images of the same scene  $\hat{I}_0, \hat{I}_1, \dots, \hat{I}_{N-1}$ , for each pixel  $(c, r)$ ,

$$\bar{I}(c, r) = \frac{1}{N} \sum_{i=0}^{N-1} \hat{I}_i(c, r)$$

$$\sigma^2(c, r) = \frac{1}{N-1} \sum_{i=0}^{N-1} [\hat{I}_i(c, r) - \bar{I}(c, r)]^2$$

see figure 2.11 [1]. Note noise averaging can reduce the noise of  $\bar{I}(c, r)$  to  $\frac{\sigma^2}{N}$ .

## Estimate $\sigma$ from a Single Image

Assume the noise for each pixel in a neighborhood  $R$  is IID distributed, i.e.,

$$\hat{I}(c, r) = I(c, r) + \epsilon$$

where  $(c, r) \in R$ .  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ .  $\sigma^2$  can then be estimated by sample variance of the pixels inside  $R$

$$\begin{aligned}\hat{I}(c, r) &= \frac{\sum_{(c,r) \in R} I(c, r)}{N} \\ \hat{\sigma}^2(c, r) &= \frac{\sum_{(c,r) \in R} (I(c, r) - \hat{I})^2}{N - 1}\end{aligned}\tag{2}$$

where  $N$  is the number pixels in  $R$ .

## Estimate $\sigma$ from a Single Image with Plane Fitting

Let  $\hat{I}(x, y)$  be the observed gray-tone value for pixel located at  $(x, y)$ . If we approximate the image gray-tone values in pixel  $(x, y)$ 's neighborhood by a plane  $\alpha x + \beta y + \gamma$ , then the image perturbation model can be described as

$$\hat{I}(x, y) = \alpha x + \beta y + \gamma + \xi$$

where  $\xi$  represents the image intensity error and follows an iid distribution with  $\xi \sim \mathcal{N}(0, \sigma^2)$ .

For a neighborhood of  $M \times N$ <sup>a</sup>, the sum of squared

---

<sup>a</sup>assume pixel noise in the neighborhood is IID distributed.

residual fitting errors

$$\epsilon^2 = \sum_{y=1}^N \sum_{x=1}^M (\hat{I}(x, y) - \alpha x - \beta y - \gamma)^2$$

follows  $\frac{\epsilon^2}{\sigma^2} \sim \chi_{M*N-2}^2$ .

As a result, we can obtain  $\hat{\sigma}^2$  following the variance of chi-square distribution as

$$\hat{\sigma}^2 = \frac{\epsilon^2}{M \times N - 2}$$

We can compute  $\hat{\sigma}^2$  for each pixel and hence can obtain a distribution of  $\hat{\sigma}^2$  over the entire image. The distribution can be used to characterize the camera noise.

Moreover, let  $\hat{\sigma}_k^2$  be an estimate of  $\sigma^2$  from the k-th

neighborhood. Given a total of  $K$  neighborhoods across the image, we can obtain the average noise level over the image

$$\hat{\sigma}^2 = \frac{1}{K} \sum_{k=1}^K \hat{\sigma}_k^2$$

## Independence Assumption Test

We want to study the validity of the independence assumption among pixel values . To do so, we compute correlation between neighboring pixel intensities. Figure 2.12 of [1] plot the results. We can conclude that neighboring pixel intensities correlate with each other and the independence assumption basically holds for pixels that are far away from each other.



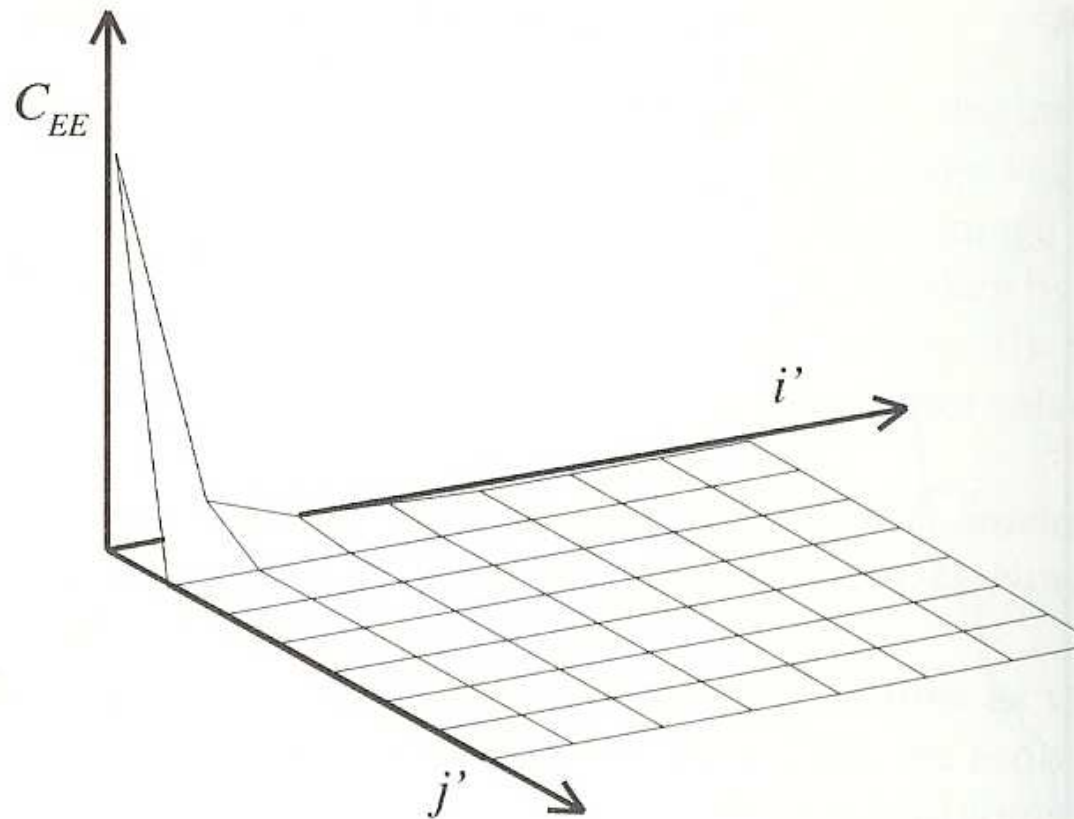


Figure 2.12 Autocovariance of the image of a uniform pattern for a typical image acquisition system, showing cross-talking between adjacent pixels along  $i'$ .

## Consequences of Image Noise

- image degradation
- errors in the subsequent computations e.g., derivatives

## Types of Image Noise

Gaussian Noise and impulsive (salt and pepper) noise.

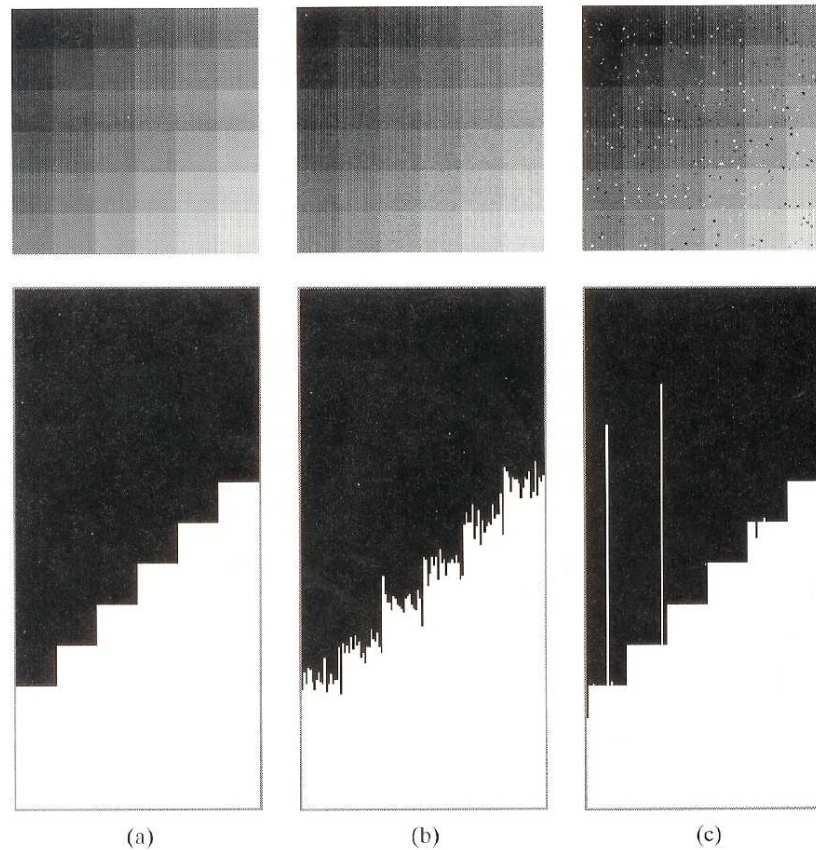


Figure 3.1 (a) Synthetic image of a  $120 \times 120$  grey-level “checkerboard” and grey-level profile along a row. (b) After adding zero-mean Gaussian noise ( $\sigma = 5$ ). (c) After adding salt and pepper noise (see text for parameters).

## Noise Removal

In image processing, intensity noise is attenuated via *filtering*. It is often true that image noise is contained in the high frequency components of an image, a low-pass filter can therefore reduce noise. The disadvantage of using a low-pass filter is that image is blurred in the regions with sharp intensity variations, e.g., near edges.

## Noise Filtering

$$I_f(x, y) = I * F = \sum_{h=-\frac{m}{2}}^{\frac{m}{2}} \sum_{k=-\frac{m}{2}}^{\frac{m}{2}} F(h, k) I(x - h, y - k)$$

where  $m$  is the window size of filter  $F$  and  $*$  indicates discrete **convolution**. The filtering process replaces the intensity of a pixel with a linear combination of neighborhood pixel intensities.

## Noise Filtering (cont'd)

- Filtering by averaging

$$F = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Gaussian filtering

$$g(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2} \left( \frac{x^2 + y^2}{\sigma^2} \right)}$$

window size  $w = 5\sigma$ .

An example of  $5 \times 5$  Gaussian filter

2.2795779e-05	0.00106058409	0.00381453967	0.00106058409	2.2795779e-05
0.00106058409	0.0493441855	0.177473253	0.0493441855	0.00106058409
0.00381453967	0.177473253	0.638307333	0.177473253	0.00381453967
0.00106058409	0.0493441855	0.177473253	0.0493441855	0.00106058409
2.2795779e-05	0.00106058409	0.00381453967	0.00106058409	2.2795779e-05

## Noise Filtering (cont'd)

Gaussian filtering has two advantages over the average filtering:

- no secondary lobes in the frequency domain ( see figure 3.3 of [?]).
- can be implemented efficiently by using two 1D Gaussian filters.



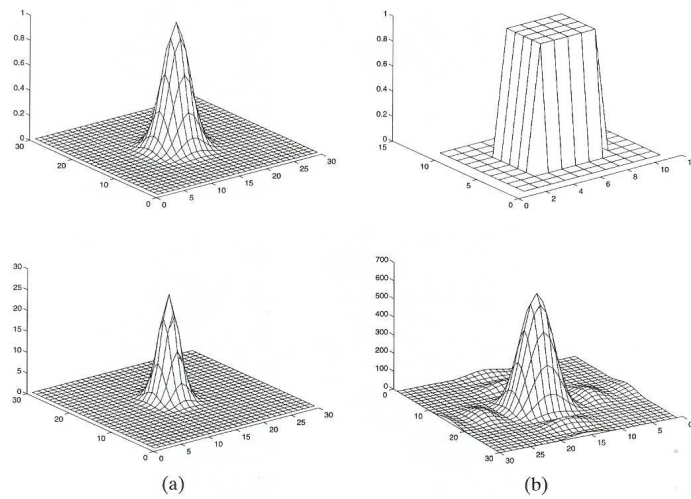


Figure 3.3 (a) The plot of a  $5 \times 5$  Gaussian kernel of width 5 (top) and its Fourier transform (bottom). (b) The same for a mean-filter kernel.

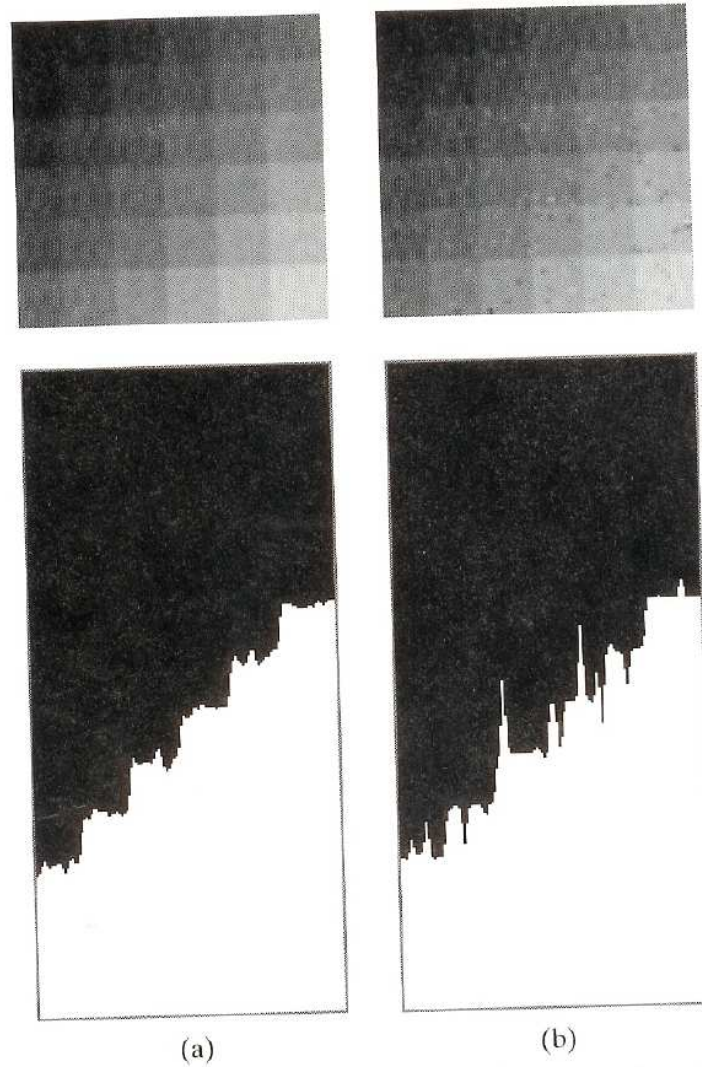


Figure 3.2 (a) Results of applying Gaussian filtering (kernel width 5 pixel,  $\sigma = 1$ ) to the “checkerboard” image corrupted by Gaussian noise, and grey-level profile along a row. (b) Same for the “checkerboard” image corrupted by salt and pepper noise.

## Non-linear Filtering

Median filtering is a filter that replaces each pixel value by the median values found in a local neighborhood. It performs better than the low pass filter in that it does not smear the edges as much and is especially effective for salt and pepper noise.

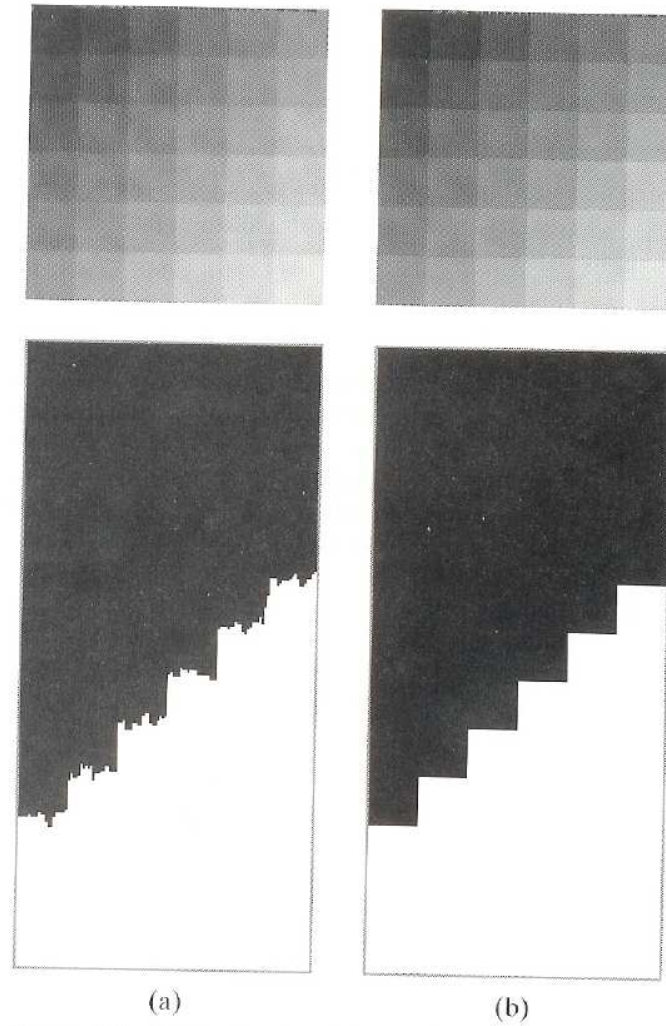


Figure 3.7 (a) Results of applying median filtering (3-pixel wide) to the “checkerboard” image corrupted by Gaussian noise, and grey-level profile along the same row of Figure 3.2. (b) Same for the “checkerboard” image corrupted by impulsive noise.

## Signal to Noise Ratio

$$SNR = 10 \log_{10} \frac{S_p}{N_p} dB$$

For image, SNR can be estimated from

$$SNR = 10 \log_{10} \frac{I}{\sigma}$$

where  $I$  is the unperturbed image intensity

## Quantization Error

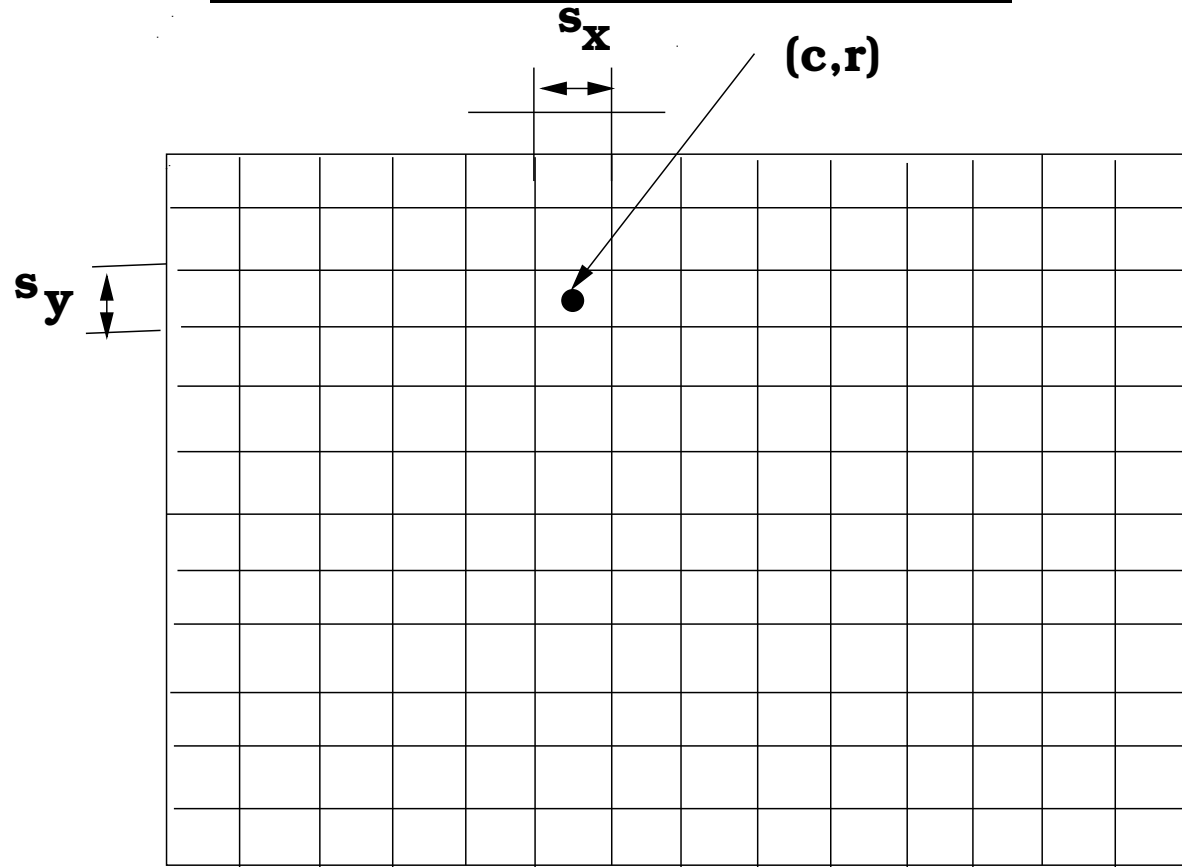
Let  $(c, r)$  be the pixel position of an image point resulted from spatial quantization of  $(x, y)$ , the actual position of the image point. Assume the width and length of each pixel (pixel/mm), i.e., the scale factors, are  $s_x$  and  $s_y$  respectively, then  $(x, y)$  and  $(r, c)$  are related via

$$c = s_x x + \xi_x$$

$$r = s_y y + \xi_y$$

where  $\xi_x$  and  $\xi_y$  represent the spatial quantization errors in  $x$  and  $y$  directions respectively.

# Quantization Error



## Quantization Error (cont'd)

Assume  $\xi_x$  and  $\xi_y$  are uniformly distributed over the range determined by  $[-0.5s_x, 0.5s_x]$  and  $[-0.5s_y, 0.5s_y]$ , i.e.,

$$f(\xi_x) = \begin{cases} \frac{1}{s_x} & -0.5s_x \leq \xi_x \leq 0.5s_x \\ 0 & \text{otherwise} \end{cases}$$

$$f(\xi_y) = \begin{cases} \frac{1}{s_y} & -0.5s_y \leq \xi_y \leq 0.5s_y \\ 0 & \text{otherwise} \end{cases}$$



## Quantization Error (cont'd)

Now let's estimate variance of row and column coordinates  $c$  and  $r$ .

$$\text{Var}(c) = \text{Var}(\xi_x) = \frac{s_x^2}{12}$$

$$\text{Var}(r) = \text{Var}(\xi_y) = \frac{s_y^2}{12}$$

# References

- [1] Emanuele Trucco and Alessandro Verri. *Introductory techniques for 3-D computer vision*, volume 201. Prentice Hall Englewood Cliffs, 1998.