Spring Semester 2012 – Final Exam

Note:

- Show your work, underline results, and always show units.
- Official exam time: 2.0 hours; an extension of at least 1.0 hour will be granted to anyone.
- Materials' parameters may be looked up from the associated table entitled: "Room temperature properties of Si, Ge, GaAs, and GaN".

1. Indicate if the following statements are true or false:
   (a) Under low-injection conditions, the minority electron concentration injected into the neutral p-type region is much smaller than the majority hole concentration in the neutral p-type region.
   True
   (b) Under high-injection conditions of a pn junction, the current-voltage (IV) characteristic of the pn junction is becoming linear.
   True
   (c) Under low-injection conditions of a pn junction, the IV characteristic of the pn junction is exponential.
   True
   (d) Minority carriers generally have a much shorter lifetime than majority carriers.
   True
   (e) The lifetime of majority carriers can be approximated to be infinitely long.
   True
   (f) In Si, the electron mobility is generally higher than the hole mobility.
   True
   (g) Under normal operation of a bipolar transistor, the BE junction and the CE junction are both operated under forward bias.
   False
   (h) In an n-channel metal-semiconductor field effect transistor (MESFET), the electron channel is pinched off at the drain end of the channel, if $V_{DS} - V_{GS} = V_{PO}$, where V_{GS} is negative quantity and V_{DS} is a positive quantity.
   True
   (i) SiO_2 is a transparent material, because its bandgap energy is greater than the energy of visible light.
   True
   (j) Si is a transparent material, because its bandgap energy is greater than the energy of visible light.
   False

2. Consider an n-channel metal-semiconductor field-effect transistor (MESFET) that is made of n-type GaAs. The MESFET has a metal-semiconductor gate with a length of L_{Gate} = 1 μm and a barrier height

of $e\Phi_B$ = 0.8 eV (can be neglected). The electron channel has an n-type doping concentration of 2×10^{17} cm^{-3} and a channel height of 200 nm and a channel width of 100 µm. Assume that the electron mobility in the channel is 2000 cm^2/(V s).

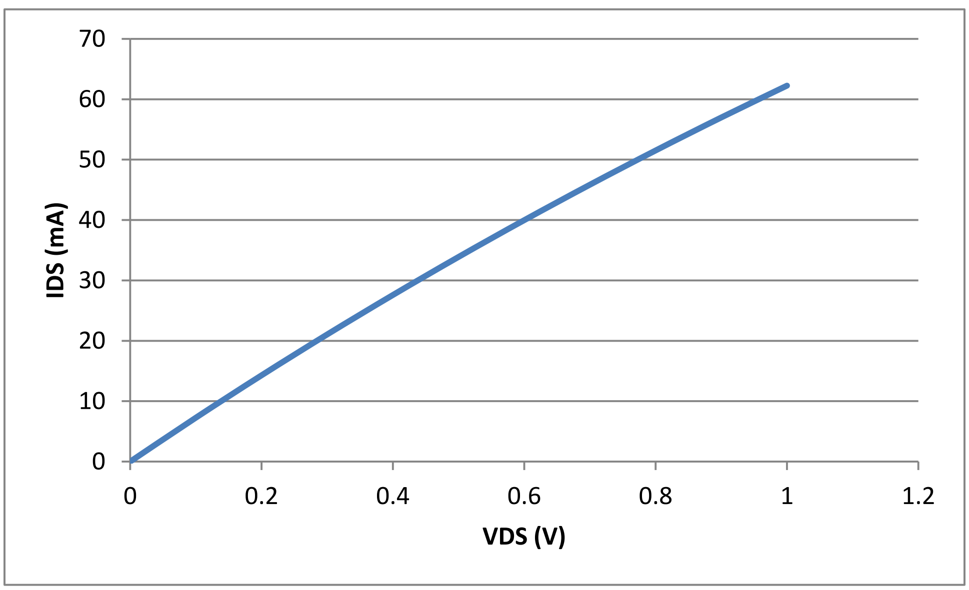
(a) Assume that the gate voltage is V_{GS} = –1 V and assume that $V_{DS} \approx 0$.
Calculate the channel resistivity.
Calculate the channel resistance.
Draw a diagram of the the IV curve (I_D-versus-V_{DS} curve) to scale. In the diagram, the ordinate, I_D, should range from 0 to 750 mA and the abscissa, V_{DS}, should range from 0 to 7.5 V.

Depletion region thickness at gate bias of –1 V: $W_D = \sqrt{\frac{2\varepsilon_r\varepsilon_0}{eN_D}(-V_{GS})} = \sqrt{\frac{2\times 13.1\times 8.85\times 10^{-14}}{1.6\times 10^{-19}\times 2\times 10^{17}}} = 85.1\ nm$

Conductivity of channel: $\sigma = en\mu = 1.6\times 10^{-19}\text{C}\times 2\times 10^{17}\text{cm}^{-3}\times 2000\,\frac{\text{cm}^2}{\text{Vs}} = 64\ \Omega^{-1}\text{cm}^{-1}$

Resistivity of channel: $\rho = \frac{1}{\sigma} = 1.56\ \times 10^{-2}\ \Omega\text{cm}$

Resistance of channel $R_{\text{Channel}} = R = \rho\frac{L_{gate}}{A} = \frac{\rho L_g}{Z(W_{ch}-W_D)} = \frac{3.68\times 10^{-3}\times 10^{-4}}{100\times 10^{-4}\times(200-85)\times 10^{-7}} = 13.6\ \Omega$

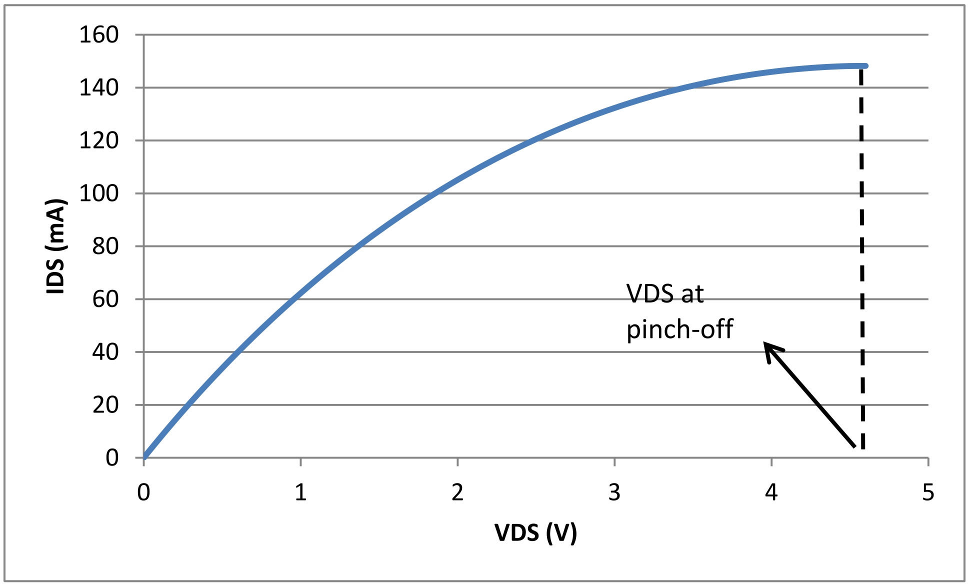


(b) Calculate the value of V_{DS} at which the drain current saturates, i.e. at which the channel is pinched off at the drain end.
Mark the calculated value of V_{DS} in the I_D-versus-V_{DS} curve.

Pinch off voltage $V_{PO} = \frac{eN_D}{2\varepsilon}{W_{ch}}^2 = \frac{1.6\times 10^{-19}\text{C}\times 2\times 10^{17}\text{cm}^{-3}\times(200\times 10^{-7}\text{cm})^2}{2\times 13.1\times 8.85\times 10^{-14}\text{C}/(\text{V cm})} = 5.52\ \text{V}$

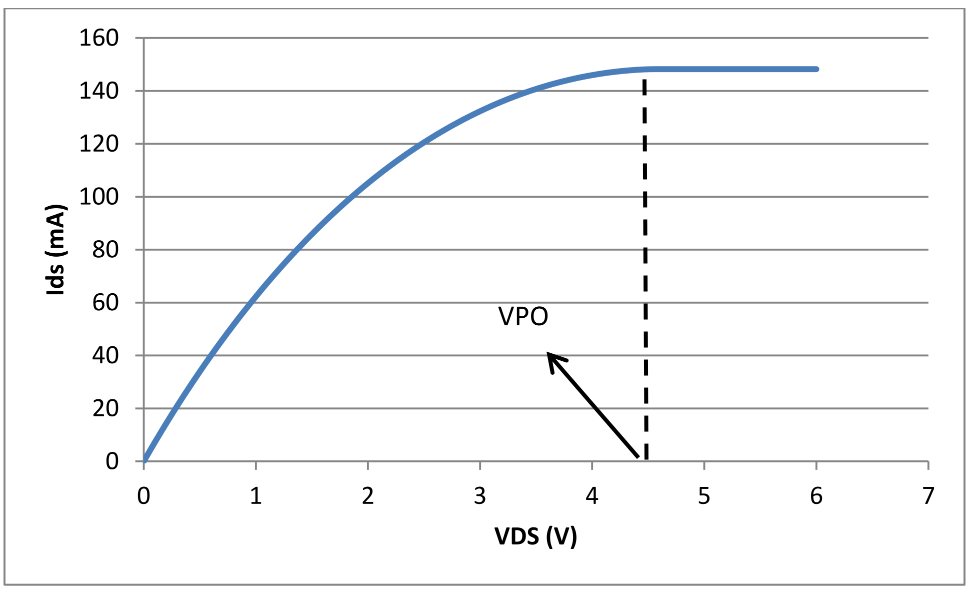
Channel is pinched off when $V_{DS} - V_{GS} = V_{PO}$.

With V_{GS} = – 1 V and V_{PO} = 5.52 V, V_{DS} = 4.52 V.



(c) Calculate the drain current at the value of V_{DS} calculated under question (b). Complete the trace of the I_D-versus-V_{DS} curve for V_{GS} = –1 V.

$$I_{D,sat} = en\mu \frac{W_{\text{channel}} Z}{L_{gate}} V_{\text{PO}} \left[\frac{V_{GS}}{V_{PO}} + \frac{2}{3} \left(\frac{-V_{GS}}{V_{PO}} \right)^{3/2} + \frac{1}{3} \right] = 0.6 \text{ A}$$



3. Consider an asymmetrically doped Si pn junction with the n-type doping concentration much greater than the p-type doping concentration, i.e. $n >> p$. Assume that $N_D = n = 5 \times 10^{18}$ cm^{-3} and $N_A = p = 1 \times 10^{17}$ cm^{-3}. Assume further that the area of the pn junction is A = 1 mm^2.

(a) Calculate the diffusion voltage of the diode, i.e. V_D (note that the diffusion voltage is frequently called the built-in voltage, i.e. V_{bi})

Diffusion voltage = $V_D = \frac{kT}{e} \ln \frac{N_A N_D}{n_i^2}$

$$V_D = (25.9\ \mathrm{meV}) \times \ln \frac{(5 \times 10^{18}\mathrm{cm}^{-3}).(1 \times 10^{17}\mathrm{cm}^{-3})}{10^{20}\mathrm{cm}^{-6}}$$

$$V_D = 0.93\ \mathrm{V}$$

(b) Consider an applied voltage that is $0.8 \times V_D$. Calculate the concentration of injected electrons into the p-type neutral region and compare it to the concentration of the hole majority carriers. Is the junction operated under low-level injection or high-level injection?

$$\Delta n_p(0) = n_{p0}\ \exp \frac{0.8 V_D}{kT/q}$$

$$n_{p0} = \frac{n_i^2}{p_{n0}} = \frac{10^{20}\mathrm{cm}^{-6}}{10^{17}\mathrm{cm}^{-3}} = 10^3\ \mathrm{cm}^{-3}$$

$$\Delta n_p(0) = 10^3\ \mathrm{cm}^{-3}\ \ \exp \frac{0.8\ \times 0.93\ \mathrm{V}}{0.0259\ \mathrm{V}} = 2.99 \times 10^{15}\mathrm{cm}^{-3}$$

Since $\Delta n_p(0) << p_{p0}$, we are under low-injection conditions.

(c) For an applied voltage of $0.8 \times V_D$, calculate the electron injection current and the hole injection current. Which of the two currents is larger? Calculate the total current of the pn junction.

$$\textit{Electron injection current} = Ae\left(\frac{D_n n_{p0}}{L_n}\right)\left(e^{eV/kT} - 1\right)$$

$$= \left(1\mathrm{mm}^2 \times 1.6 \times 10^{-19}\mathrm{C}\right)\left(\frac{39\frac{\mathrm{cm}^2}{s}.10^3\ \mathrm{cm}^{-3}}{\sqrt{39} \times 10^{-6}\mathrm{cm}}\right)\left(e^{0.8\times 0.93/0.0259} - 1\right)$$

$$= 29.8\ \mathrm{mA}$$

$$\textit{Hole injection current} = Ae\left(\frac{D_p p_{n0}}{L_p}\right)\left(e^{eV/kT} - 1\right)$$

$$= \left(1\mathrm{mm}^2 \times 1.6 \times 10^{-19} C\right)\left(\frac{12\frac{\mathrm{cm}^2}{s}.20\ \mathrm{cm}^{-3}}{\sqrt{12} \times 10^{-6}\mathrm{cm}}\right)\left(e^{0.8\times 0.93/0.0259} - 1\right)$$

$$= 0.33\ \mathrm{mA}$$

Electron injection current is greater than hole injection current.

Total current = (29.8 + 0.33) mA = 30.1 mA

(d) Consider that the pn junction would be used as the Base-Emitter (BE) junction of a bipolar transistor.

Calculate the emitter efficiency if the n-type layer would be the emitter.

Emitter efficiency = I_n/I_{total} **= 29.8 mA / 30.1 mA = 0.99**

Calculate the emitter efficiency if the p-type layer would be the emitter.

Emitter efficiency = I_p/I_{total} **= 0.33 mA / 30.1 mA = 0.01**

If the pn junction would be used as the BE junction of a bipolar transistor, would this transistor advantageously be a pnp transistor or an npn transistor? Explain your choice.

The transistor should be an npn transistor because the Emitter should have a higher doping concentration than the base. If the transistor would be a pnp transistor the emitter efficiency would be very low.

4. Consider an n-type Si sample under thermal equilibrium at 300 K. The donor concentration varies from left to right as $N_D(x) = N_{D0} \exp(-ax)$ (where $N_{D0} = 10^{17}\ \text{cm}^{-3}$ and $a = 1\ \mu\text{m}^{-1}$). Assume that donor concentration equals electron concentration (donors are fully ionized). The electron mobility in this sample is 1200 cm^2/ (V s).

(a) What is the diffusion constant of electrons in this sample?

Using Einstein relation:

$$D = \frac{kT}{q}\mu = 0.0259 \times 1200\ \frac{\text{cm}^2}{\text{s}} = 31\ \frac{\text{cm}^2}{\text{s}}$$

(b) What is the diffusion current density of electrons at x = 0? Plot the diffusion current as a function of distance.

Diffusion current density is given by:

$$J_{\text{diffusion}} = q\, D_n \frac{dn}{dx} = q\, D_n \frac{d(N_{\text{D0}} e^{-ax})}{dx} = -q\, D_n\, a\, N_{\text{D0}}\, e^{-ax}$$

$$J_{\text{diffusion}}(x = 0) = -q\, D_n\, a\, N_{\text{D0}} = -1.6 \times 10^{-19} \times 31 \times 10^{17} \times 10^4 = 4960\ \frac{\text{A}}{\text{cm}^2}$$

(c) What is the total current density at any point *x* in the sample?

Since the sample is under thermal equilibrium, the total current density is 0 at any point *x*.

(d) Plot the drift current density as a function of distance to scale.

$$J_{total} = J_{\text{diffusion}} + J_{\text{drift}} = 0$$
$$J_{\text{drift}} = -J_{\text{diffusion}}$$

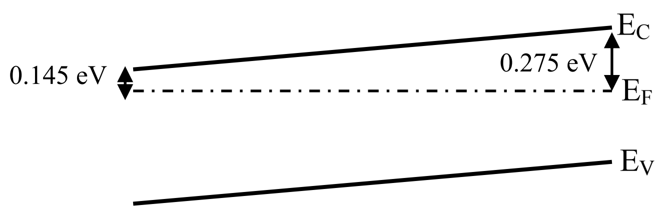
(e) Plot the band diagram from *x* = 0 to *x* = 5 µm.

$$n = N_{\text{D0}} \exp(-ax) = N_{\text{C}} \exp\left(-\frac{E_{\text{C}} - E_{\text{F}}}{kT}\right)$$

$$E_C - E_F = kT\left(\ln\left(\frac{N_{\text{C}}}{N_{\text{D0}}}\right) + ax\right)$$

$$E_C - E_F\ \ (at\ x = 0\ \mu\text{m}) = kT\left(\ln\left(\frac{N_{\text{C}}}{N_{\text{D0}}}\right) + ax\right) = 0.145\ eV$$

$$E_C - E_F\ \ (at\ x = 5\ \mu\text{m}) = kT\left(\ln\left(\frac{N_{\text{C}}}{N_{\text{D0}}}\right) + ax\right) = 0.275\ eV$$



X = 0 X = 5 μm

Based on the last two equations, the potential difference is 0.13 eV.

Under thermal equilibrium, Fermi level is invariant.

(f) What is the potential difference between $x = 0$ to $x = 5$ μm.

As shown above, from the band diagram, potential difference is 0.13 V.

(g) What is the direction of electric field? Briefly explain your answer.

The electric field points from left to right. This can be inferred from the electron drift current which is positive (towards right). This implies that electrons (negatively charged) drift towards the left. Thus the electric field points to the right.

5. Assume an npn bipolar junction transistor operating in forward active mode at 300 K has the following properties: $\mu_n = 1200\ \text{cm}^2/(\text{V s})$ (electron mobility in base); $\mu_p = 425\ \text{cm}^2/(\text{V s})$ (hole mobility in emitter). The base doping is $N_A = p = 10^{16}/\text{cm}^3$ and the base width is $W_{Base} = 40$ μm. While being fabricated, the device was contaminated, and, as a result, the minority carrier lifetimes are shortened. Assume the minority carrier lifetime is the only parameter of the device affected by this contamination, such that they are greatly reduced to $\tau_n = 50$ ns (electron minority carrier lifetime in base) and $\tau_p = 100$ ns (hole minority carrier lifetime in emitter).

(a) What is the diffusion length of electrons in the base?

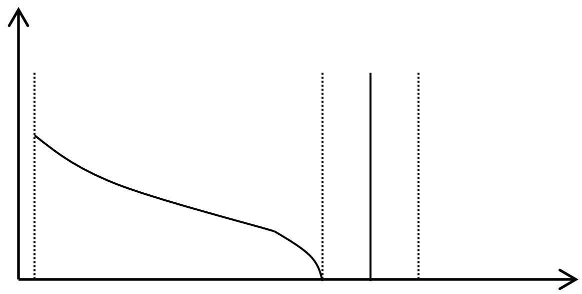
Diffusion length of electrons in base:

$$D_n = \mu_n \frac{kT}{q} = 1200\ \text{cm}^2/\text{Vs} \cdot 0.026\ \text{V} = 31.2\ \ \text{cm}^2/\text{s}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{\left(31.2\ \text{cm}^2/\text{s}\right) \cdot \left(50 \cdot 10^{-9}\ \text{s}\right)} \approx 12.5\ \mu\text{m}$$

(b) Assume a low level bias exists such that there is an injected minority carrier concentration in the base (at the emitter end) of $10^{10}/\text{cm}^3$. Plot a rough sketch of the minority carrier concentration profile in the base and explain the slope of the line. The plot need not be to scale.

Minority carrier concentration (i.e. electrons) in base:



The base width is over 3 × the diffusion length long, so the above curve is exponential (due to strong recombination). A drop-off occurs at the reversely biased BC junction depletion region.

(c) In one or two sentences, please explain qualitatively how the plot in part (b) would look differently if the sample had not been contaminated.

The carrier lifetime would be greatly increased. Therefore, the plot in part (b) would be much more linear (i.e. almost a straight line) instead of exponential, due to significantly less recombination in the base.