Exam-02

1. A capacitor is formed by two open concentric metal cylinders with radius & charge of r_1 & Q_1, and r_2 & Q_2, respectively, where $r_1 < r_2$, $Q_1 = -Q$, and $Q_2 = Q$. The metal cylinders are located in air, have a length ℓ, and a negligibly small metal thickness.
   (a) Draw the experimental setup and label all objects appropriately. What is the charge density, ρ^{2D} (units of C/m^2), on each of the two open cylinders?
   (b) Calculate the electric field for $r < r_1$, $r_1 < r < r_2$, and $r > r_2$. Neglect any fringe fields.
   (c) Derive a symbolic expression for the voltage drop between the two cylinders. Calculate the voltage drop for $Q = 10^{-8}$ C, $r_1 = 1$ cm, $r_2 = 2$ cm, and $\ell = 1.0$ m.
   (d) Derive a symbolic expression for the capacitance of the capacitor. What is the numerical value of the capacitance?
   (e) Give a symbolic expression for the capacitance for $r_2 = r_1 + \Delta r$ where $\Delta r << r_1$ (you may use $\ln(1 + x) \approx x$ which is valid for $x << 1$). Is the obtained expression reminiscent of a parallel plate capacitor?
   (f) Give a symbolic expression for the energy density stored in the electric field as a function of r for $r_1 < r < r_2$.

2. A split parallel-plate capacitor has two metal plates with area $A = 1$ m^2 and a distance between the plates of $2d = 2$ mm. The gap between the plates is filled with two dielectrics, $\varepsilon_{r1} = 1.0$, on LHS[1], and $\varepsilon_{r2} = 20.0$, on RHS, each dielectric having an area $A = 1$ m^2 and a thickness of $d = 1$ mm.
   (a) Draw a diagram of the experimental configuration and label all objects appropriately. The capacitor is charged with a current pulse of 1 mA and pulse duration of 1 ms. What is the charge Q, and the charge per unit area, ρ^{2D}, of the capacitor?
   (b) Assume that the LHS plate is "–" charged and the RHS plate is "+" charged. Neglect fringe fields. Starting with Maxwell's first equation, calculate the electric flux density $\vec{D}$ and the electric field intensity $\vec{E}$ inside the two dielectric materials (symbolic expressions & numerical values).
   (c) What is the voltage drop (numerical value) across the capacitor?
   (d) What is the energy stored (numerical value) in each of the two regions between the two metal plates?
   (e) Which one of the two regions could be neglected? Justify your answer.

3. Determine if the following statements are (i) true, (ii) false, or (iii) impossible to determine due to lack of information. Explain each of your answers with a few words.
   (a) Dielectric media contain only bound charge but no free charge.
   (b) Metals contain free charge. Bound charge that is present in a metal is irrelevant in the context of the present course (ECSE-2100, Fields and Waves I).
   (c) A grounded metal sheet screens an electric field better than a non-grounded metal sheet.
   (d) The boundary condition $E_{t1} = E_{t2}$ always applies, whereas the boundary condition $D_{n1} = D_{n2}$ only applies in the absence of free interface charges (free boundary charges).

[1] LHS = Left hand side; RHS = Right hand side