

Exam - 03 - Solution

Q1 High-voltage transmission line

(a) Maxwell 4 $H \cdot 2\pi r = I$

$$\Rightarrow H = \frac{I}{2\pi r} = \frac{1000 \text{ A}}{2\pi \cdot 10 \text{ m}} = 15.92 \frac{\text{A}}{\text{m}}$$

$$\begin{aligned} B &= \mu H = \mu_0 H = 12.57 \times 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 15.92 \frac{\text{A}}{\text{m}} \\ &= 20 \times 10^{-6} \frac{\text{Vs}}{\text{m}^2} = \underline{\underline{20 \mu\text{T}}} \end{aligned}$$

Earth's magnetic flux density is about $50 \mu\text{T}$. We expect no significant effect from the transmission line's magnetic field.

(b) Reliability of compass compromised, if B-field due to transmission line is equal to the Earth's magnetic field.

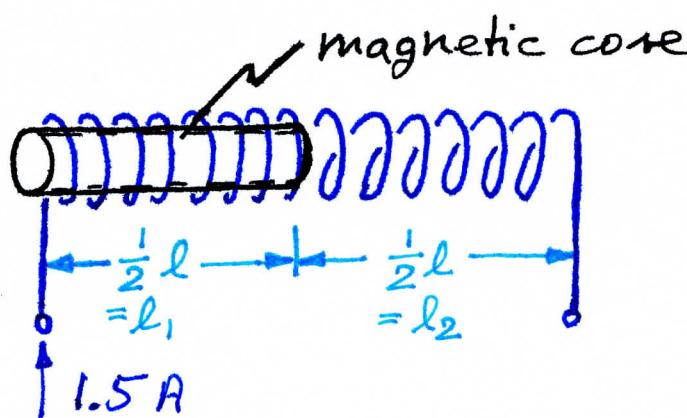
$$B = \mu_0 H = \mu_0 \frac{I}{2\pi r}$$

$$\begin{aligned} \Rightarrow r &= \mu_0 \frac{I}{2\pi B} = 12.57 \times 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot \frac{1000 \text{ A}}{2\pi \cdot 50 \times 10^{-6} \text{ Vs}} \\ &= \underline{\underline{4.0 \text{ m}}} \end{aligned}$$

Q2

Solenoid

(a)



Maxwell's 4th eqn.

$$H_1 l_1 + H_2 l_2 = NI$$

$$B_1 = \mu_{\text{core}} H \quad B_2 = \mu_0 H_2$$

$$\Phi_B = \text{const.} \Rightarrow B_1 A = B_2 A$$

$\xrightarrow{\pi r^2}$
 $\xrightarrow{1 \text{ cm}}$

... these are 4 eqns. with 4 unknowns

$$\frac{B_1}{\mu_{\text{core}}} l_1 + \frac{B_2}{\mu_0} l_2 = NI$$

$$B_1 = B_2 = \frac{NI}{\frac{l_1}{\mu_{\text{core}}} + \frac{l_2}{\mu_0}}$$

$$H_1 = \frac{B_1}{\mu_{\text{core}}} \quad H_2 = \frac{B_1}{\mu_0}$$

$$(b) B_1 = B_2 = \frac{\mu_0 NI}{\frac{l_1}{\mu_{r, \text{core}}} + l_2} \approx \mu_0 NI / l_2$$

$$= 12.57 \times 10^{-7} \frac{Vs}{Am} 200 \frac{1.5A}{0.1m} = \underline{\underline{3.77 \text{ mT}}}$$

(3)

$$H_1 = \frac{B_1}{\mu_{\text{core}}} = \frac{3.77 \times 10^{-3} \frac{\text{Vs}}{\text{m}^2} \frac{\text{Am}}{\text{Vs}}}{1000 \times 12.57 \times 10^{-7} \frac{\text{Vs}}{\text{A}}} = 3.0 \frac{\text{A}}{\text{m}}$$

$$H_2 = \frac{B_2}{\mu_0} = \frac{3.77 \times 10^{-3} \frac{\text{Vs}}{\text{m}^2} \frac{\text{Am}}{\text{Vs}}}{12.57 \times 10^{-7} \frac{\text{Vs}}{\text{A}}} = 2999 \frac{\text{A}}{\text{m}}$$

(c) Inductance of solenoid

$$V_{\text{ind}} = L \dot{I}$$

$$V_{\text{ind}} = -N \dot{\Phi}_m = -N A \dot{B}$$

$$= -N A \underbrace{\mu_0 \frac{N}{l_2} \dot{I}}_L$$

$$\Rightarrow L = \mu_0 \frac{N^2 A}{l_2}$$

$$= 12.57 \times 10^{-7} \frac{\text{Vs}}{\text{Am}} \frac{(200)^2 \pi (0.01 \text{m})^2}{0.1 \text{m}}$$

$$= 0.158 \times 10^{-3} \frac{\text{V}}{\text{A}} \text{s} = 158 \times 10^{-6} \Omega \text{s} = 158 \mu\text{H}$$

$$(d) \text{ Force } F = -\frac{d}{dl_2} \text{ Energy}$$

$$= -\frac{d}{dl_2} \frac{1}{2} H_2 B_2 A l_2$$

$$= \underline{\underline{-\frac{1}{2} H_2 B_2 A}}$$

(4)

(e) The force is attractive

$$F = \frac{1}{2} H_2 B_2 A$$

$$= \frac{1}{2} 2999 \frac{A}{m} 3.77 \times 10^{-3} \frac{Vs}{m^2} \pi (0.01\cancel{m})^2$$

$$= 1.776 \times 10^{-3} \frac{VAs}{m} = \underline{\underline{1.776 \text{ mN}}}$$

Q3 Transformer

(a) Winding ratio $\frac{N_1}{N_2} = \frac{10kV}{120V} = \frac{10000}{120} = 83.3$

Secondary power $= P_2 = 120V \cdot 100A = 12kW$

$\Rightarrow P_1 \approx 12kW$

$\Rightarrow I_1 = \frac{P_1}{V_1} = \frac{12000W}{10000V} = 1.2A$

(b) Ohmic losses in a wire: $P = I^2 R$

$$\frac{I_2}{I_1} = \frac{100A}{1.2A} = 83.3 \Rightarrow \frac{I_2^2}{I_1^2} = 6944$$

\Rightarrow Secondary winding must have lower resistance and larger cross section.

Otherwise ohmic losses (\Rightarrow heat) would be too large in the secondary winding.

(c) Maxwell 4: $H_1 l = N_1 I_1$

$$\frac{B_1}{\mu_{core}} l = N_1 I_1 \quad \text{Vs/m}^2$$

$$\Rightarrow N_1 = \frac{B_1}{\mu_{core}} \frac{l}{I} = \frac{1T}{1000 \cdot 12.57 \times 10^{-7} \frac{\text{Vs}}{\text{Am}}} \cdot \frac{0.5m \cdot 1.2A}{\frac{\text{m}}{\text{m}}} = 331$$

$$\Rightarrow N_2 = \frac{331}{83.3} = 4$$

(6)

- (d) Exceeding B_{residual} means that the effective μ_r drops and, accordingly, the core of the transformer is less permeated by the magnetic field. That is, the advantageous properties of the core are reduced, so that the transformer works less efficiently.

Q4 Multiple choice & short explanation

(a) True The voltage dropping across an inductor is $V_{\text{ind}} = L \dot{I}$. For a constant DC current I , it is $\dot{I} = 0 \Rightarrow V_{\text{ind}} = 0$. An ideal inductor has no resistance, $R=0$. Therefore, no voltage drops across the inductor.

(b) False Secondary voltage will be reduced and secondary current will be reduced.