

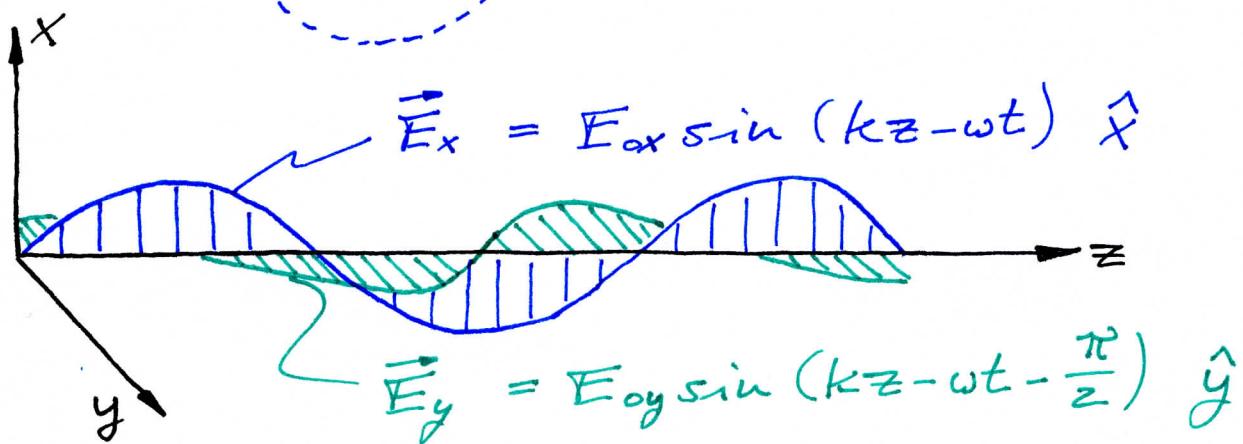
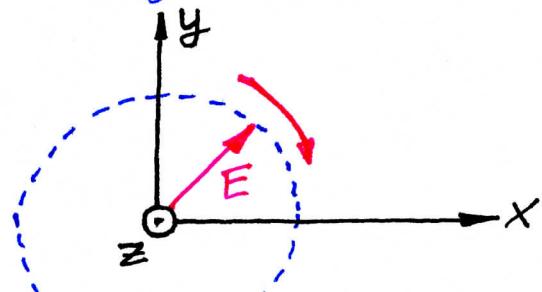
Exam - 04 - Solution

Q1

Superposition of two waves

⇒ Circularly polarized wave

(a)



Mathematical expression:

$$\vec{E} = E_0 \sin(kz - wt) \hat{x} + E_0 \sin(kz - wt - \frac{\pi}{2}) \hat{y}$$

(where $E_{0x} = E_{0y}$)

(b) Magnetic field

$$\vec{H} = \frac{1}{Z} \vec{k} \times \vec{E}$$

 E_x field has an associated H_y field E_y field has an associated $-H_x$ field

Therefore:

$$\underline{\underline{\vec{H}}} = \frac{1}{z} \hat{k} \times \underline{\vec{E}} = \frac{1}{z} E_{ox} \sin(kz - \omega t) \hat{j} + \\ + \frac{1}{z} E_{oy} \sin(kz - \omega t - \frac{1}{2}\pi) (-1) \hat{x}$$

Rotation of \vec{H} -vector: Rightward rotation
(clockwise rotation)

- (c) The polarizer transmits only the y-polarized EM-wave. $\Rightarrow \vec{E}$ -field after the polarizer:

$$\underline{\underline{\vec{E}}} = E_{oy} \sin(kz - \omega t - \frac{\pi}{2}) \hat{j}$$

(3)

Q2

Plane wave

$$\text{P/A} = 10 \text{mW/cm}^2$$

$$\text{Weakly conducting material } \sigma = \frac{1}{\rho} = 10^{-5} \frac{\text{A}}{\text{Vm}}$$

$$\epsilon_r = 5.0$$

$$f = 10^9 \text{ Hz}$$

(a) Amplitude reflection coefficient τ

$$\tau = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$Z_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega \quad (\text{air})$$

$$Z_2 = \sqrt{\frac{\mu}{\epsilon'}} = \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j \frac{\epsilon''}{\epsilon'} \right)^{-1/2}$$

$$\epsilon'' = \frac{\sigma}{\omega} = \frac{10^{-5}}{2\pi \cdot 10^9} \frac{\text{As}}{\text{Vm}} = 1.59 \times 10^{-15} \frac{\text{As}}{\text{Vm}}$$

$$\epsilon' = \epsilon = 5 \times 8.85 \times 10^{-12} \frac{\text{As}}{\text{Vm}} = 4.43 \times 10^{-11} \frac{\text{As}}{\text{Vm}}$$

$$\Rightarrow \frac{\epsilon''}{\epsilon'} = 3.59 \times 10^{-5}$$

$$\Rightarrow \left(1 - j \frac{\epsilon''}{\epsilon'} \right)^{-1/2} = \frac{1}{\sqrt{1 - j 3.59 \times 10^{-5}}} \approx \frac{1}{\sqrt{1}} = 1$$

$$\Rightarrow Z_2 = \sqrt{\frac{\mu_0}{5\epsilon_0}} = \frac{1}{\sqrt{5}} 377 \Omega = 169 \Omega$$

$$\Rightarrow \tau = \frac{169 \Omega - 377 \Omega}{169 \Omega + 377 \Omega} = \underline{\underline{-0.381}}$$

$$R = \tau^2 = 0.145 = \underline{\underline{14.5\%}}$$

$$(b) T = 1 - R = 0.855 = 85.5\%$$

$$\text{Power density} = \frac{P}{A} = \frac{P_0}{A} T = 10 \frac{\text{mW}}{\text{cm}^2} 0.855 \\ = \underline{\underline{8.55 \frac{\text{mW}}{\text{cm}^2}}}$$

(c) Amplitude attenuation constant

$$\alpha = \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon}} = \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{1}{2} 10^{-5} \frac{1}{\text{Vm}} \frac{1}{\sqrt{5}} 377 \frac{\text{V}}{\text{A}} \\ = \underline{\underline{0.843 \times 10^{-3} \frac{1}{\text{m}}}}$$

(d) Amplitude of wave:

$$E_o(z) = E_o(z=0) e^{-\alpha z} \quad \xrightarrow{\text{Location of boundary}}$$

$$\frac{E_o(z)}{E_o(z=0)} = \frac{E_o(\ell_{10\%})}{E_o(z=0)} = e^{-\alpha z} = 0.10 = 10\%$$

$$\Rightarrow \ln e^{-\alpha z} = \ln 0.1 \Rightarrow -\alpha z = \ln 0.1$$

$$\Rightarrow z = \ell_{10\%} = -\frac{1}{\alpha} \ln 0.1 = \frac{-1}{0.843 \times 10^{-3}} \ln 0.1 \\ = \underline{\underline{2731 \text{ m}}}$$

$$(e) \text{Amplitude} \propto E_o(z) \quad \text{Power} \propto E_o(z)^2$$

\Rightarrow At a distance of 2731 m, the amplitude has decreased to 10% and the power to 1%.

\Rightarrow At a distance of 2731 m, 99% of the wave's power has been absorbed.

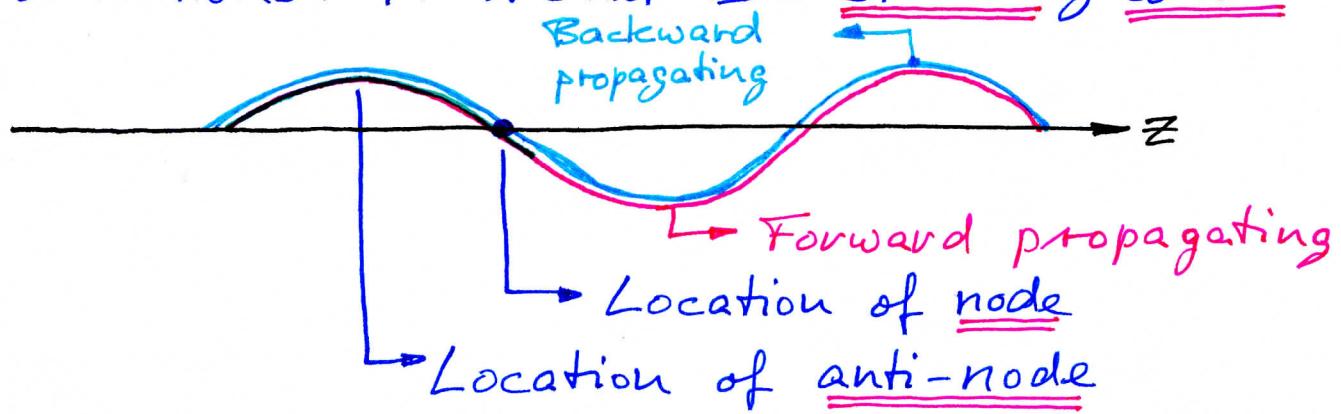
Q3

Pairs of waves

$$(a) E_x = E_{x0} \sin(kz - \omega t)$$

$$E_x = E_{x0} \sin(kz + \omega t)$$

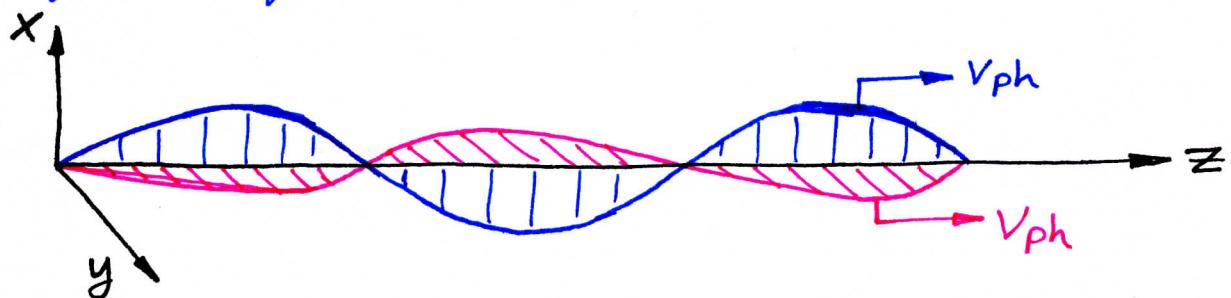
These are two waves propagating in opposite directions. The result is a standing wave.



(b)

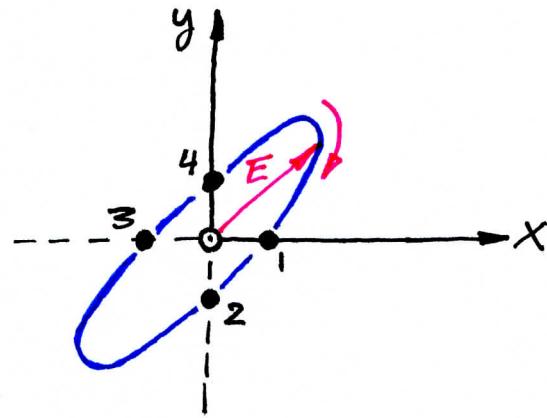
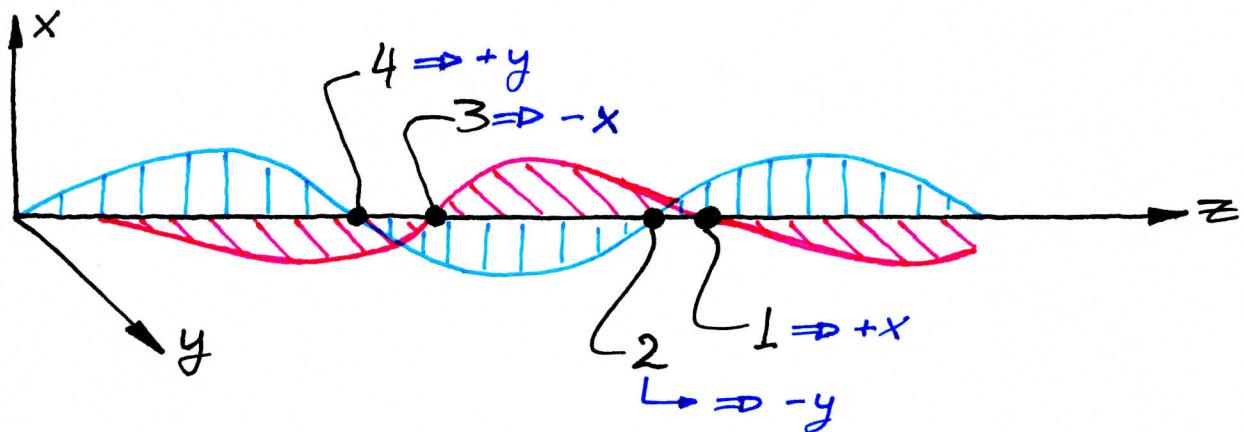
$$E_x = E_{x0} \sin(kz - \omega t)$$

$$E_y = E_{y0} \sin(kz - \omega t)$$



These are two waves, both forward propagating. The resulting wave is a forward propagating wave that is polarized at 45° , i.e. between the x & y axes.

(c) $E_x = E_{x_0} \sin(kz - \omega t)$
 $E_y = E_{y_0} \sin(kz - \omega t - \pi/4)$



\vec{E} -field rotates rightward.

This is an elliptically polarized wave.

Q4

- (a) False. Plane waves have an \vec{E} - and \vec{H} -field along the transverse direction.
If a wave propagates along the x -direction, \vec{E} and \vec{H} can point along the y -and z -direction
- (b) False. It is reasonable to neglect reflection of light at the air-to-outer-space boundary, since ϵ_r and μ_r of air and outer space (vacuum) are very similar.
- (c) False. The Fresnel eqns. for oblique incidence and the Fresnel eqns. for normal incidence are fully consistent.