Exam-04

1. Assume that we create a circularly polarized EM-wave by superposition of two linearly polarized EM-waves (propagating along the +*z*-direction) each one having a sin-function $\vec{E}$-field. The circularly polarized EM-wave has an electric field vector that rotates clockwise when looking along the –*z*-direction, i.e. opposite the propagation direction.
   (a) Give a mathematical expression for the electric field of an EM-wave that meets the above requirements. Schematically illustrate the wave.
   (b) Give a mathematical expression for the $\vec{H}$-field of the wave. Give the rotation direction if the $\vec{H}$-field when looking along the opposite propagation direction (i.e. the –*z*-direction).
   (c) Assume that the circularly polarized EM-wave is transmitted through a polarizer that blocks waves whose $\vec{E}$-field is polarized along the *x*-direction. Describe the wave after having been transmitted through the polarizer. Give the mathematical expression of the wave.

2. A plane wave (f = 1 GHz) propagating along the +*z*-direction in air, having a power density of 10 mW/cm^2 is incident on a weakly conducting, non-magnetic, solid material. The surface-normal vector of the material is pointed along the –*z*-direction. The relative permittivity of the material is ε_r = 5.0 and its resistivity is $\rho = 10^5\ \Omega$ m.
   (a) Determine the amplitude reflection coefficient, *r*, and the power reflection coefficient, *R*, at the boundary. Make approximations as appropriate.
   (b) Determine the power density that is transmitted through the boundary by using the equation $T = 1 - R$.
   (c) Determine the amplitude attenuation constant α for the weakly conducting material.
   (d) Calculate the distance, $\ell_{10\%}$, at which the *amplitude* of the wave decreased to 10% of the initial value (i.e. the value it had upon having just entered the weakly conducting material).
   (e) What is the fraction of the wave's *power* that has been absorbed after having propagated for the distance $\ell_{10\%}$ in the weakly conducting material?

3. Sketch the following pairs of EM-waves and describe the kind of wave obtained when super-imposing the pairs. Give a detailed answer; be as specific as possible; assume that $|E_{x0}| = |E_{y0}|$.
   (a) $E_x = E_{x0} \sin(k z - \omega t)$ and $E_x = E_{x0} \sin(k z + \omega t)$
   (b) $E_x = E_{x0} \sin(k z - \omega t)$ and $E_y = E_{y0} \sin(k z - \omega t)$
   (c) $E_x = E_{x0} \sin(k z - \omega t)$ and $E_y = E_{y0} \sin(k z - \omega t - \pi/4)$

4. Determine if the following statements are (i) true, (ii) false, or (iii) impossible to determine due to lack of information. Explain each of your answers with a few words.
   (a) A plane EM-wave propagating along the x-direction cannot have a magnetic field vector that points along the z direction.
   (b) When considering the boundary between the Earth's atmosphere (air) and outer space (vacuum), it would be unreasonable to neglect the reflection of solar light at this boundary because the wave impedance of air and vacuum are different.
   (c) The calculation of the amplitude reflection coefficient, *r*, for $\theta_1 = 0°$, using Fresnel's equation for *oblique* incidence, gives a different result than the calculation of *r* using Fresnel's equation for *normal* incidence.