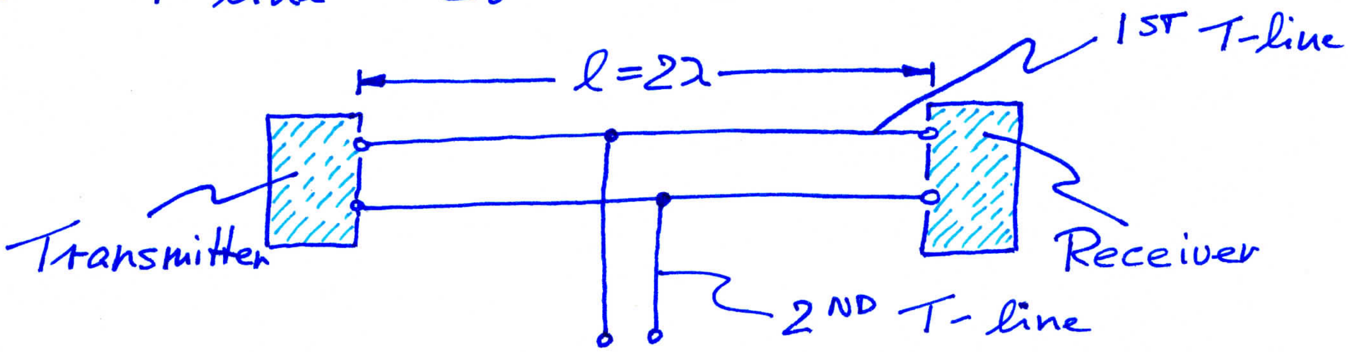
Exam - 01 - Solution

Q1 1^{ST} T-line $Z_0 = 50\,\Omega$ $\ell = 2\lambda$

2^{ND} T-line $Z_0 = 50\,\Omega$

(a)



(b) 2^{ND} T-line is lossless $\ell = \frac{1}{4}\lambda$

OC (open circuit) termination

Signal change at receiver = ?

2^{ND} T-line has length $\frac{1}{4}\lambda$ so that signal travels distance $\frac{1}{2}\lambda$ (forward & backward).

Termination = OC

$$\Rightarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1$$

$\hookrightarrow$ Phase shift $\Delta\phi = 0$

$$\text{Total phase shift} = \underbrace{\pi}_{\text{Due to distance } \frac{1}{2}\lambda} + \underbrace{0}_{\text{Termination}} = \pi$$

$\Rightarrow$ Destructive interference.

$\Rightarrow$ Voltage at beginning of 2^{ND} T-line is zero.

$\Rightarrow$ $\frac{\lambda}{4}$ line acts as a short circuit.

⇒ Signal at Receiver will be zero or strongly attenuated.

(c) There will not be a significant change at the receiver. The 2^{ND} T-line is lossy, so that no standing-wave effect occurs. (We note that the Receiver signal will be somewhat smaller due to the 2^{ND} T-line being in parallel with the 1^{ST} T-line.)

(d) The 2^{ND} T-line is very short, has a phase shift π (SC termination) so that destructive interference results.

⇒ Signal at Receiver will be zero or will strongly decrease.

Q2 $f = 1\,GHz$ $\qquad \lambda = 10\,cm$

(a) Phase velocity $v_{ph} = \frac{\lambda}{T} = \lambda f = 0.1m\, 10^9 \frac{1}{s}$

$$= 10^8 \frac{m}{s}$$

(this is about $\frac{1}{3}$ of c where $c = 3\times10^8 \frac{m}{s}$)

$\alpha = 0$ since the T-line is lossless

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.1m} = 20\pi \frac{1}{m} = 62.8 \frac{1}{m}$$

$$\gamma = \alpha + j\beta = 0 + j\,62.8 \frac{1}{m} = j\,62.8\frac{1}{m}$$

(b) Inductance per unit length $= L' = ?$

For a lossless T-line, we have

$$\beta = \omega\sqrt{L'C'} \quad \text{or} \quad \beta^2 = \omega^2 L'C'$$

$$\Rightarrow L' = \frac{\beta^2}{\omega^2 C'} = \frac{\left(62.8 \frac{1}{m}\right)^2}{\left(2\pi\, 10^9 \frac{1}{s}\right)^2 150\times10^{-12} \frac{F}{m}}$$

$$= \frac{62.8^2 \frac{1}{m^2}\, m\, s^2\, V}{(2\pi\times10^9)^2\, 150\times10^{-12}\, A s} = 666\times10^{-9} \underbrace{\frac{Vs}{A}}_{H} \frac{1}{m}$$

$$= 666 \frac{nH}{m}$$

(c) In order to improve the transmission line (T-line), I would propose to...

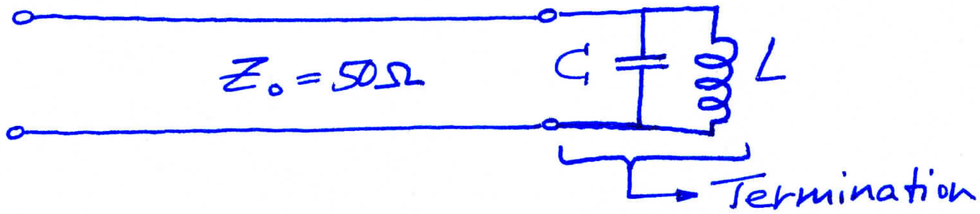
* Use a better insulator between the inner and outer conductor of the coaxial line

* Use a larger-cross-section metal conductor for the inner and/or outer conductor

* Use a more conductive metal material, e.g. Ag, for the inner and/or outer conductor.

$\boxed{\text{Q3}}$ Transmission line termination

(a)



$$Z_L = \left(\frac{1}{j\omega C}\right) \parallel (j\omega L)$$

$\parallel$ → parallel

$$= \frac{\frac{1}{j\omega C}\, j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{1 + j^2\omega^2 LC}$$

$$= \frac{j\omega L}{1 - \omega^2 LC}$$

(b) For $\underline{\underline{f = 0\,\text{Hz}}}$ $\Rightarrow$ $Z_L = 0$

$$\Rightarrow \underline{\underline{\Gamma}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \underline{\underline{-1}}$$

For $\underline{\underline{f = \infty\,\text{Hz}}}$ $\Rightarrow$ $Z_L = 0$

$$\Rightarrow \underline{\underline{\Gamma = -1}}$$

For $\underline{\underline{\omega}} = \underline{\underline{100\,\text{MHz}}} = 10^8\,\text{Hz}$

$L = 10\,\text{nH} = 10^{-8}\,\frac{Vs}{A}$

$C = 10\,\text{nF} = 10^{-8}\,\frac{As}{V}$

$$\Rightarrow Z_L = \frac{j\, 10^8 \frac{1}{\cancel{s}}\, 10^{-8} \frac{V\cancel{s}}{A}}{1 - 10^{16} \frac{1}{\cancel{s^2}}\, 10^{-8} \frac{V\cancel{s}}{A}\, 10^{-8} \frac{A\cancel{s}}{V}}$$

$$= \frac{j\,\Omega}{1 - 1} = \infty$$

$$\Rightarrow \underline{\underline{\Gamma}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \underline{\underline{1}}$$

* At $\omega = 0$ Hz, the load is a short circuit (due to inductor). Therefore $\Gamma = -1$

* At $\omega = \infty$ Hz, the load is a short circuit (due to capacitor). Therefore $\Gamma = -1$

* At $\omega = 100$ MHz $= 1/\sqrt{LC}$, i.e. at resonance frequency, $Z_L = \infty$ so that $\Gamma = 1$

(c) When adding a resistive load, R, the voltage reflection coefficient $\underline{\underline{\Gamma}}$ <u>decreases</u> in magnitude. That is, by adding a resistive load, impedance matching is improved.

$\boxed{Q4}$ (a) T-line: $\ell = \frac{7}{4}\lambda \Rightarrow 2\ell = \frac{14}{4}\lambda = \frac{7}{2}\lambda$

OC $\Rightarrow \Gamma = 1 \Rightarrow$ Phase shift $= \Delta\phi = 0$

Total phase shift $= \frac{7}{2}\lambda \frac{2\pi}{\lambda} + 0 = 7\pi \Rightarrow \pi$

$\Rightarrow$ Destructive interference $\Rightarrow$ Input behaves like a SC $\Rightarrow$ True

(b) True

Transmission line is lossy $\Rightarrow$ Wave is attenuated $\Rightarrow$ No interference $\Rightarrow$ Effect at input side is minimal.