ECSE-2100 Fields & Waves I 2016 Spring

Exam-02

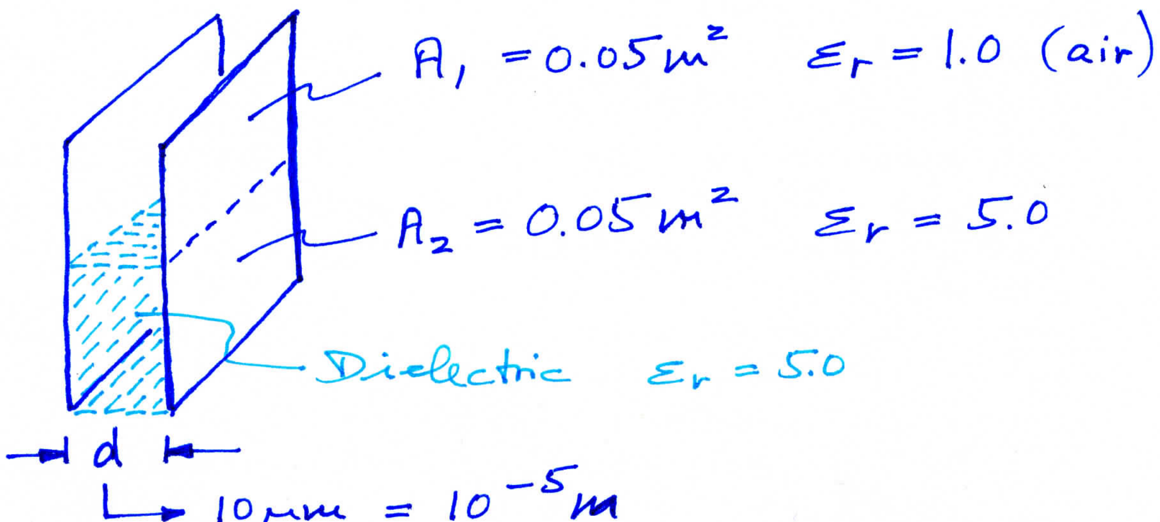
Q1 Capacitor with $d = 10\,\mu m = 0.01\,mm$

$A = 0.1\,m^2$ $\quad A_1 = 0.05\,m^2$ $\quad A_2 = 0.05\,m^2$

$\varepsilon_{r1} = 1.0$ $\quad \varepsilon_{r2} = 5.0$

$Q = 10^{-5}\,C$

(a)



$$C = C_1 + C_2 = \varepsilon_1 \frac{A_1}{d} + \varepsilon_2 \frac{A}{d}$$

$$= 8.85\times10^{-12}\,\frac{As}{Vm}\,\frac{0.05\,m^2}{10^{-5}\,m} + 5.0\times8.85\times10^{-12}\,\frac{As}{Vm}\,\frac{0.05\,m^2}{10^{-5}\,m}$$

$$= 265\times10^{-9}\,\frac{As}{V} = \underline{265.5\,nF}$$

$$Q = CV \Rightarrow V = \frac{Q}{C} = \frac{10^{-5}\,C}{265.5\times10^{-9}\,C}\,V$$

$$= \underline{37.66\,V}$$

(b) $C_1 = \varepsilon_1 \frac{A_1}{d} = 8.85 \times 10^{-12} \frac{As}{Vm} \frac{0.05\,m^2}{10^{-5}\,m} = \underline{\underline{44.25\,nF}}$

$C_2 = \varepsilon_2 \frac{A_2}{d} = 5.0 \times 8.85 \times 10^{-12} \frac{As}{Vm} \frac{0.05\,m^2}{10^{-5}\,m} = \underline{\underline{221.3\,nF}}$

$V_1 = 37.66\,V \Rightarrow \underline{\underline{Q_1}} = C_1 V = 44.25\,nF \times 37.66\,V$
$= \underline{\underline{1.666 \times 10^{-6}\,C}}$

$V_2 = 37.66\,V \Rightarrow \underline{\underline{Q_2}} = C_2 V = 221.3\,nF \times 37.66\,V$
$= \underline{\underline{8.332 \times 10^{-6}\,C}}$

(c) Energy stored in C_1

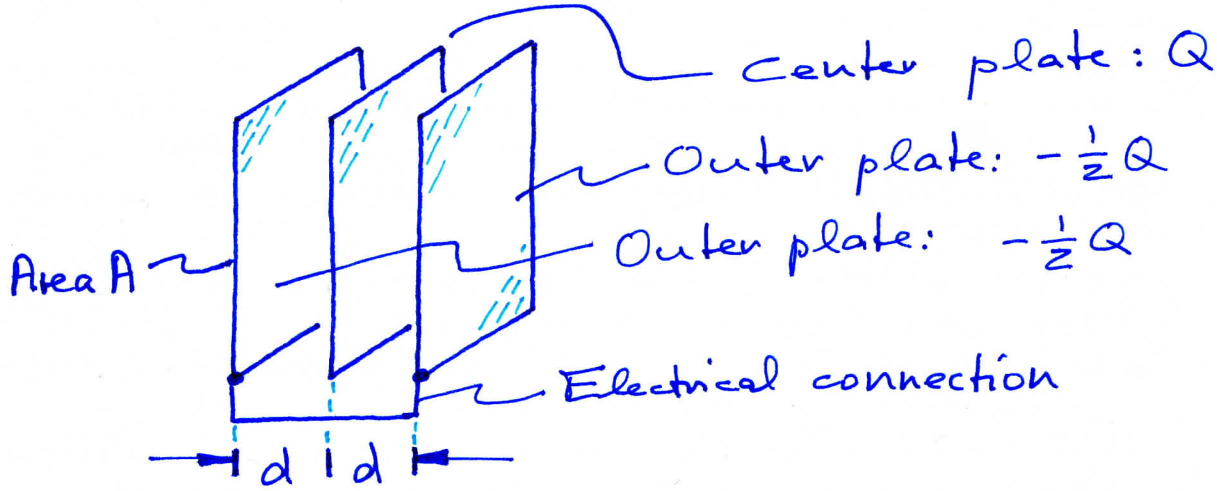
$\underline{\underline{E_1}} = \frac{1}{2} C_1 V^2 = \frac{1}{2}\, 44.25 \times 10^{-9} \frac{As}{V} (37.66\,V)^2$
$= 3.138 \times 10^{-5}\, VAs = \underline{\underline{31.38\,\mu J}}$

$\underline{\underline{E_2}} = \frac{1}{2} C_2 V^2 = \frac{1}{2}\, 221.3 \times 10^{-9} \frac{As}{V} (37.66\,V)^2$

$= \underline{\underline{156.9\,\mu J}}$

(d) "Permittivity" originates from the word "permit". The electric field's energy density ($E/V = \frac{1}{2} \vec{E} \cdot \vec{D}$) is much greater in the high-permittivity material where $\varepsilon_r = 5.0$, illustrating that the field is more energy-relevant in the high-ε_r material. In other words, for a given $\vec{E}$, the high-ε_r material has the greater $\vec{D}$ and thus the greater energy density E/V.

Q2 (a) Experimental setup



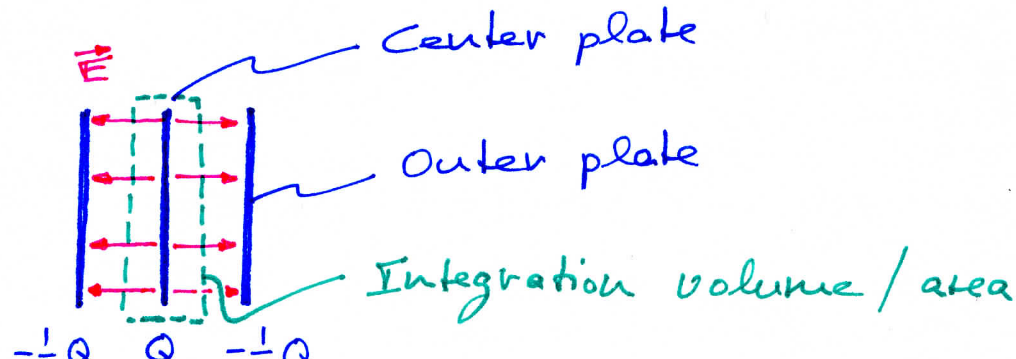
$\vec{E}$ - field = ?

Maxwell 1: $\operatorname{div} \vec{D} = \rho$

Integral form $\oint_A \vec{D} \cdot d\vec{A} = \int_V \rho \, dV$

(A → Area: LHS; V → Volume: RHS)

Choice of Volume V and Integration area A



$$\left.\begin{aligned} LHS &= \vec{D}\, 2\vec{A} = 2DA \\ RHS &= Q \end{aligned}\right\} \Rightarrow D = \frac{Q}{2A}$$

$$|\vec{E}| = \frac{|\vec{D}|}{\varepsilon} = \frac{Q}{2\varepsilon A}$$

(b) $V = \vec{E} \cdot \vec{d} = |\vec{E}|\, d = \frac{Q}{2\varepsilon A} d$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\varepsilon A} d} = 2\varepsilon \frac{A}{d}$$

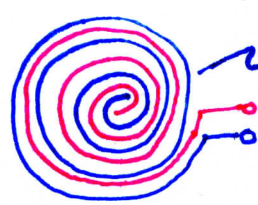
(c) The tri-plate capacitor has...

... more plates (disadvantage)

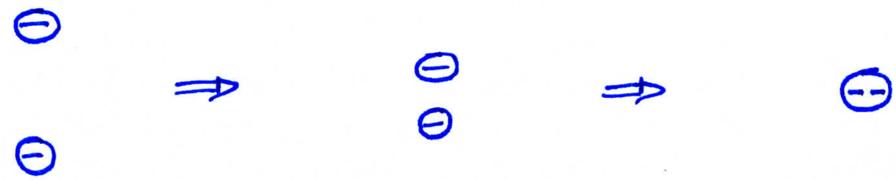
... greater capacitance (advantage)

Overall the tri-plate capacitor has 1.5 times the number of plates (2 → 3) but 2 times the capacitance. ⇒ As a consequence, the tri-plate capacitor is better, more favorable.

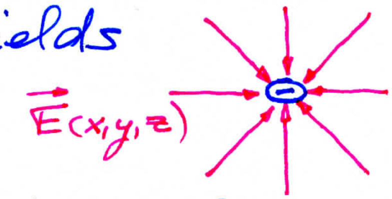
Comment: Spirally wound capacitors are similar to tri-plate capacitors, since each "plate" has one opposing plate next to it.



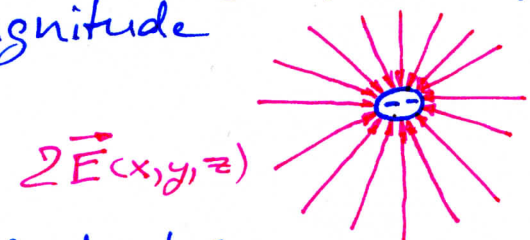
Q3 (a) Experimental setup



Initial fields: Two isolated Coulomb-charge fields



Final field: One Coulomb charge field of double magnitude



During experiment $\vec{E}$ will double.

⇒ Energy density $\left(\frac{E}{V} = \frac{1}{2}\varepsilon E^2\right)$ quadruples

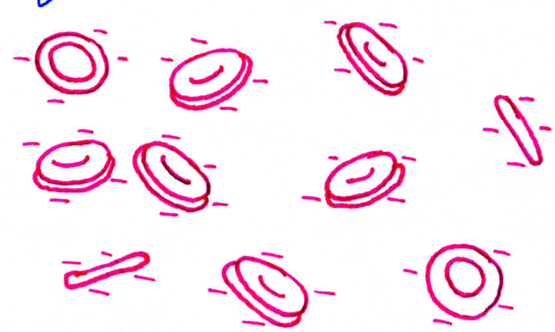
⇒ Number of charges: $2 \to 1$

⇒ Total energy doubles during experiment.

(b) The system of the two charges strives to minimize the $\vec{E}$-field energy. This is accomplished by the two charges moving apart. For this reason, like charges (−− or ++) repel one another.

(c) If I were to design red blood-cells, I would give each blood cell a negative charge so that the blood-cells repel one another. (Of course a + charge would work equally well.)

Comment: Repulsive coulombic interaction is indeed the mechanism by which red blood cells repel each other. Red blood cells are all negatively charged.



⇒ No agglomeration
⇒ No sticking
⇒ No blood clot

Q4 (a) False. An open convertible car is open and thus will not screen the electric field of a lightning spark.

(b) False. A grounded metal sphere can carry any charge.

(c) True. Q = No. of electrons × elementary charge

$= 6.242 \times 10^{18} \times 1.602 \times 10^{-19}\, C$

$= 1\, C$

(d) True. Any solid material has atoms ~~and~~ consisting of a core (nucleus) and electrons. They will get polarized in the presence of an electric field.