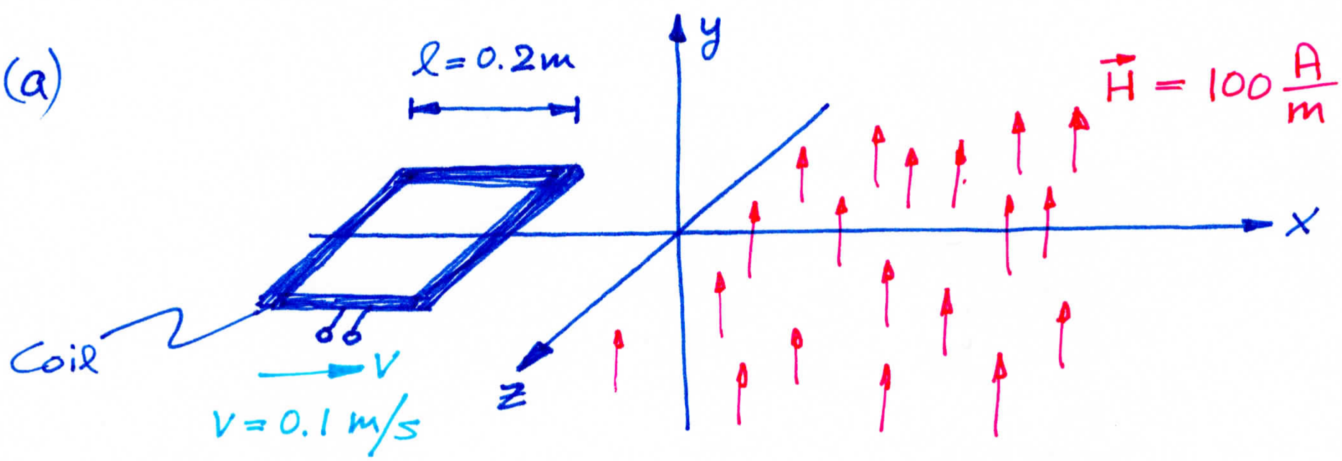
ECSE-2100 Fields & Waves I 2016 Spring

Exam - 03

Q1 OG coil $N = 500$ $l = 0.2\,m$ $l^2 = 0.04\,m^2$

(a)

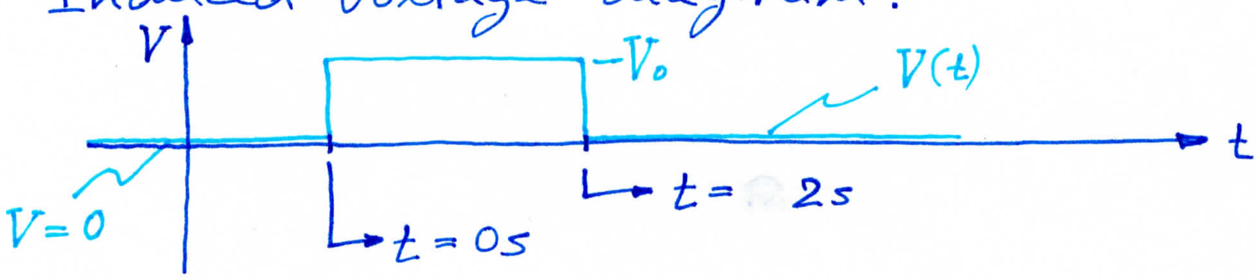


(b) $V = 0.1\,\frac{m}{s}$

At $t = 0$, coil enters $\vec{H}$-field

At $t = 2\,s$, coil is fully located in $\vec{H}$-field

Induced voltage diagram:



Calculation of V_0:

$$V_{ind} = -N\dot{\Phi}_m = -N\frac{d}{dt}\int_A B\,dA = -NB\frac{d}{dt}\int_A dA$$

$$= -NB\frac{d}{dt}A(t)$$

Area penetrated by $\vec{B}$-field as a function of t

$$A(t) = l v t$$

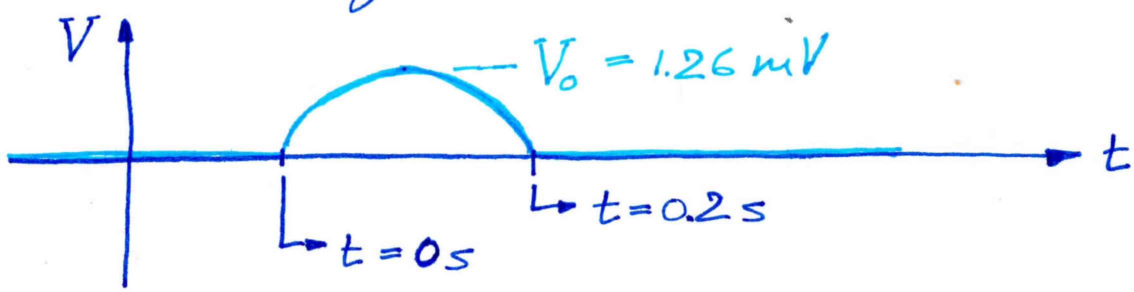
$$\Rightarrow |V_{ind}| = + N B \frac{d}{dt} l v t = N B l v$$

$$= N \mu_0 H l v$$

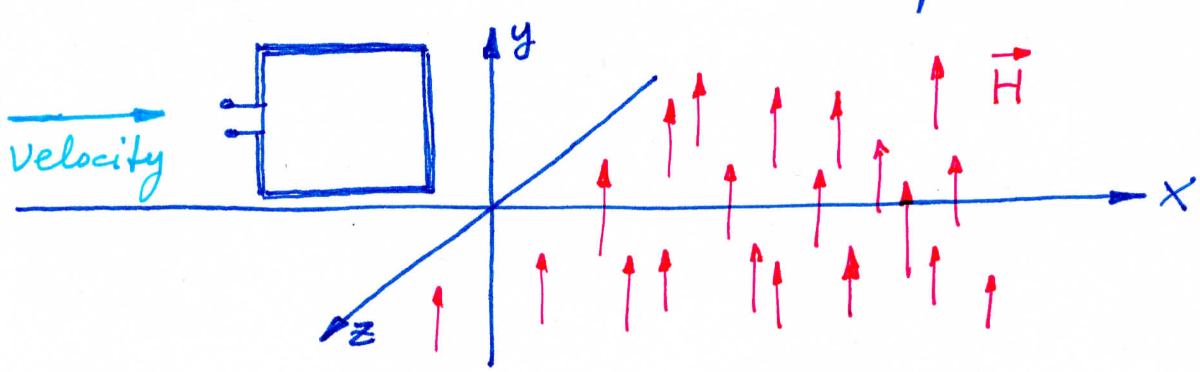
$$= 500 \times 12.56 \times 10^{-7} \frac{Vs}{Am} \, 100 \frac{A}{m} \, 0.2\,m \; 0.1 \frac{m}{s}$$

$$= 0.001257\,V = 1.26\,mV \Rightarrow \underline{V_0} = \underline{1.26\,mV}$$

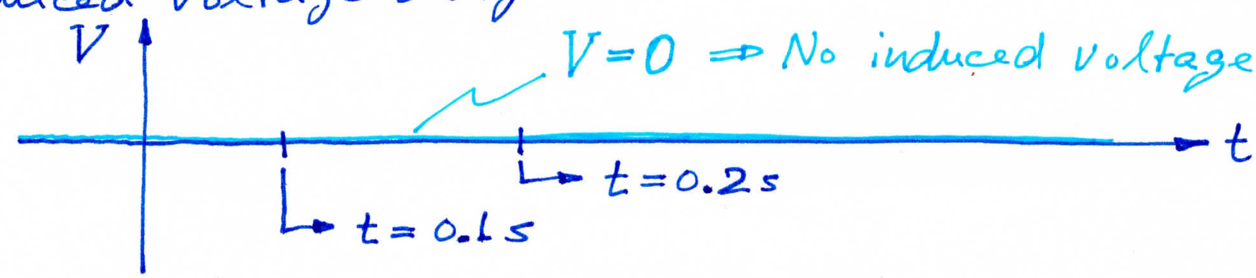
(c) Circular coil with $r = 0.1m$. Draw V_{ind}-versus-t diagram.



(d) Coil is now oriented to lie in the plane $z=0$



Induced voltage diagram:



Q2 Square-shaped loop with $N = 100$

$l = 0.5\,m \quad A_0 = l^2 = 0.25\,m^2 \quad B = 0.5\,T$

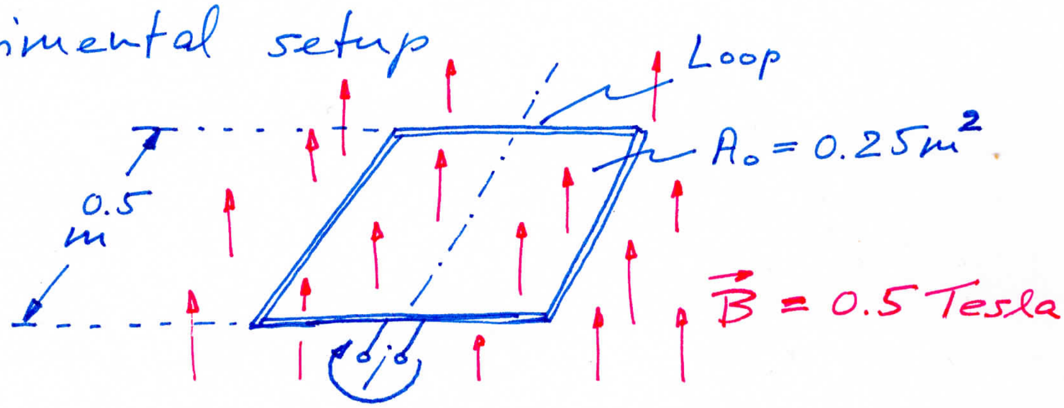
Rotation 600 rpm = 10 rotations per s.

$\Rightarrow f = 10\,Hz$

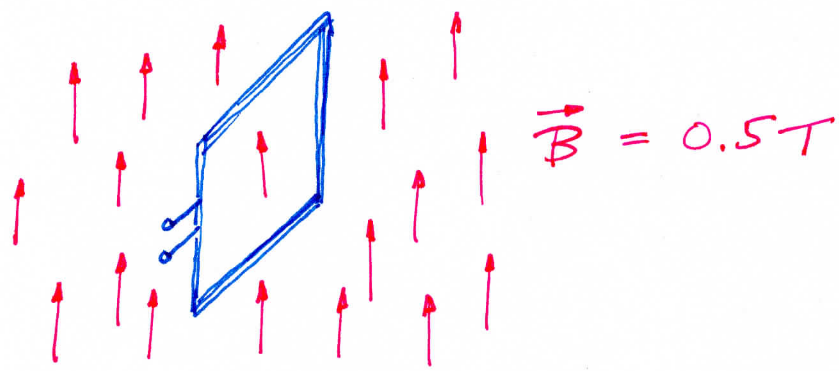
$t = 0 \Rightarrow$ Penetrating magnetic flux is maximal

(a) Experimental setup

t = 0



t = 25 ms



Area penetrated by magnetic flux:

$$A(t) = A_0 \cos(2\pi f t)$$

$f \rightarrow 10\,Hz$

$A_0 \rightarrow = l^2 = 0.25\,m^2$

(b) V_{ind} versus time t = ?

$$V_{ind} = -N\dot{\Phi}_m = -N\frac{d}{dt}\Phi_m = -N\frac{d}{dt}B\,A(t)$$

$$= -NB\frac{d}{dt}A(t)$$

$$= -NB\frac{d}{dt}A_0\cos(2\pi f t)$$

(A_0 → ℓ^2)

$$= -NBA_0(-\sin 2\pi f t)\,2\pi f$$

$$= \underbrace{NBA_0 2\pi f}\,\sin(2\pi f t)$$

→ Amplitude of voltage V_0

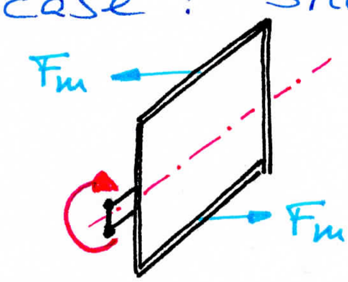
$$V_0 = NBA_0 2\pi f$$

$$= 100 \times 0.5T\;0.25m^2\;2\pi\;10\frac{1}{s}$$

(T → $\frac{Vs}{m^2}$)

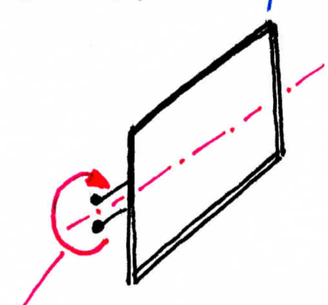
$$= 785.4V$$

(c) First case: Short circuit (SC)



Magnetic force F_m opposes rotation.

Second case: Open circuit (OC)



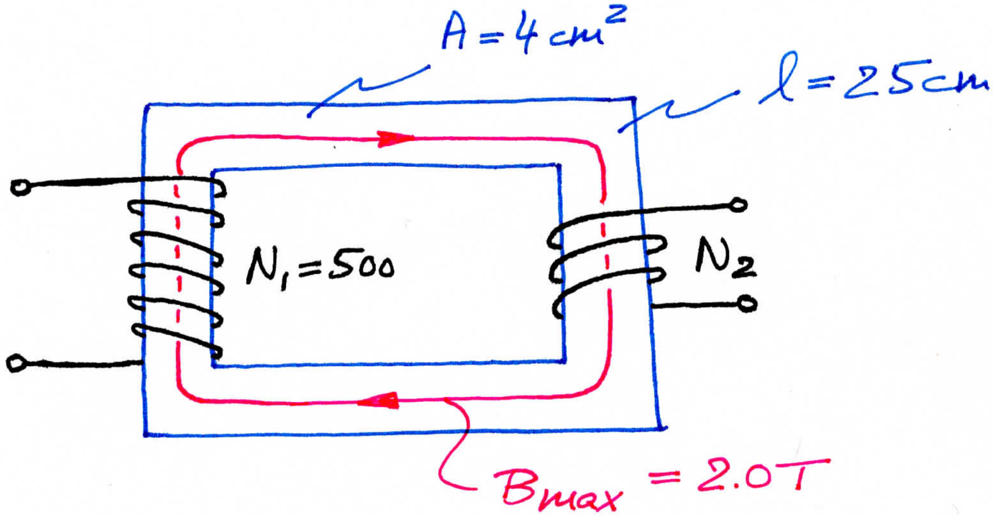
No current. ⇒
No magnetic force.
$F_m = 0$

Q3 Transformer $f = 60\ Hz$ $N_1 = 500$

$\mu_r = 10\,000$ $A = 4\ cm^2$ $l = 25\ cm$

$B_{max} = B_{residual} = 2.0\,T = 2.0\ \frac{Vs}{m^2}$

(a)



$$B_{max} = B_{residual} = 2.0\,T = 2.0\ Vs/m^2$$

$$H_{max} = \frac{B_{max}}{\mu} = \frac{B_{residual}}{\mu_r \mu_0}$$

$$= \frac{2.0\ Vs\ \frac{1}{m^2}\ \ Am}{10\,000\ \ 12.56 \times 10^{-7}\ Vs} = \underline{\underline{159.1\ \frac{A}{m}}}$$

Maxwell's 4th eqn.

$$\text{rot}\ \vec{H} = \vec{J} \qquad \text{(Differential)}$$

$$\int_C \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{A}$$

$$\Rightarrow \quad H l = N I$$

$$\Rightarrow I = \frac{H l}{N} = \frac{159.1\ \frac{A}{m}\ \ 0.25\,m}{500} = 0.07955\,A = \underline{\underline{79.6\ mA}}$$

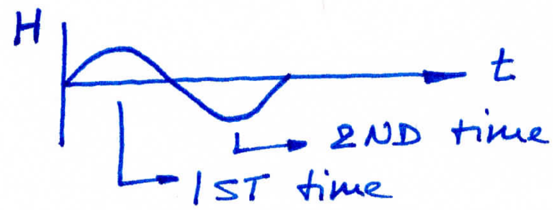
(b) Magnetic field energy in core:

$$\text{Energy} = \underbrace{\frac{1}{2} HB}_{\text{Energy density}} \; \underbrace{l A}_{\text{Volume}}$$

$$= \frac{1}{2}\, 159.1 \frac{A}{m}\; 2.0 \underbrace{\frac{Vs}{m^2}}_{\text{Tesla}}\; 0.25\,m\; 0.0004\,m^2$$

$$= \underline{\underline{0.01591\ Ws}}$$

Per period T, magnetic energy is stored twice:



Power transmitted per half-cycle (per $T/2$)

$$\Rightarrow P = \frac{\text{Energy}}{T/2} = \text{Energy}\ 2f$$

$$= 0.01591\ Ws\ 2 \times 60\ Hz = \underline{\underline{1.91\ W}}$$

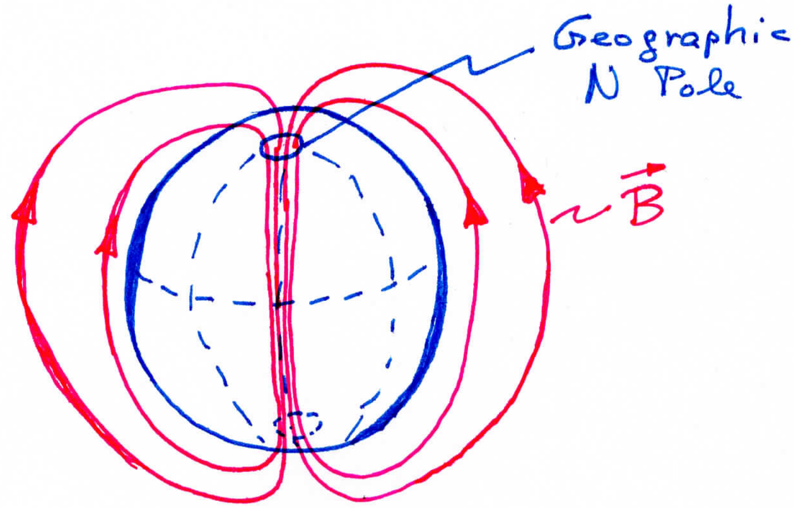
Q4

(a) Earth's magnetic flux density

True

$\text{div}\,\vec{B} = 0$

$\vec{B}$ has no beginning and no end.



(b) **True**

If a permanent magnet is used, no electrical power is expended to generate the magnetic field of the stator. ⇒ Motor is more energy efficient. ⇒ For this reason, electric cars use permanent magnet motors.