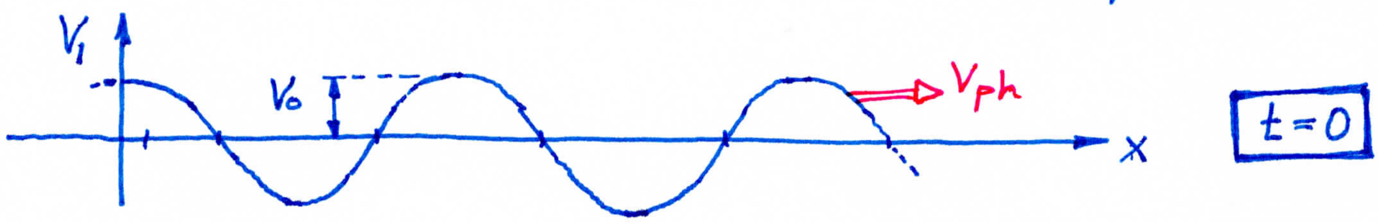
Fields & Waves I Midterm exam - 01 - Solution

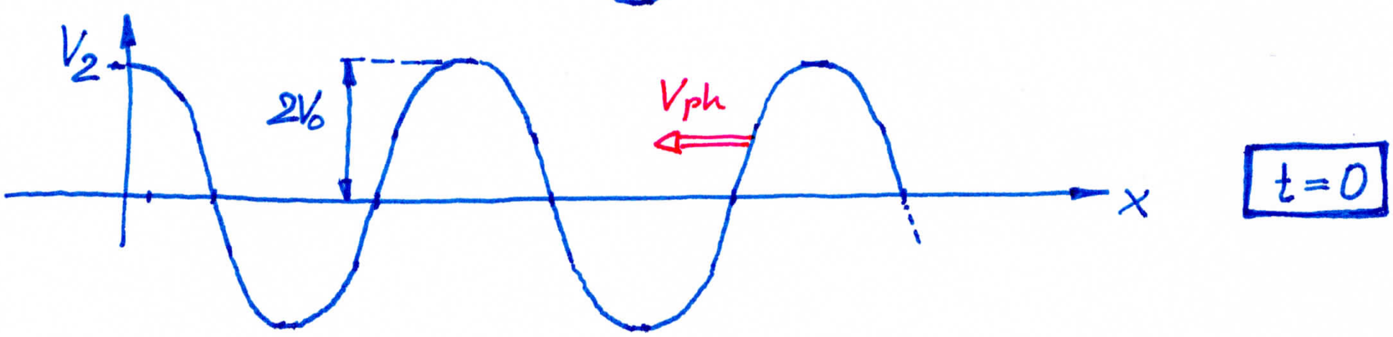
Transmission Lines

Problem 1

(a) 1st wave: $V_1(x,t) = V_0 \cos(\beta x - \omega t)$ — Forward propagating, Amplitude V_0

2nd wave: $V_2(x,t) = 2V_0 \cos(\beta x + \omega t)$ — Backward propagating, Amplitude $2V_0$





(b) $V = V_1 + V_2$

$$= V_0 \cos(\beta x - \omega t) + V_0 \cos(\beta x + \omega t) + V_0 \cos(\beta x + \omega t)$$

Using the trigonometric formula, one obtains:

$$V = \underbrace{2 \cos \beta x \cos \omega t}_{\text{Standing wave}} + \underbrace{V_0 \cos(\beta x + \omega t)}_{\text{Backward propagating wave}}$$

(c) The resulting wave is a superposition of (i) a standing wave and (ii) a backward moving wave. The resulting wave does not have nodes.

(d) α = Attenuation constant

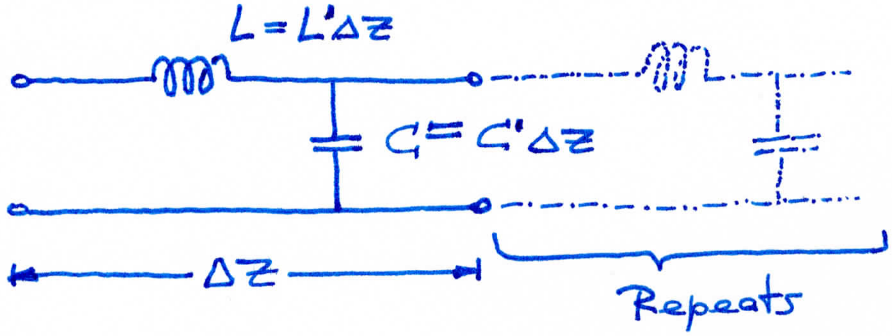
$1/\alpha$ = Length after which the amplitude of the wave has decreased to $e^{-1} = 1/e$ of its initial value.

$1/\alpha$ has the units of length (meters)

Problem 2

$C' = 2\,nF/m \qquad L' = 10\,nH/m \qquad f = 100\ MHz$

(a) Lumped circuit model



Phase constant of wave $= \beta = \frac{2\pi}{\lambda} = \omega\sqrt{L'C'}$

$$\beta = 2\pi \; 100\times10^{6}\,\frac{1}{s}\,\sqrt{10\times10^{-9}\,\frac{Vs}{Am}\;2\times10^{-9}\,\frac{As}{Vm}}$$

$$= 2\pi\;10^{-1}\sqrt{20}\,\frac{1}{m} = 2\pi\;0.447\,\frac{1}{m} = 2.81\,\frac{1}{m}$$

$$\beta = 2\pi/\lambda \Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.81}\,m = 2.24\,m$$

(b) Phase velocity in free space $= c = 3\times10^{8}\,\frac{m}{s}$

Phase velocity on transmission line $= v_{ph} =$

$$v_{ph} = \frac{\lambda}{T} = \lambda f = 2.24\,m\;10^{8}\,\frac{1}{s} = 2.24\times10^{8}\,\frac{m}{s}$$

(c) Wires become resistive due to aging.

Insulator still in perfect condition.

Attenuation of wave:

$$V(x) = V_0(x=0)\ e^{-\alpha x}$$

$$V(x_0) = V_0(x=0)\ (1/2)$$

$\hookrightarrow$ 50%

$$\Rightarrow e^{-\alpha x_0} = 1/2 \quad \Rightarrow \quad -\alpha x_0 \underbrace{\ln e}_{1} = \ln 1/2$$

$$\Rightarrow \underline{\underline{\alpha}} = \frac{-1}{x_0} \ln\frac{1}{2} = \frac{-1}{500\,m} \ln 1/2 = \underline{\underline{0.00139\ \frac{1}{m}}}$$

(d) $\underline{\underline{R' \neq 0}}$ (Wires have become resistive)

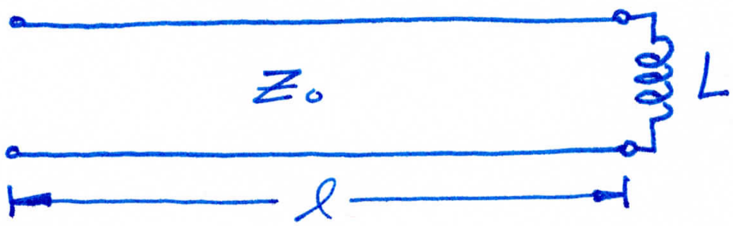
$\underline{\underline{G' = 0}}$ (Insulator is perfect)

Problem 3

$$V(x,t) = V_0 \sin(\beta x - \omega t)$$

Transmission line with length l and wave impedance Z_0. Termination with L.

(a)



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j\omega L - Z_0}{j\omega L + Z_0}$$

(b) $$\Gamma = \frac{(j\omega L - Z_0)\,(j\omega L - Z_0)}{(j\omega L + Z_0)\,(j\omega L - Z_0)} = \frac{(j\omega L - Z_0)^2}{(j\omega L)^2 - Z_0^2}$$

$$= \frac{-1}{\omega^2 L^2 + Z_0^2}\left(j^2\omega^2 L^2 - 2j\omega L Z_0 + Z_0^2\right)$$

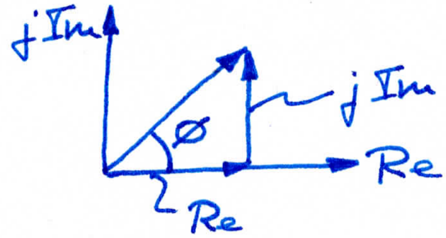
$$= \frac{-1}{\omega^2 L^2 + Z_0^2}\left(-2j\omega L Z_0 + Z_0^2 - \omega^2 L^2\right)$$

Recall $\quad Z = Re + j\,Im$

$$\Rightarrow Re_\Gamma = \frac{-1}{\omega^2 L^2 + Z_0^2}\left(Z_0^2 - \omega^2 L^2\right)$$

$$\Rightarrow j\,Im_\Gamma = \frac{-1}{\omega^2 L^2 + Z_0^2}\left(-j2\omega L Z_0\right)$$

Complex numbers



Phase angle $\tan\phi = \dfrac{Im}{Re}$

$$\Rightarrow \phi = \arctan\frac{Im}{Re} = \arctan\frac{-2\omega L Z_0}{Z_0^2 - \omega^2 L^2}$$

$f \to 0 \Rightarrow \omega \to 0 \Rightarrow \underline{\underline{\Gamma = -1}} \Rightarrow \underline{\underline{SC}} \Rightarrow \phi = -\pi$
$= \underline{\underline{-180^\circ}}$

$f \to \infty \Rightarrow \omega \to \infty \Rightarrow \underline{\underline{\Gamma = 1}} \Rightarrow \underline{\underline{OC}} \Rightarrow \underline{\underline{\phi = 0 = 0^\circ}}$

(c) $f = 0.1\ MHz \quad Z_0 = 50\,\Omega \quad L = 50\,\mu H \quad \phi = ?$

$$\phi = \arctan\frac{-2\omega L Z_0}{Z_0^2 - \omega^2 L^2}$$

$$= \arctan\frac{-2\ 2\pi\ 10^5 \frac{1}{s}\ 50\times 10^{-6}\frac{Vs}{A}\ 50\frac{V}{A}}{50^2\frac{V^2}{A^2} - (2\pi)^2\ 10^{10}\frac{1}{s^2}\ (50\times10^{-6})^2\frac{V^2 s^2}{A^2}}$$

$$= \arctan\frac{-3142}{2500 - 987} = \arctan(-2.077)$$

$$\Rightarrow \underline{\underline{\phi}} = \underline{\underline{-64.3^\circ}}$$

Note: **SC** = Short Circuit
OC = Open Circuit

Problem 4

(a) True

It is $v_{ph,\,T\text{-line}} \leq c$

with $v_{ph} = \frac{\lambda}{T} = \lambda f$

Since $v_{ph,\,T\text{-line}} \leq c$

$\Rightarrow \quad \lambda_{T\text{-line}} \leq \lambda_{\text{free space}}$

(b) False

Γ depends on Z_L. For example, if $Z_L = Z_0$ then $\Gamma = 0$.

(c) True

The transmission line is strongly attenuated so that Z_L becomes irrelevant.