Exam-01 – Transmission lines

1. Consider two waves, a 1st one: $V_1(x, t) = V_0 \cos(\beta x - \omega t)$ and a 2nd one: $V_2(x, t) = 2 V_0 \cos(\beta x + \omega t)$.
   (a) Describe these two waves with a sketch and a few words.
   (b) Mathematically superimpose these two waves. Feel free to make use of the trigonometric formula: $\cos a \cos b = ½ [\cos(a - b) + \cos(a + b)]$.
   (c) Describe the resulting wave. Does the resulting wave have nodes?
   (d) A transmission line has the propagation constant $\gamma = \alpha + j\beta$. What is the meaning of α? What is the meaning of $(1/\alpha)$? What are the units of $(1/\alpha)$?

2. Consider a lossless transmission line having two wires with $C' = 2$ nF/m and $L' = 10$ nH/m. A sinusoidal voltage signal with $f = 100$ MHz is applied to the input of the transmission line.
   (a) Draw the lumped circuit model of the transmission line. Determine the wavelength of the signal propagating on the transmission line.
   (b) Give the wave's phase velocity v_{phase} when propagating (i) in free space and (ii) on the transmission line.
   (c) As a result of age, one of the wires becomes resistive. Despite the age, the other wire is perfectly conducting and the insulator located between the wires is perfectly insulating. The amplitude of the voltage signal drops to 50% of its input value at a distance of $x_0 =$ 500 m. Give the value of the attenuation constant α.
   (d) Determine if R' is zero or non-zero. Determine if G' is zero or non-zero.

3. Consider a sine wave $V(x, t) = V_0 \sin(\beta x - \omega t) = V_0 \sin(\beta x - 2\pi f t)$ propagating transmission line with length ℓ having the wave impedance Z_0 that is terminated by an inductor L.
   (a) Draw the experimental setup and appropriately label all objects. Give a symbolic expression for the voltage reflection coefficient Γ of the transmission line.
   (b) Determine the (complex) voltage reflection coefficient of the transmission line as a function of f. The reflection coefficient Γ can be expressed by a magnitude and phase angle, that is $\Gamma = |\Gamma| \exp(j\phi)$. What is $|\Gamma|$ and ϕ for $f = 0$? What is $|\Gamma|$ and ϕ for $f \longrightarrow \infty$?
   (c) Assume that $f = 0.1$ MHz, $Z_0 = 50\ \Omega$ and $L = 50$ µH. Determine the phase angle ϕ.

4. Determine if the following statements are (i) true or (ii) false. Explain each answer with a few words.
   (a) The wavelength of a sinusoidal signal with frequency f propagating on a transmission line cannot be longer than the wavelength of the signal propagating in free space.
   (b) A transmission line that has no resistive losses has a magnitude of the voltage reflection coefficient of $|\Gamma| = 1.0$
   (c) Assume a lossy transmission line with propagation constant $\gamma = \alpha + j\beta$ and length ℓ such that that $\ell >> 1/\alpha$. In this case, the input impedance of the transmission line is independent of the transmission line's termination (i.e. load).