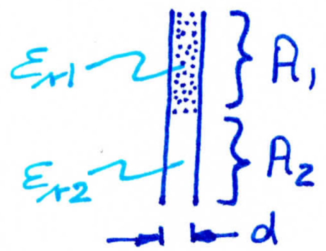
Exam - 02 Electrostatics Fields and Waves

Problem 1 Split capacitor $A = A_1 + A_2$

$Q = Q_1 + Q_2$ $\varepsilon_{r1} = 10.0$ $\varepsilon_{r2} = 1.0$

(a) Experimental setup:



① Assume $Q = Q_1$

$$\text{Energy } 1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} C_1 \frac{Q_1^2}{C_1^2} = \frac{1}{2} Q^2 \frac{d}{\varepsilon_1 A_1}$$

② Assume $Q = Q_2$

$$\text{Energy } 2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} C_2 \frac{Q_2^2}{C_2^2} = \frac{1}{2} Q^2 \frac{d}{\varepsilon_2 A_2}$$

Which of the two calculated energies is lower?

$$\frac{\text{Energy } 1}{\text{Energy } 2} = \frac{\frac{1}{2} Q^2 \frac{d}{\varepsilon_1 A_1}}{\frac{1}{2} Q^2 \frac{d}{\varepsilon_2 A_2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} = \frac{1}{10}$$

$$\Rightarrow \underline{\underline{\text{Energy } 1 \ll \text{Energy } 2}}$$

(b) Assuming that the charge Q can freely distribute itself between the areas A_1 and A_2, we expect that it will distribute in such a way that minimizes the energy of the capacitor, that is, we expect $\underline{\underline{Q_1 \gg Q_2}}$.

(c) Total capacitance $C = C_1 + C_2 = \varepsilon_1 \frac{A_1}{d} + \varepsilon_2 \frac{A_2}{d} =$

$$= \frac{1}{2} \varepsilon_1 \frac{A}{d} + \frac{1}{2} \varepsilon_2 \frac{A}{d} = \underline{\underline{\frac{1}{2} \frac{A}{d} (\varepsilon_1 + \varepsilon_2)}}$$

Assume a voltage V is applied to the capacitor

$\Rightarrow Q_1 = C_1 V \qquad Q_2 = C_2 V$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{\varepsilon_1 \ \frac{1}{2} A/d}{\varepsilon_2 \ \frac{1}{2} A/d} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \frac{10}{1}$$

$\Rightarrow Q_1 = 10\, Q_2$ or $\underline{\underline{Q_1 \gg Q_2}}$

$\Rightarrow$ This is consistent with our expectation.

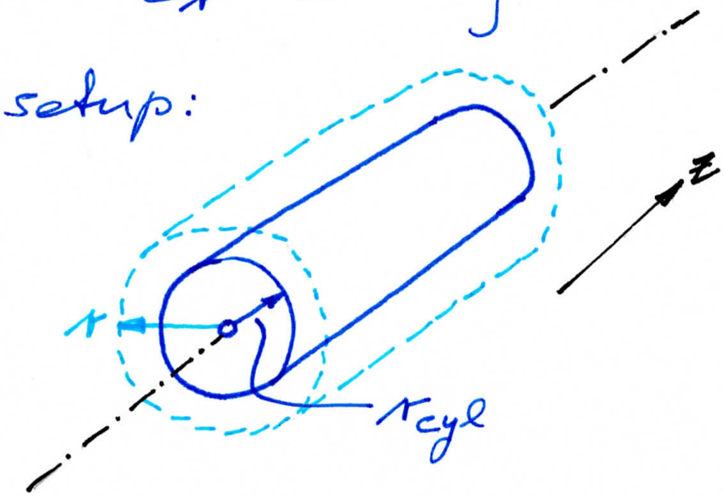
(d) ε is called "<u>permittivity</u>" since regions of high ε_r will have an electric field of higher energy density, if the system has the freedom to adjust itself (e.g. by redistributing charge, as is possible in the present capacitor).

Note: "permittivity" is associated with words such as "permission" or "permit".

Problem 2 Dielectric charged cylinder

$r_{cyl} = 1\ \text{cm} \qquad \varepsilon_r = 5.0 \qquad \rho = +10^{-9}\ \frac{C}{cm^3}$

(a) Experimental setup:



Maxwell 1 $\qquad \oint_A \vec{D} \cdot d\vec{A} = \int_V \rho \, dV$

Case 1: $\qquad r \leq r_{cyl}$

$$\Rightarrow \quad D_r \, 2\pi r z = \rho \, \pi r^2 z$$

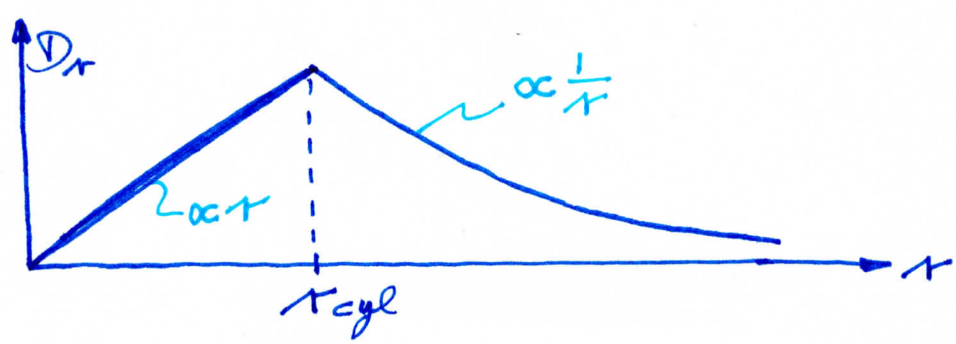
$$\Rightarrow \quad D_r = \frac{1}{2} \rho r$$

Case 2: $\qquad r \geq r_{cyl}$

$$\Rightarrow \quad D_r \, 2\pi r z = \rho \, \pi r_{cyl}^2 z$$

$$\Rightarrow \quad D_r = \frac{1}{2} \rho \frac{r_{cyl}^2}{r}$$

Sketch:



(b) Electric field $\Rightarrow E_r$

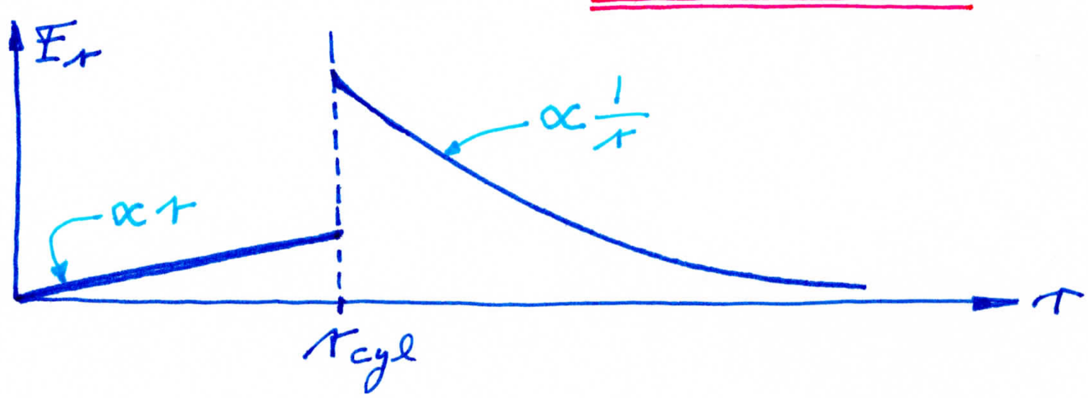
Case 1: $r \leq r_{cyl}$

$$\Rightarrow D_r = \frac{1}{2}\rho r \Rightarrow \underline{\underline{E_r}} = \frac{D_r}{\varepsilon_{cyl}} = \underline{\underline{\frac{1}{2\,\varepsilon_{cyl}}\rho r}}$$

Case 2: $r \geq r_{cyl}$

$$\Rightarrow D_r = \frac{1}{2}\rho \frac{r_{cyl}^2}{r} \Rightarrow \underline{\underline{E_r}} = \frac{D_r}{\varepsilon_{air}} = \underline{\underline{\frac{1}{2\,\varepsilon_{air}}\rho \frac{r_{cyl}^2}{r}}}$$

Sketch:



(c) D_r is maximal at r_{cyl}

$$\Rightarrow \underline{\underline{D_r}} = \frac{1}{2}\rho r_{cyl} = \frac{1}{2}\; 10^{-9}\frac{C}{cm^3}\; 1\,cm$$

$$= 0.5 \times 10^{-9}\frac{C}{cm^2} = \underline{\underline{0.5 \times 10^{-5}\frac{C}{m^2}}}$$

E_r is maximal at $r = r_{cyl}$ (just outside the cylinder, i.e. in air)

$$\Rightarrow \quad E_r = \frac{1}{2\,\varepsilon_{air}}\,\rho\,\frac{r_{cyl}^2}{r}$$

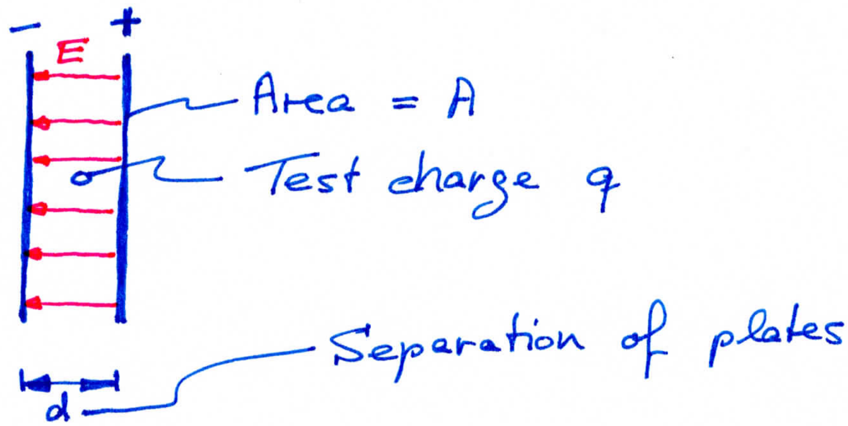
$$= \frac{1}{2}\,\frac{1}{8.85\times 10^{-12}}\,\frac{Vm}{As}\;10^{-9}\,\frac{C}{cm^3}\;\frac{(1\,cm)^2}{1\,cm}$$

$$= \frac{1}{2}\,\frac{1}{8.85}\,10^{3}\,\frac{V\,m}{cm^2} = 5.65\times 10^{5}\,\frac{V}{m}$$

Problem 3 Capacitor $A = 10\,cm^2$ $d = 0.0\cancel{0}1\,mm$

$q = 10^{-9}C$ = Test charge

(a)



$$\vec{F} = q\vec{E} = 10^{-9}C\,\frac{100V}{10^{-5}m} = 10^{-2}\,\frac{Nm}{m} = \underline{10^{-2}N}$$

Direction of $\vec{F}$: Along $(-x)$ direction

└→ negative x

(b) Calculate Q of capacitor

$$Q = CV = \varepsilon_r\varepsilon_0\frac{A}{d}V = 1.0 \quad 8.85\times10^{-12}\frac{As}{Vm}\;\frac{10cm^2}{10^{-5}m}\;100V$$

$$= 8.85\times10^{-4}\;\frac{C\,cm^2}{m^2} = 8.85\times10^{-8}\,C$$

Voltage of capacitor after dielectric fluid $(\varepsilon_r = 5)$ fills the gap d:

$$V = \frac{Q}{C} = \frac{Q\quad d}{\varepsilon_r\varepsilon_0\,A} = \frac{8.85\times10^{-8}C\quad Vm\quad 10^{-5}m}{5\times8.85\times10^{-12}As\quad 10cm^2}$$

$$= \frac{1}{5}\,10^{-2}\,V\frac{m^2}{cm^2} = \frac{1}{5}\,10^2\,V = 20V$$

Electric field in capacitor:

$$E = \frac{20V}{0.01mm} = \frac{20V}{10^{-5}m} = 20 \times 10^{5} \frac{V}{m}$$

Force on test charge:

$$\underline{\underline{\vec{F}}} = q\vec{E} = 10^{-9}C \times 20 \times 10^{5} \frac{V}{m} = 2 \times 10^{-3} \frac{Nm}{m}$$

$$= \underline{\underline{2 \times 10^{-3} N}}$$

Direction of $\vec{F}$: Along $\underline{\underline{-x}}$-$\underline{\underline{\text{direction}}}$

Problem 4

(a) True. A grounded metal sphere acts as a Faraday cage that screens electric fields.

(b) False. A good size of an antenna is $\lambda/2$ or $\lambda/4$, but not $\gg \lambda$.