Exam-02 – Electrostatics[1]

1. Consider a parallel plate capacitor with plate area A and distance d between the plates. For an area A_1 = ½ A the gap between the plates is completely filled with a dielectric material (ε_{r1} = 10). For an area A_2 = ½ A the gap between the plates is completely filled with air (ε_{r2} = 1). The capacitor carries the total charge Q; the Areas A_1 and A_2 carry the charge Q_1 and Q_2, respectively. Furthermore, $Q = Q_1 + Q_2$.
   (a) Draw the experimental setup and label all objects. Assume that we force all charge into area A_1 so that $Q = Q_1$ and $Q_2 = 0$. Calculate the energy carried by the capacitor. Next, assume that we force all charge into area A_2 so that $Q = Q_2$ and $Q_1 = 0$. Calculate the energy carried by the capacitor. Which of the two calculated energies is lower?
   (b) Given the results obtained above, and assuming that the charge Q can freely distribute between the areas A_1 and A_2, which area would you expect to carry most charge?
   (c) Give a symbolic expression for the capacitance of the capacitor. Calculate the ratio Q_1 / Q_2. Is the calculated distribution of Q between areas A_1 and A_2 consistent with your expectation?
   (d) Given that $\varepsilon = \varepsilon_r \varepsilon_0$ is called "permittivity", can you explain why it is called that way?

2. A uniformly charged dielectric cylinder of infinite length having a radius of 1 cm and ε_r = 5.0 is positively charged with a charge density of $+10^{-9}$ C/cm^3. The cylinder is surrounded by air.
   (a) Draw the experimental setup and label all objects. Derive a symbolic expression for the electric flux density ***D*** (i) inside and (ii) outside the cylinder. Sketch ***D***(r).[2]
   (b) Give a symbolic expression for the electric field ***E*** (i) inside and (ii) outside the cylinder. Sketch ***E***(r).
   (c) Calculate ***D*** and ***E*** at the distance from the symmetry axis where ***D*** and ***E*** are maximal.

3. A voltage of 100 V is applied to two parallel capacitor plates (area 10 cm^2) that are separated by a 0.01 mm wide air gap. Assume that the LHS and RHS[3] plate is negatively and positively charged, respectively (the left-to-right direction is the x-direction). Assume that a test charge with $q = 10^{-9}$ C is placed in the gap.
   (a) Draw the experimental setup and label all objects appropriately. Calculate the magnitude and the direction of the force ***F*** acting on the test charge.
   (b) Next, the capacitor is disconnected from the 100 V power supply. Then the air gap is filled with a dielectric fluid having a relative dielectric constant of ε_r = 5.0. Calculate the magnitude and the direction of the force ***F*** acting on the test charge.

4. Determine if the following statements are (i) true or (ii) false. Explain your answers.
   (a) If a point charge is located inside a grounded metal sphere, then there will not be an electric field outside the metal sphere.
   (b) An antenna having a spatial dimension (size) that is >> λ (λ = wavelength of an electromagnetic signal received by the antenna) will have good reception.

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[1] Always show your work, always give units, and write your name on first page.
[2] Bold italic letters are meant to be vectors, i.e. ***D*** is a vector.
[3] LHS = Left hand side; RHS = Right hand side