Exam-03 - Magnetostatics - Solution

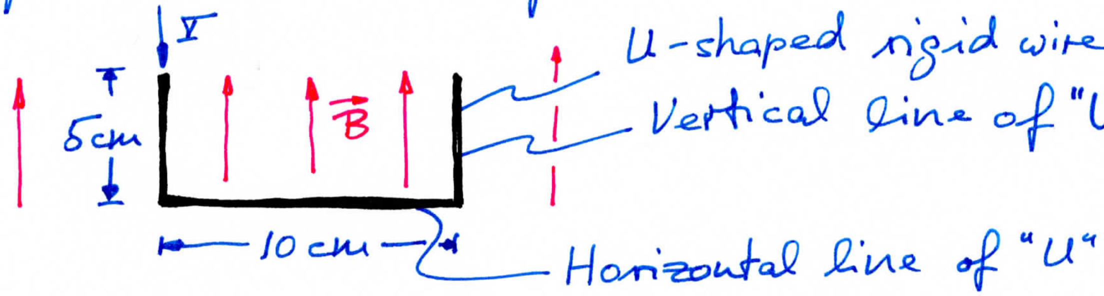
Problem 1

Uniform $\vec{B}$ field $B = 1.0\,T$

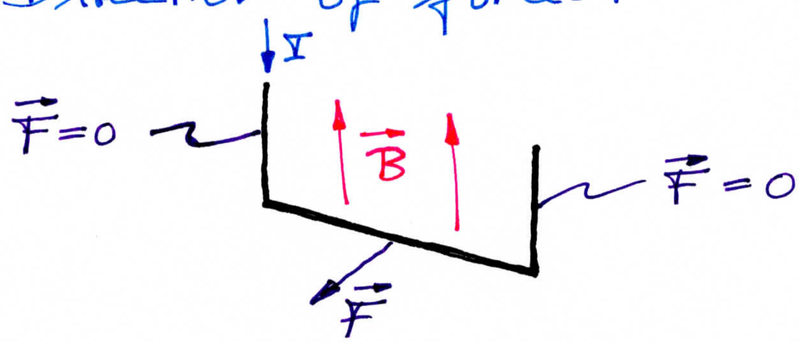
Rigid U-shaped wire ⇒ ⊔

Current = I = 10 A

(a) Experimental setup



Direction of forces:



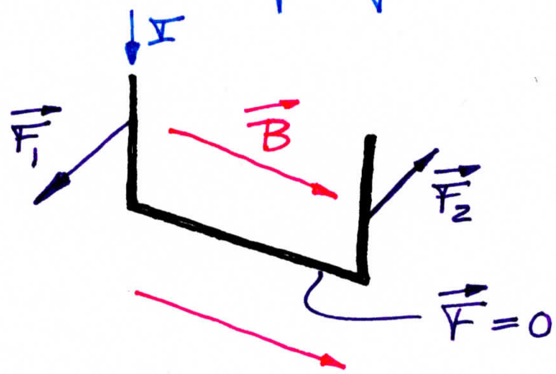
Magnitude of force:

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

$$= 10\,A \;\; 0.1\,m \;\; 1.0\,T$$

$$= 1\;Am\,\frac{Vs}{m^2} = 1\;\frac{VAs}{m} = 1\;J/m = \underline{\underline{1\,N}}$$

(b) Direction of forces

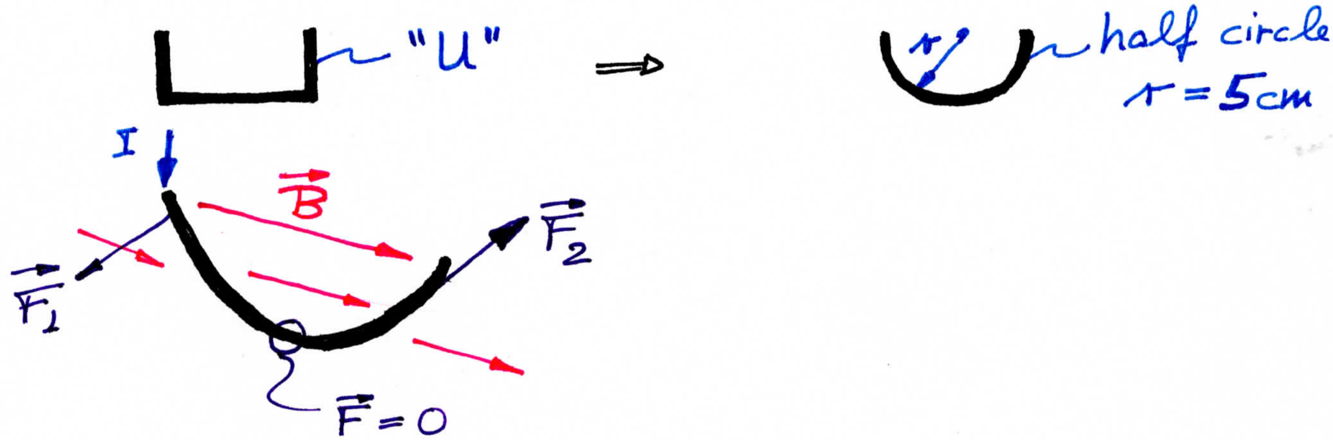


⇒ Forces cause rotation of "U" but no translational (linear) movement.

$$F_1 = I\,\vec{l} \times \vec{B} = \underline{\underline{0.5\,N}}$$

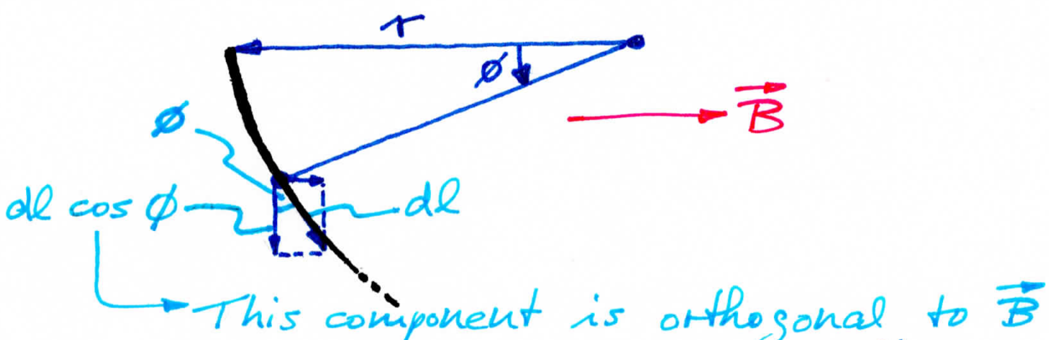
$$F_2 = \underline{\underline{0.5\,N}}$$

(c) Replacement of "U" by half circle



⇒ Current flow not orthogonal to $\vec{B}$

⇒ For cross product, only orthogonal component counts



$$|\vec{F}| = I B \int \cos\phi \underbrace{dl}_{r\,d\phi} = I B \int_{\phi=0}^{\pi/2} \cos\phi \; r \, d\phi$$

$$= I B r \left[\sin\phi \right]_0^{\pi/2} = I B r \left[1 - 0 \right]$$

$$= I B r$$

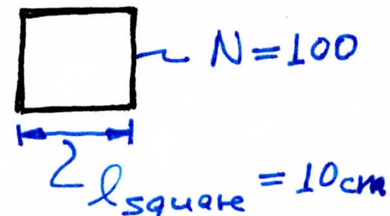
⇒ This is the same value (since $r = l = 5cm$) as obtained for the "U" shape.

⇒ Same magnitude and same direction of $\vec{F}$

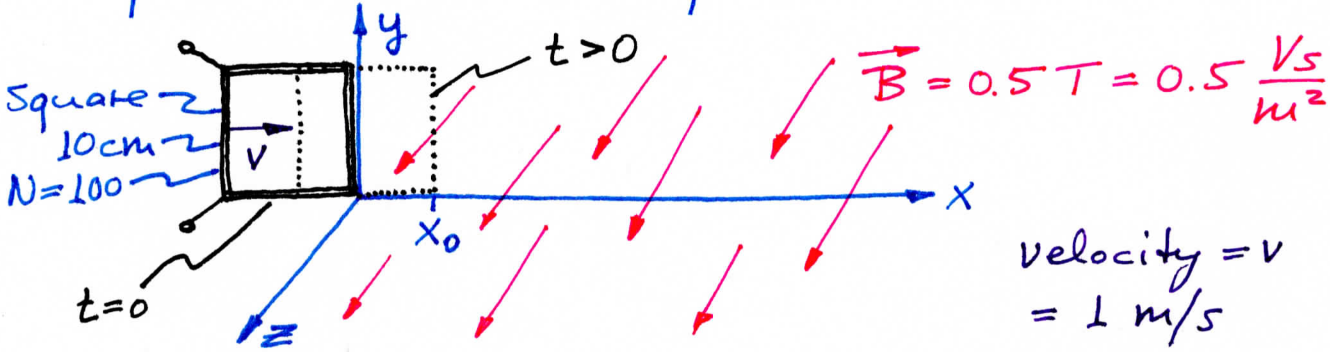
Problem 2

Square-shaped wire loop

$B = 0.5\ T$



(a) Experimental setup at $t=0$



Maxwell 2 (Faraday's law)

$$V_{ind} = -N \dot{\Phi}_m = -N \frac{d}{dt} \vec{B} \cdot \vec{A}$$

($\vec{B}$, $\vec{A}$: parallel vectors)

$$= -N B \frac{d}{dt} A(t)$$

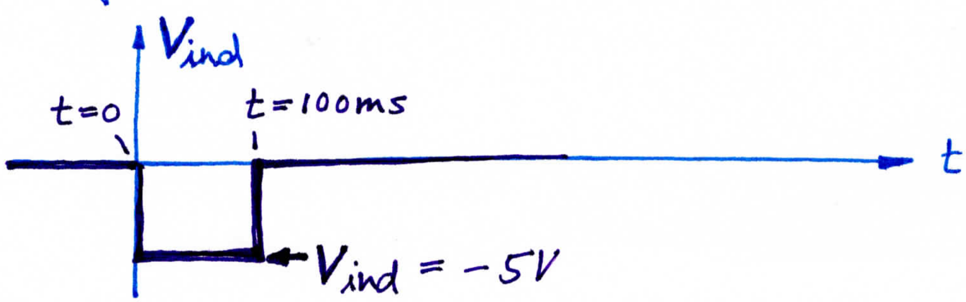
$$= -N B \frac{d}{dt} l_{square} x_0 \qquad (x_0 = vt)$$

$$= -N B \frac{d}{dt} l_{square} v t$$

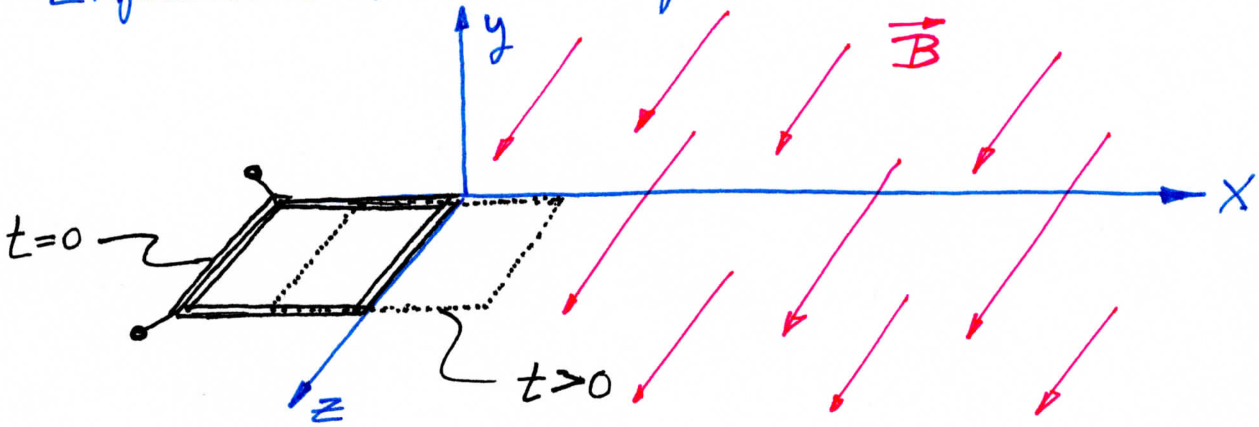
$$= -N B l_{square} v = -100 \; 0.5 \frac{Vs}{m^2} \; 0.1 m \; 1 \frac{m}{s}$$

$$= -5V$$

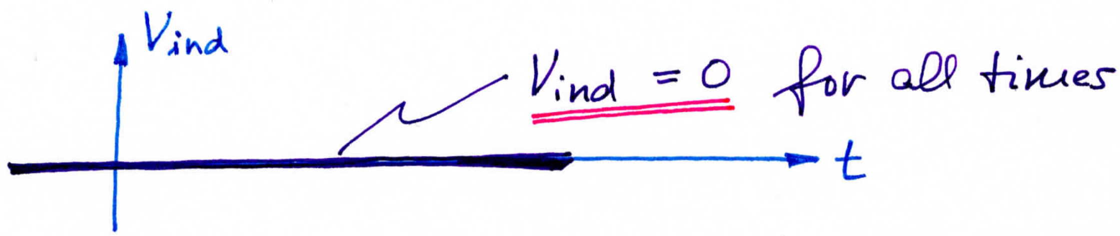
Plot of V_{ind} versus time



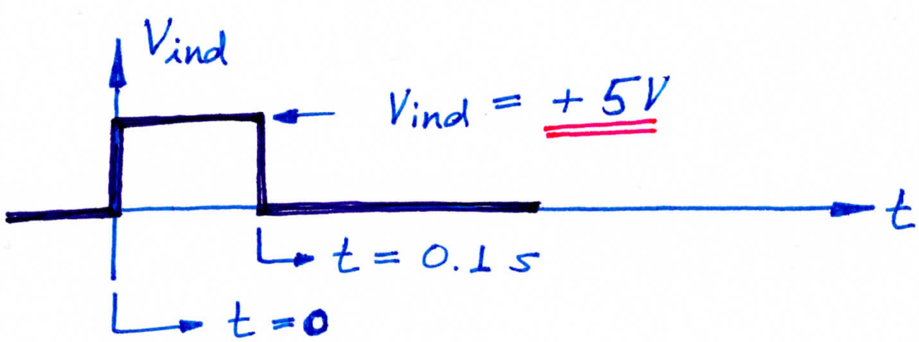
(b) Experimental setup



Plot of V_{ind} versus time

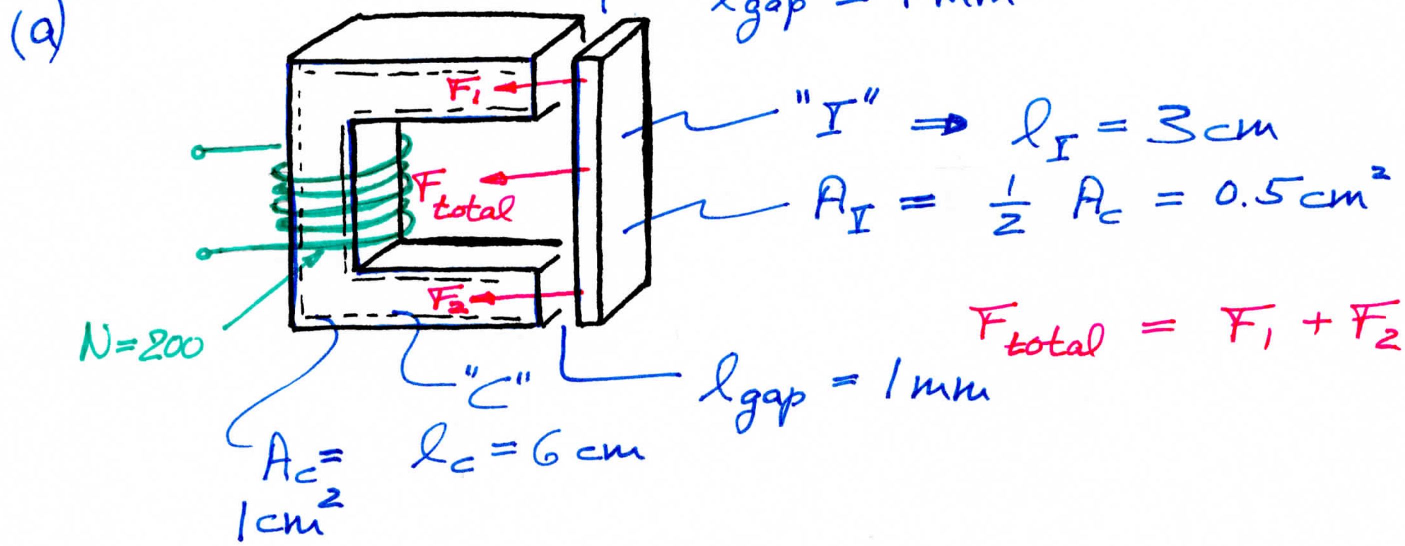


(c) Plot of V_{ind} versus time



Problem 3 Ferromagnetic core with "C" and "I"-shaped part

(a)



Total force $F_{total} = F_1 + F_2 = 0.1\,N$

$$\Rightarrow F_1 = F_2 = 0.05\,N$$

Magnetic force $|F| = +\frac{1}{2} \frac{B_{gap}^2}{\mu_0} A_{gap}$, $A_{gap} \rightarrow = A_c$

$$\Rightarrow B_{gap} = \sqrt{2 \mu_0 F / A}$$

$$= \sqrt{2 \times 12.56 \times 10^{-7} \frac{Vs}{Am}\, 0.05 \frac{CV}{m} / 10^{-4} m^2}$$

$$= \sqrt{0.001256 \frac{Vs}{Am} \frac{As}{m} \frac{V}{m^2}}$$

$$= 0.0355 \frac{Vs}{m^2} = \underline{\underline{35.5\,mT}}$$

(b) Calculation of current

Maxwell 4 : $\oint_C \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{A}$

Maxwell 4:

$$H_c l_c + 2 H_{gap} l_{gap} + H_I l_I = N I$$

Materials eqns.:

$$B_c = \mu H_c \qquad B_I = \mu H_I \qquad B_{gap} = \mu_0 H_{gap}$$

Φ_m = const.

$$\underbrace{B_c A_c = B_{gap} A_{gap}}_{\Rightarrow B_{gap} = B_c} \qquad \underbrace{B_c A_c = B_I A_I}_{\text{with } A_I = \frac{1}{2} A_c \Rightarrow B_c = \frac{1}{2} B_I}$$

⇒ We have 6 eqns. and 6 unknowns

⇒ We can solve the problem

$$\frac{B_c}{\mu_{core}} l_c + 2 \frac{B_c}{\mu_0} l_{gap} + \frac{2 B_c}{\mu_{core}} l_I = N I$$

Using $B_{gap} = B_c$ and solving for I yields

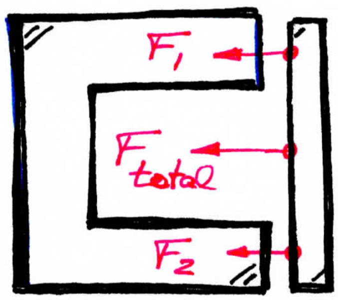
$$I = \frac{1}{N} B_{gap} \left(\frac{l_c}{\mu_{core}} + 2 \frac{l_{gap}}{\mu_0} + 2 \frac{l_I}{\mu_{core}} \right)$$

$$= \frac{1}{N} B_{gap} \frac{1}{\mu_0} \left(\frac{l_c}{\mu_{r,core}} + 2 l_{gap} + 2 \frac{l_I}{\mu_{r,core}} \right)$$

$$= \frac{1}{200}\, 0.0355 \frac{Vs}{m^2} \frac{Am}{12.56 \times 10^{-7} Vs} \left(\frac{0.06m}{1000} + 0.002m + \frac{0.06m}{1000} \right)$$

$$= \frac{1}{2}\, 0.0355 \frac{A}{12.56 \times 10^{-5}} (0.00201) = \underline{\underline{0.284\,A}}$$

(c) Magnitudes and direction of all forces



$F_1 = F_2 = 0.05\ N$

$F_{total} = F_1 + F_2 = 0.1\ N$

The C-shaped part is subject to the same-magnitude forces, as the I-shaped part, just in the opposite direction.