Exam-03 – Magnetostatics[1]

1. Consider a spatially uniform magnetic flux density[2] ***B*** = 1.0 T and an approximately "U-shaped" wire that is injected with a current I = 10 A. The "U-shaped" wire has a horizontal bottom line (10 cm long) and the two vertical side lines (each 5 cm long). There are two 90° angles where the horizontal bottom line of the "U" meets the vertical side lines of the "U".
   (a) Assume that the uniform magnetic flux density is directed upwards and parallel to the vertical side lines of the "U-shaped" wire. Draw the experimental setup and label all objects appropriately. Calculate magnitude of the force(s) ***F*** acting on the wire.[3] Show the directions of ***B***, I, and ***F*** by means of a drawing.
   (b) Next assume that the magnetic flux density is parallel to the horizontal bottom line of the "U-shaped" wire. Calculate magnitude of the force(s) acting on the wire. Show the directions of ***B***, I, and ***F*** by means of a drawing.
   (c) Next assume that the "U-shaped" wire is replaced by a "half-circle-shaped" wire with radius 5 cm. Give the directions of ***B***, I, and ***F***. Will the magnitude of the force(s) change?
2. A square-shaped wire loop (a winding with N = 100 turns) with side length 10 cm is moved along the x axis at a rate of v = 1 m/s. The plane formed by the wire loop is identical with the plane z = 0. At t = 0, the RHS[4] of the wire loop is located at x = 0. A uniform and time-invariant magnetic flux density of ***B*** = 0.5 T exists only for $x > 0$ and the ***B*** vector points in the z direction.
   (a) Draw the experimental setup at t = 0 in a cartesian coordinate system and label all objects. Calculate the voltage induced into the wire loop. Plot the induced voltage V_{ind} versus time and give quantitative values for all significant voltages and times.
   (b) The wire loop is turned by 90° so that the plane of the loop coincides with the plane y = 0. Starting at t = 0, the loop moves with v = 1 m/s. Draw the experimental setup. Plot V_{ind} versus time and give quantitative values for all significant voltages and times.
   (c) Assume that the wire loop is turned again in the same rotational direction as previously, again by 90°. Starting at t = 0, the loop moves with v = 1 m/s. Plot V_{ind} versus time and give quantitative values for all significant voltages and times.
3. A ferromagnetic circuit core (μ_r = 1000) consists of a "C-shaped" part (ℓ_C = 6 cm) and an "I-shaped" part (ℓ_I = 3 cm) so that the "I" perfectly fits the two ends of the "C". Two air gaps occur where the two ends of the "C" meet the "I" (ℓ_{gap} = 1 mm). The "C" and the "I" have cross sections of A_C = 1 cm² and A_I = 0.5 cm². The "C" has a wire winding with N = 200 and is injected with current I. The total force on the "I" is measured to be ***F*** = 0.1 N.
   (a) Draw the experimental setup. Calculate the magnetic flux density $\boldsymbol{B}_{gap}$ in the two gaps.[5]
   (b) Calculate the current I. (10 points)
   (c) Give magnitudes and directions of all forces ***F*** acting on the "I" by means of a drawing.

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[1] Always give units and show your work! Credit: 5 points per question unless noted otherwise.
[2] Vectors are indicated by bold italic font, i.e. ***B*** is a vector.
[3] Assume that the magnetic fields caused by the current I are much smaller than ***B*** = 1.0 T (and can be neglected).
[4] LHS = Left-hand side; RHS = Right-hand side.
[5] Neglect any ***B***-field fringing effects.