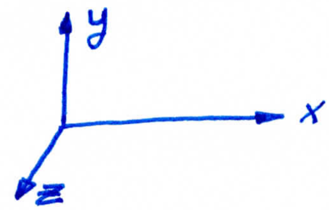
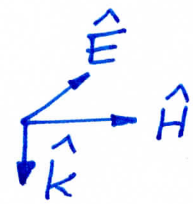
2017 - Electrodynamics - Exam 04 - Solution

Question 1 : EM wave with $\hat{E}$, $\hat{H}$, and $\hat{k}$

(a) Right-hand cartesian coordinate system

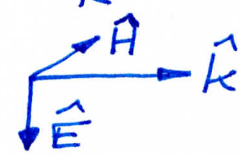


(i) $\vec{E} \parallel -\hat{z}$ $\quad \vec{H} \parallel \hat{x}$ $\quad \Rightarrow \hat{k} \parallel -\hat{y}$

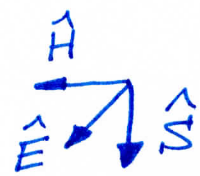


(ii) $\hat{k} \parallel -\hat{y}$ $\quad \vec{H} \parallel \hat{z}$ $\quad \Rightarrow \vec{E} \parallel \hat{x}$

(iii) $\hat{k} \parallel \hat{x}$ $\quad \vec{E} \parallel -\hat{y}$ $\quad \Rightarrow \vec{H} \parallel -\hat{z}$



(b) (i) $\vec{E} \parallel \hat{z}$ $\quad \vec{H} \parallel -\hat{x}$ $\quad \Rightarrow \vec{S} \parallel -\hat{y}$



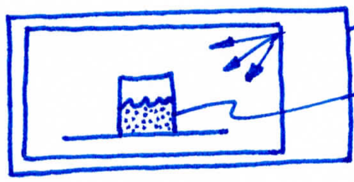
(ii) $\hat{k} \parallel -\hat{x}$ $\quad \vec{E} \parallel \hat{y}$ $\quad \Rightarrow \vec{S} \parallel \hat{k} \parallel -\hat{x}$

$\Rightarrow \vec{H} \parallel -\hat{z}$

(iii) $\vec{S} \parallel \hat{y}$ $\quad \vec{H} \parallel -z$ $\quad \Rightarrow \vec{E} \parallel \hat{x}$

Question 2 Microwave oven with cup filled with liquid ($\varepsilon_r = 80$) $f = 2.45\ GHz$

$\rho = 2\ \Omega m \Rightarrow \sigma = 0.5 \frac{1}{\Omega m}$



(a) For good conductor $\varepsilon'' \gg \varepsilon'$

For weak conductor $\varepsilon'' \ll \varepsilon'$

$$\varepsilon'' = \sigma/\omega = \frac{0.5\ A}{2\pi\ Vm\ 2.45\times10^{9}\ s^{-1}} = 3.25\times10^{-11}\ \frac{As}{Vm}$$

$$\varepsilon' = \varepsilon = 80\times8.85\times10^{-12}\ \frac{As}{Vm} = 7.08\times10^{-10}\ \frac{As}{Vm}$$

$\Rightarrow \varepsilon'' \ll \varepsilon' \Rightarrow$ Weak conductor

(b) Absorption constant (for weak conductor)

$$\alpha = \frac{1}{2}\sigma\sqrt{\mu/\varepsilon} = \frac{1}{2}\ 0.5\frac{A}{Vm}\sqrt{\frac{12.6\times10^{-7}\ Vs\ Vm}{80\times8.85\times10^{-12}\ Am\ As}}$$

$$= 10.5\ \frac{1}{m}$$

$\Rightarrow \frac{1}{\alpha}$ = Absorption length = 9.52 cm

↳ 1/e length

(c) We would like most of the EM radiation to be absorbed by the liquid. ⇒ Desirable relationship:

$$\boxed{2r \geq 1/\alpha}$$

Given that $\frac{1}{\alpha} = 9.52$ cm and $2r = 10$ cm, the desired relationship is satisfied.

(d) In the liquid, the phase constant is given by: $\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon}$

$$\beta = 2\pi \; 2.45\times10^{9} \frac{1}{s} \sqrt{12.6\times10^{-7} \frac{Vs}{Am} \; 80\times8.85\times10^{-12} \frac{As}{Vm}}$$

$$= 459.8 \frac{1}{m}$$

Wavelength in air:

$$c = \frac{\lambda}{T} = \lambda f \Rightarrow \lambda = c/f$$

$$\lambda = \frac{3\times10^{8} \; m \; s}{2.45\times10^{9} \; s} = 12.2 \text{ cm}$$

Question 3

Cell phone with $P_{RF} = 1.0\,W$

Magnitude of Poynting vector at distance of 20 cm = ?

$$|\vec{S}| = \frac{\text{Power}}{\text{Area}} = \frac{1W}{4\pi r^2} = \frac{1W}{4\pi\,(0.2m)^2} = 1.99\,\frac{W}{m^2}$$

Solar radiation on Earth $|\vec{S}| = 1\,\frac{kW}{m^2} = 1000\,\frac{W}{m^2}$

Comparison:

$$|\vec{S}_{\text{cell phone}}| \ll |\vec{S}_{\text{solar}}|$$

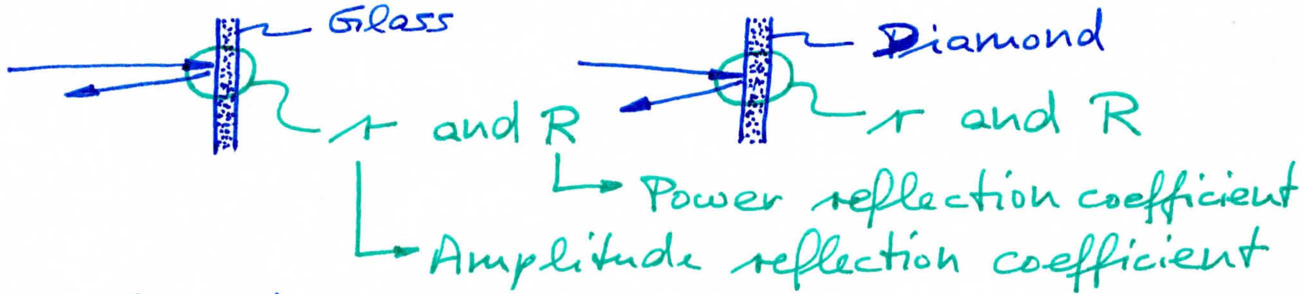
$$1.99\,\frac{W}{m^2} \ll 1000\,\frac{W}{m^2}$$

Question 4

SiO_2 → Glass → $n = 1.45$ $\varepsilon_r = 2.10$

Diamond → C → $n = 2.2$ $\varepsilon_r = 4.84$

(a)



Air - glass boundary:

$$r = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{1 - 1.45}{1 + 1.45} = \underline{\underline{-0.183}}$$

Air - diamond boundary:

$$r = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{1 - 2.2}{1 + 2.2} = \underline{\underline{-0.375}}$$

(b) Power reflection coefficient

Air - glass: $R = r^2 = 0.0335 = \underline{\underline{3.35\%}}$

Air - diamond $R = r^2 = 0.141 = \underline{\underline{14.1\%}}$

⇒ $\underline{\underline{R_{diamond} \gg R_{glass}}}$

⇒ This is consistent with experience

Question 5

(a) True

Electrons oscillate along direction of electric field of the incident EM wave.
The oscillation causes the reflected EM wave.
⇒ Incident and reflected EM wave have the same polarization.

(b) True

Example: