## Exam-02 – Electrostatics[1]

1. A uniformly charged dielectric pipe of infinite length, having an inner radius of $r_1$ = 1 cm, an outer radius of $r_2$ = 2 cm, and $\varepsilon_r$ = 5.0, is negatively charged with a charge density of $-10^{-9}$ C/m$^3$. The dielectric pipe is concentrically surrounded by a grounded metal cylinder with radius of $r_3$ = 5 cm. Air is located inside and outside the dielectric pipe.
   (a) Draw the experimental setup and label all objects. Derive symbolic expressions for the electric flux density ***D*** for $r = 0$ to $r = \infty$. Sketch ***D***($r$).[2]
   (b) Give symbolic expressions for ***E*** for $r = 0$ to $r = \infty$. Sketch ***E***($r$).
   (c) Give the $r$ values where ***D*** and ***E*** are maximal. Calculate ***D*** and ***E*** at these values of $r$.

2. First, consider a ***two***-plate capacitor with the plates having an area $A$ and a separation $d$ with one plate carrying a charge $+Q_1$ and the other plate carrying the charge $-Q_1$. Second, consider a ***three***-plate capacitor with all plates having an area $A$ and a separation $d$. The middle plate carries the charge $+2Q_1$ and the outer two plates each carry the charge $-Q_1$. The two outer plates are electrically connected (shorted). Air is located between all plates.
   (a) Draw the two capacitors and label all objects. Derive the electric field ***E*** in the ***two***-plate capacitor using Maxwell's 1st equation.
   (b) Derive the electric field ***E*** in the ***three***-plate capacitor using Maxwell's 1st equation. Are the ***E*** values identical for both capacitors? Are the voltages identical for both capacitors?
   (c) Determine the capacitances of the two capacitors ($C_{2\text{-plate}}$ and $C_{3\text{-plate}}$) using $C = Q / V$.
   (d) Compare the two capacitance values. Is one of the capacitors ($C_{2\text{-plate}}$ or $C_{3\text{-plate}}$) more favorable in terms of materials usage? Explain your answer.

3. A metal sphere (very small radius $r_0$ = 1 nm = $10^{-9}$ m) carries the charge $Q$ and is located in free space ($\varepsilon_r$ = 1.0). Consider a concentric spherical shell ($\varepsilon_r$ = 1.0) surrounding the metal sphere having a radius $r$ and a shell thickness d$r$, with d$r \neq 0$ and d$r << r$.
   (a) Draw the experimental setup and label all objects. Calculate the electric field surrounding the metal sphere.
   (b) The electric field energy density is given by (½) $\varepsilon$ ***E***$^2$. Calculate the electric field energy contained inside the thickness d$r$ of the shell.
   (c) Calculate the electric field energy within the entire space surrounding the metal sphere.
   (d) Calculate the numerical value of the electric field energy for $Q = 10^{-10}$ C.
   (e) Consider two metal sphere charges $+Q$ and $-Q$. Calculate the sum of the electric field energies (i) if the two charges are separated by a large distance ("Configuration 1"), and (ii) if the two charges would be located in the exactly same location ("Configuration 2").
   (f) What is the energy gained (i.e. the change in energy) by moving the two point charges from "Configuration 1" to "Configuration 2"?

4. Determine if the following statements are (i) true or (ii) false. Explain your answers.
   (a) The electric field energy density has the unit J/m$^3$.
   (b) The electric flux density ***D*** can be smaller than the electric field ***E***, that is, ***D*** < ***E***.

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[1] Always show your work, always give units, and write your name on first page.
[2] Bold italic letters are meant to be vectors, i.e. ***E*** and ***D*** are vectors.