Exam-03 – Magnetostatics[1]

1. Consider an inductor made of a magnetic circuit made of a ferromagnetic material (μ_r = 1000) that is shaped like a toroid (doughnut). The toroid inductor has a center length of ℓ_{core} = 20 cm and a cross-sectional area of A_{core} = 1 cm^2. The toroid inductor is equipped with a winding having N = 150 turns, injected with a current of I = 3 A.
   - (a) Draw the experimental setup and label all objects. Calculate the numerical value of the magnetic field in the core, $\boldsymbol{H}_{core}$.
   - (b) Derive a symbolic expression for the magnetic-field energy stored in the magnetic circuit.
   - (c) Calculate the numerical value of the magnetic field energy stored in the core.
   - (d) The energy stored in the magnetic circuit is given by $E = (½) LI^2$ where L is the inductance of the inductor. Derive a symbolic expression for the inductance L of the inductor.
   - (e) Give the numerical value of the inductance L of the inductor.
   - (f) Next consider that the toroid is cut with a saw through its cross-sectional area so that an air gap with length 1 mm is created. Due to the cut, the length of the core material shrinks to 19.9 cm. Draw the experimental setup. Derive symbolic expressions for $\boldsymbol{H}$ and $\boldsymbol{B}$ in the core material and the air gap.
   - (g) Calculate the numerical value of $\boldsymbol{H}_{core}$ and $\boldsymbol{H}_{gap}$.
   - (h) Give the numerical value of the magnetic field energy stored in the inductor.
   - (i) Compare the numerical values of the magnetic field energy of the toroid inductor before and after the cut. What is your conclusion regarding magnetic forces acting at the air gap of the toroid inductor?
   - (j) Assume that the toroid inductor is driven by an AC current with frequency f = 60 Hz. A humming sound is heard. What is the frequency of the humming sound?

2. A toroid-shaped transformer, having a primary winding (N_1 = 300) and a secondary winding (N_2 = 200), uses a "soft magnetic material" (μ_r = 100) that has a very small coercive magnetic field $\boldsymbol{H}_{coercive}$ so that $\boldsymbol{H}_{coercive}$ is nearly zero. Furthermore, $\boldsymbol{B}_{remnant}$ = 0.4 T. The toroid core has a center length of ℓ_{core} = 10 cm and a cross-sectional area of A_{core} = 1 cm^2.
   - (a) Draw the experimental setup. Draw the hysteresis of the core material and indicate the values of $\boldsymbol{H}_{coercive}$ and $\boldsymbol{B}_{remnant}$.
   - (b) At a value $\boldsymbol{H}_{sat}$, the magnetic flux density $\boldsymbol{B}$ is equal to the remnant magnetic flux density $\boldsymbol{B}_{remnant}$. Draw the hysteresis of the magnetic material and denote the value of $\boldsymbol{H}_{coercive}$, $\boldsymbol{H}_{sat}$, and $\boldsymbol{B}_{remnant}$.
   - (c) Determine the numerical value of $\boldsymbol{H}_{sat}$.
   - (d) Determine the primary current I at which the magnetic field has a value of $\boldsymbol{H}_{sat}$.
   - (e) We can operate the transformer at a current value below the just-calculated one, or above the just-calculated current value. State the consequences of operating the transformer at current values higher than the one calculated above. Accordingly, what range of current values is desirable when operating the transformer?

[1] Always show your work, always give units, and write your name on first page. Bold italic letters are meant to be vectors, i.e. ***H*** is a vector.