Chapter 2-4. Equilibrium carrier concentrations

Equilibrium electron concentration is given by:

$$n_0 = \int_{E_C}^{E_{\text{top}}} g_C(E)\ f(E)\,\mathrm{d}E$$

Equilibrium hole concentration is given by:

$$p_0 = \int_{E_{\text{Bottom}}}^{E_V} g_V(E)\left(1 - f(E)\right)\mathrm{d}E$$

This integral cannot be solved in closed form, but we can make some approximations to get a closed-form solution.

Approximate solutions

When $E_C - E_F > 3kT$ or $E_F < E_C - 3kT$, the solution for the total free electron concentration can be expressed as:

$$n = N_c e^{-\left(\frac{E_C - E_F}{kT}\right)} \quad \text{where} \quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \qquad \text{Eq 2.16a}$$

When $E_F - E_V > 3kT$ or $E_F > E_V + 3kT$, the solution for the total free hole concentration can be expressed as:

$$p = N_v e^{-\left(\frac{E_F - E_V}{kT}\right)} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \qquad \text{Eq. 2.16b}$$

Effective density of states

N_c is called effective density of states in the conduction band
N_v is called effective density of states in the valence band

In Si:

$$N_C = 2.51 \times 10^{19} (m_n^*/m_0)^{3/2} \text{ cm}^{-3} = 2.8 \times 10^{19} \text{ cm}^{-3}$$

$$N_V = 2.51 \times 10^{19} (m_p^*/m_0)^{3/2} \text{ cm}^{-3} = 1.0 \times 10^{19} \text{ cm}^{-3}$$

What do you get when you multiply n with p?

The result gives an intrinsic property of the semiconductor.

Degenerate and non-degenerate semiconductors

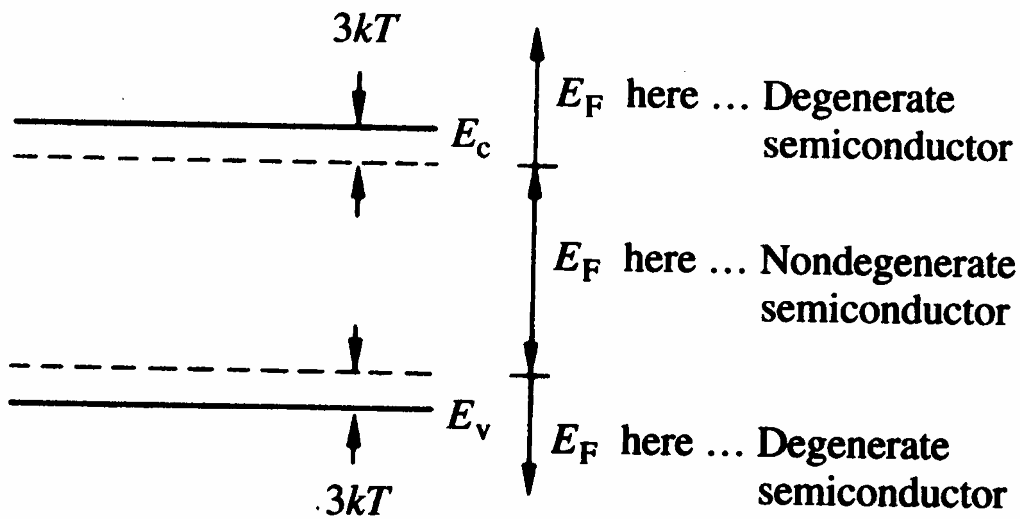


Figure 2.19

Alternative expression for *n* and *p*

Manipulation of the previous expressions will give us more useful expressions as given below:

$$n = n_i e^{(E_F - E_i)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

(2.19a) (2.19b)

$$np = n_i^2$$

(2.22)

Note from previous class that:

At $T = 300\text{K}$, $n_i = 2 \times 10^6\ \text{cm}^{-3}$ in GaAs
$= 1 \times 10^{10}\ \text{cm}^{-3}$ in Si
$= 2 \times 10^{13}\ \text{cm}^{-3}$ in Ge

} An intrinsic property

Show that

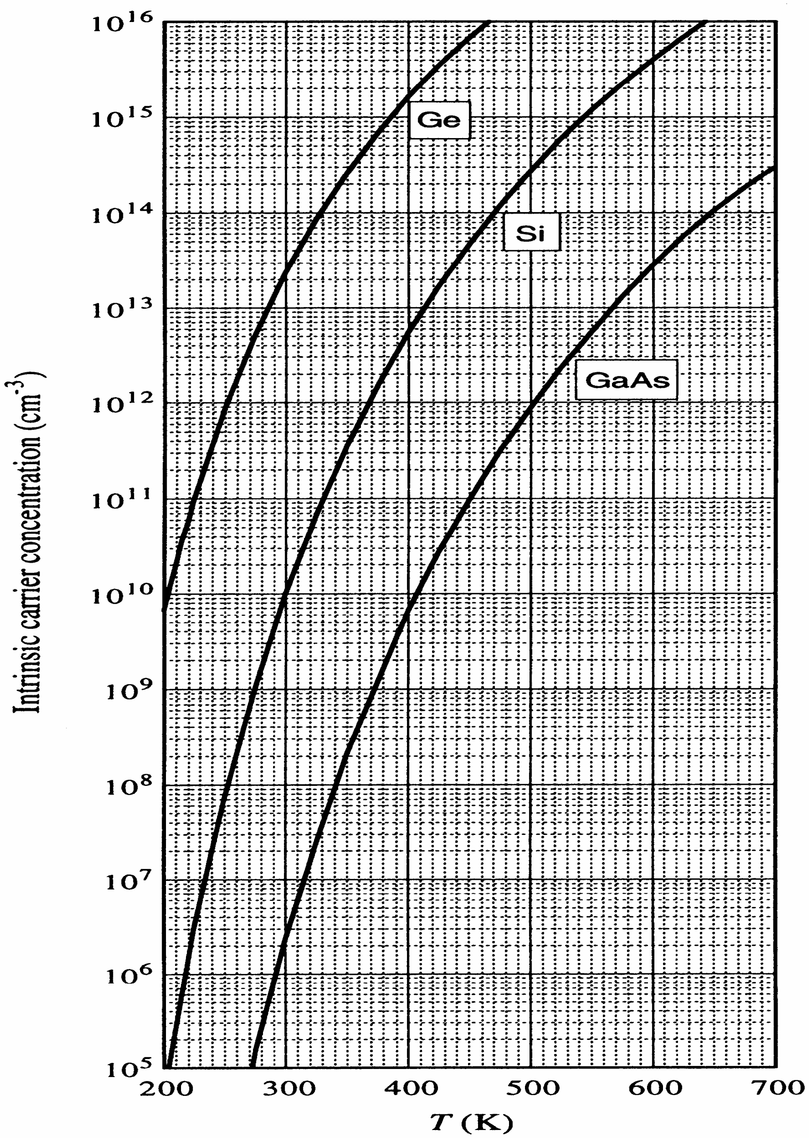
$$n = n_i e^{(E_F - E_i)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

(2.19a) (2.19b)

Hint: Start with Eq. 2.16 in text

Intrinsic carrier concentrations in Ge, Si, and GaAs vs. *T*



Si

T(°C)	n_i(cm^{-3})
0	8.86×10^8
5	1.44×10^9
10	2.30×10^9
15	3.62×10^9
20	5.62×10^9
25	8.60×10^9
30	1.30×10^{10}
35	1.93×10^{10}
40	2.85×10^{10}
45	4.15×10^{10}
50	5.97×10^{10}
300 K	1.00×10^{10}

GaAs

T(°C)	n_i(cm^{-3})
0	1.02×10^5
5	1.89×10^5
10	3.45×10^5
15	6.15×10^5
20	1.08×10^6
25	1.85×10^6
30	3.13×10^6
35	5.20×10^6
40	8.51×10^6
45	1.37×10^7
50	2.18×10^7
300 K	2.25×10^6

Figure 2.20

Charge neutrality relationship

So far, we haven't discussed any relationship between the dopant concentration and the free carrier concentrations. Charge neutrality condition can be used to derive this relationship.

The net charge in a small portion of a uniformly doped semiconductor should be zero. Otherwise, there will be a net flow of charge from one point to another resulting in current flow (that is against out assumption of thermal equilibrium).

Charge/cm^3 $= q\,p - q\,n + q\,N_D^+ - q\,N_A^- = 0$ or $p - n + N_D^+ - N_A^- = 0$
where N_D^+ = # of ionized donors/cm^3 and N_A^- = # of ionized acceptors per cm^3.

Assuming total ionization of dopants, we can write:

$$p - n + N_D - N_A = 0$$

(2.25)

Carrier concentration calculations

Assume a non-degenerately doped semiconductor and assume total ionization of dopants. Then,

$n\,p = n_i^2$; electron concentration $\times$ hole concentration = n_i^2

$p - n + N_D - N_A = 0$; net charge in a given volume is zero.

Solve for n and p in terms of N_D and N_A

We get:

$$(n_i^2 / n) - n + N_D - N_A = 0$$

$$n^2 - n\,(N_D - N_A) - n_i^2 = 0$$

Solve this quadratic equation for the free electron concentration, n.
From $n\,p = n_i^2$ equation, calculate free hole concentration, p.

Special cases

Intrinsic semiconductor:

$$N_D = 0 \text{ and } N_A = 0 \quad \rightarrow \quad p = n = n_i$$

Doped semiconductors where $| N_D - N_A | >> n_i$

$$n = N_D - N_A \;;\; p = n_i^2 / n \qquad \text{if} \quad N_D > N_A$$

$$p = N_A - N_D \;;\; n = n_i^2 / p \qquad \text{if} \quad N_A > N_D$$

Compensated semiconductor

$$n = p = n_i \text{ when } \; n_i >> | N_D - N_A |$$

When $| N_D - N_A |$ is comparable to $n_{i,}$, we need to use the charge neutrality equation to determine n and p.

Fermi level in Si at 300 K vs. doping concentration

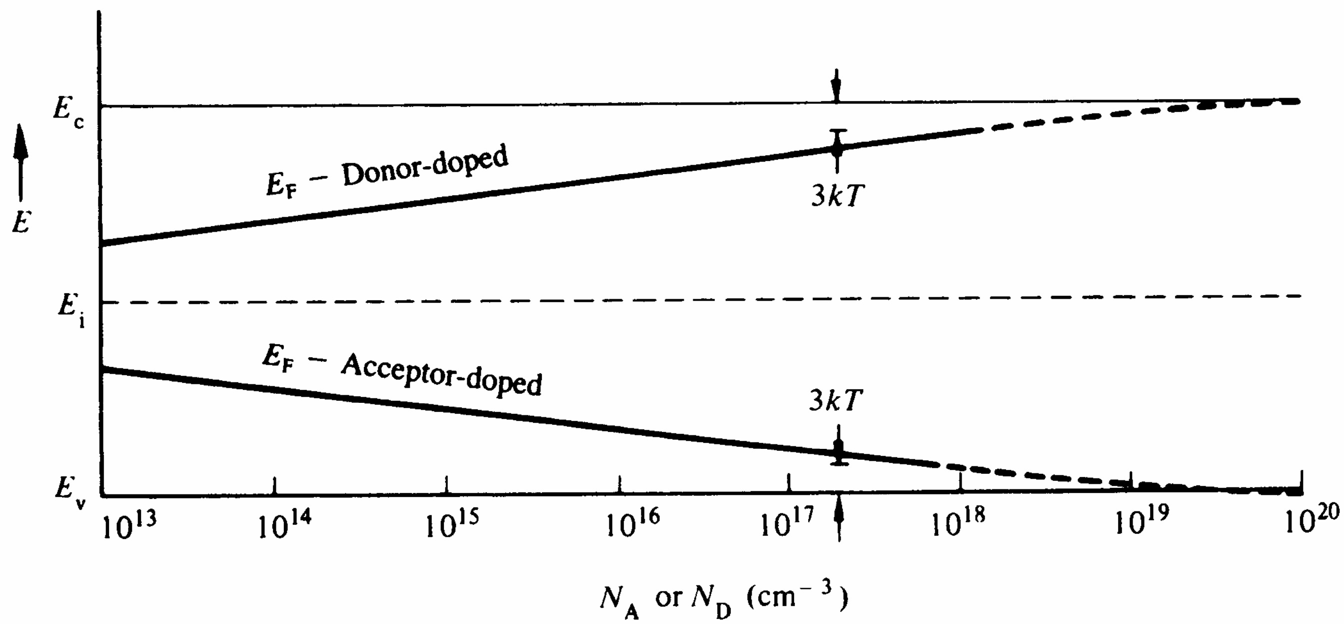


Figure 2.21

Majority-carrier temperature-dependence

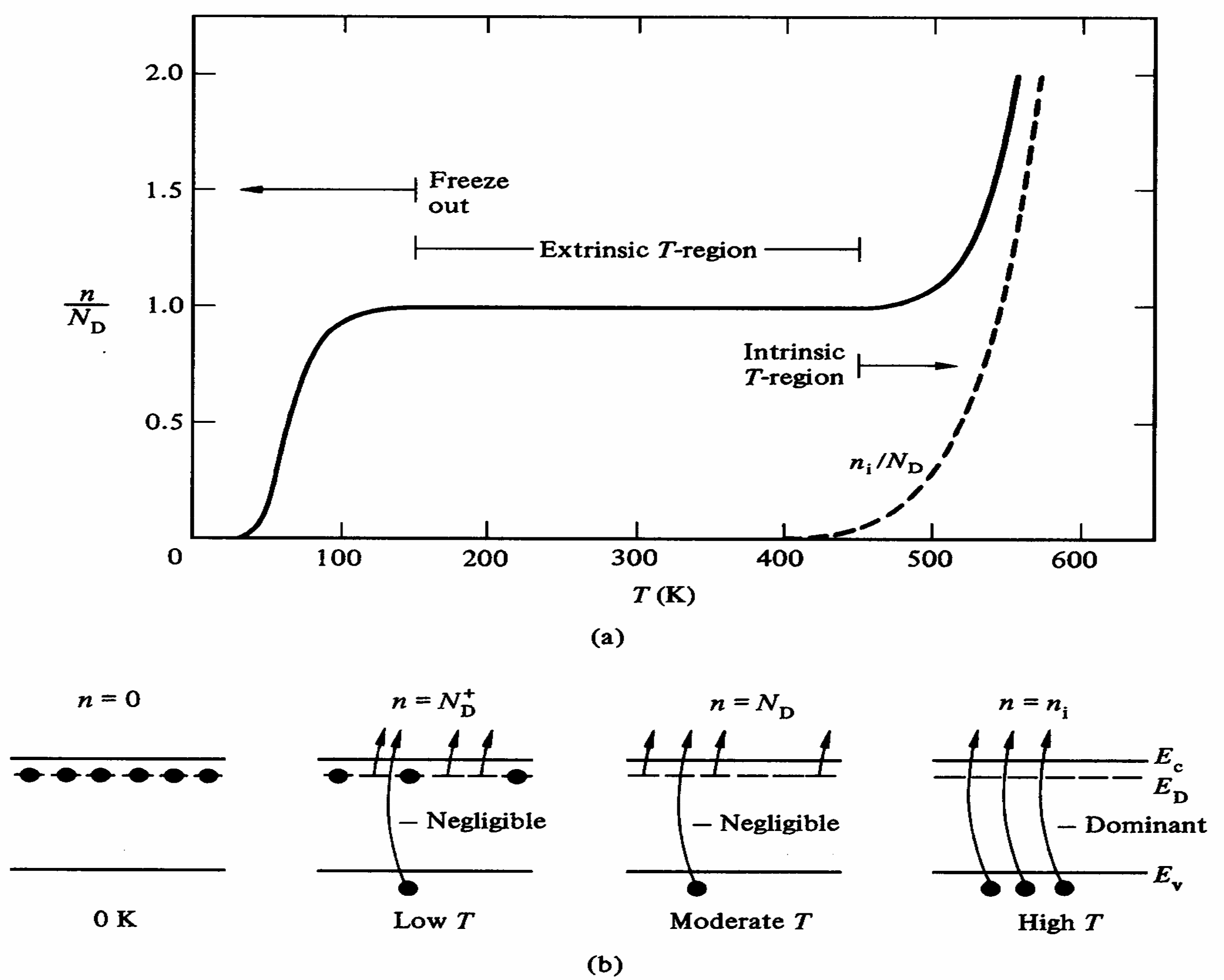


Figure 2.22

Equations to remember

$$n = n_i e^{(E_F - E_i)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

(2.19a) (2.19b)

$$np = n_i^2$$

(2.22)

$$p - n + N_D - N_A = 0$$

(2.25)

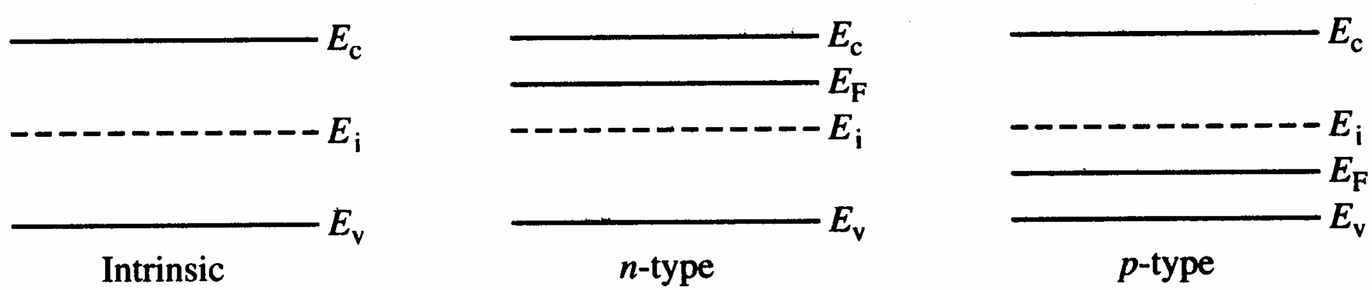


Figure 2.18

Note: Our interest was in determining *n* and *p*. Free carriers strongly influence the properties of semiconductors.

Example 1

(a) Consider Si doped with 10^{14} cm^{-3} boron atoms. Calculate the carrier concentration (n and p) at 300 K.

(b) Determine the position of the Fermi level and plot the band diagram.

(c) Calculate the the carrier concentration (n and p) in this Si material at 470 K. Assume that intrinsic carrier concentration at 470 K in Si is 10^{14} cm^{-3}. (Refer to figure 2.20).

(d) Determine the position of the Fermi level with respect to E_i at 470 K.

Example 2

Consider a Si sample doped with 3×10^{16} cm^{-3} of phosphorous (P) atoms and 10^{16} cm^{-3} of boron (B) atoms.

(a) Is the semiconductor n-type or p-type?

(b) Determine the free carrier concentration (hole and electron concentrations, or p and n) at 300K.

(c) Determine the position of the Fermi level and draw the band diagram.