Chapter 3-5. Continuity equation

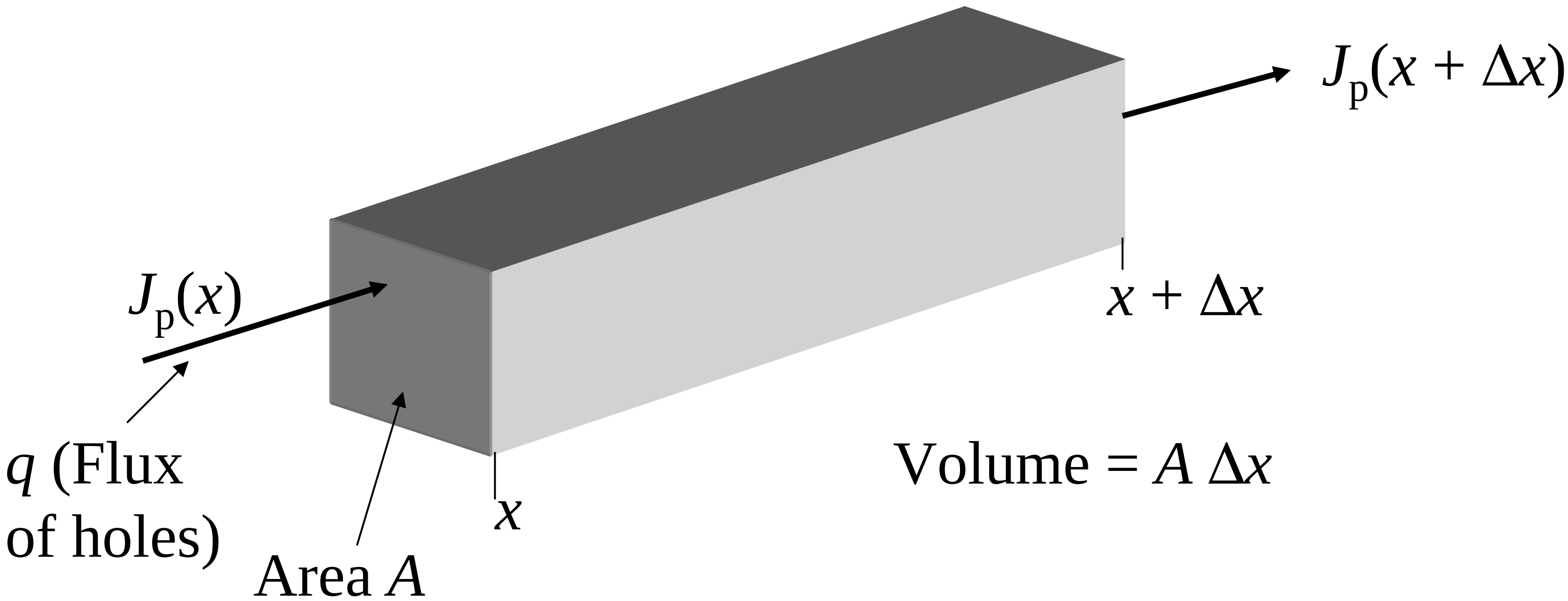
The continuity equation satisfies the condition that particles should be conserved! Electrons and holes cannot mysteriously appear or disappear at a given point, but must be transported to or created at the given point via some type of carrier action.

Inside a given volume of a semiconductor,

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial p}{\partial t}\right|_{\text{diffusion}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R}-\text{G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{others}\\ \text{light etc.}}}$$

There is a corresponding equation for electrons.

Continuity equation - consider 1D case



$$\frac{\partial p}{\partial t} A\Delta x = \frac{A}{q} J_{\mathrm{p}}(x) - \frac{A}{q} J_{\mathrm{p}}(x+\Delta x) + A\Delta x \left(\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal R-G}\\ \text{light etc.}}} \right)$$

$$= \frac{A}{q} J_{\mathrm{p}}(x) - \frac{A}{q}\left[J_{\mathrm{p}}(x) + \frac{\partial J_{\mathrm{p}}(x)}{\partial x}\Delta x \right] + A\Delta x \left(\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal R-G}\\ \text{light etc.}}} \right)$$

$$\frac{\partial p}{\partial t} A\Delta x = -\frac{A}{q}\frac{\partial J}{\partial x}\Delta x + A\Delta x\left(\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal R}-\text{G,}\\ \text{light etc.}}} \right)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J}{\partial x} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal R}-\text{G,}\\ \text{light etc.}}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J}{\partial x} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R}-\text{G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{others}\\ \text{light} \ldots}}$$

Continuity eqn. for holes

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J}{\partial x} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R}-\text{G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{others}\\ \text{light} \ldots}}$$

Continuity eqn. for electrons

These are general equations for one dimension, indicating that particles are conserved.

Minority carrier diffusion equations

Apply the continuity equations to minority carriers, with the following assumptions:

- Electric field $\mathcal{E} = 0$ at the region of analysis
- Equilibrium minority carrier concentrations are not functions of position, i.e., $n_0 \neq n_0(x)$; $p_0 \neq p_0(x)$
- Low-level injection
- The dominant R-G mechanism is thermal R-G process
- The only external generation process is photo generation

Minority carrier diffusion equations

Consider electrons (for p-type) and make the following simplifications:

$$J_{\mathrm{n}} = q\mu_{\mathrm{n}} n \mathcal{E} + qD_{\mathrm{n}} \frac{\partial n}{\partial x} \approx qD_{\mathrm{n}} \frac{\partial n}{\partial x}$$

$$\frac{\partial n}{\partial x} = \frac{\partial}{\partial x}\left(n_0 + \Delta n\right) = \frac{\partial \Delta n}{\partial x}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\text{thermal } R-G} = -\frac{\Delta n}{\tau_{\mathrm{n}}} \quad \text{and} \quad \left.\frac{\partial n}{\partial t}\right|_{\text{light etc.}} = G_{\mathrm{L}}$$

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial t}\left(n_0 + \Delta n\right) = \frac{\partial \Delta n}{\partial t}$$

Minority carrier diffusion equations

$$\frac{\partial \Delta n_{\mathrm{p}}}{\partial t} = D_{\mathrm{n}} \frac{\partial^2 \Delta n_{\mathrm{p}}}{\partial x^2} - \frac{\Delta n_{\mathrm{p}}}{\tau_{\mathrm{n}}} + G_{\mathrm{L}}$$

$$\frac{\partial \Delta p_{\mathrm{n}}}{\partial t} = D_{\mathrm{p}} \frac{\partial^2 \Delta p_{\mathrm{n}}}{\partial x^2} - \frac{\Delta p_{\mathrm{n}}}{\tau_{\mathrm{p}}} + G_{\mathrm{L}}$$

The subscripts refer to type of materials, either n-type or p-type.

Why are these called “diffusion equations”?
Why are these called “minority carrier” diffusion equations?

Example 1

Consider an n-type Si uniformly illuminated such that the excess carrier generation rate is G_L e-h pairs / (s cm^3). Use MCDE to predict how excess carriers decay after the light is turned-off.

$t < 0$: uniform ➔ d/dx is zero; steady state ➔ d/dt = 0
So, applying to holes, $\Delta p(t < 0) = G_L \tau_P$

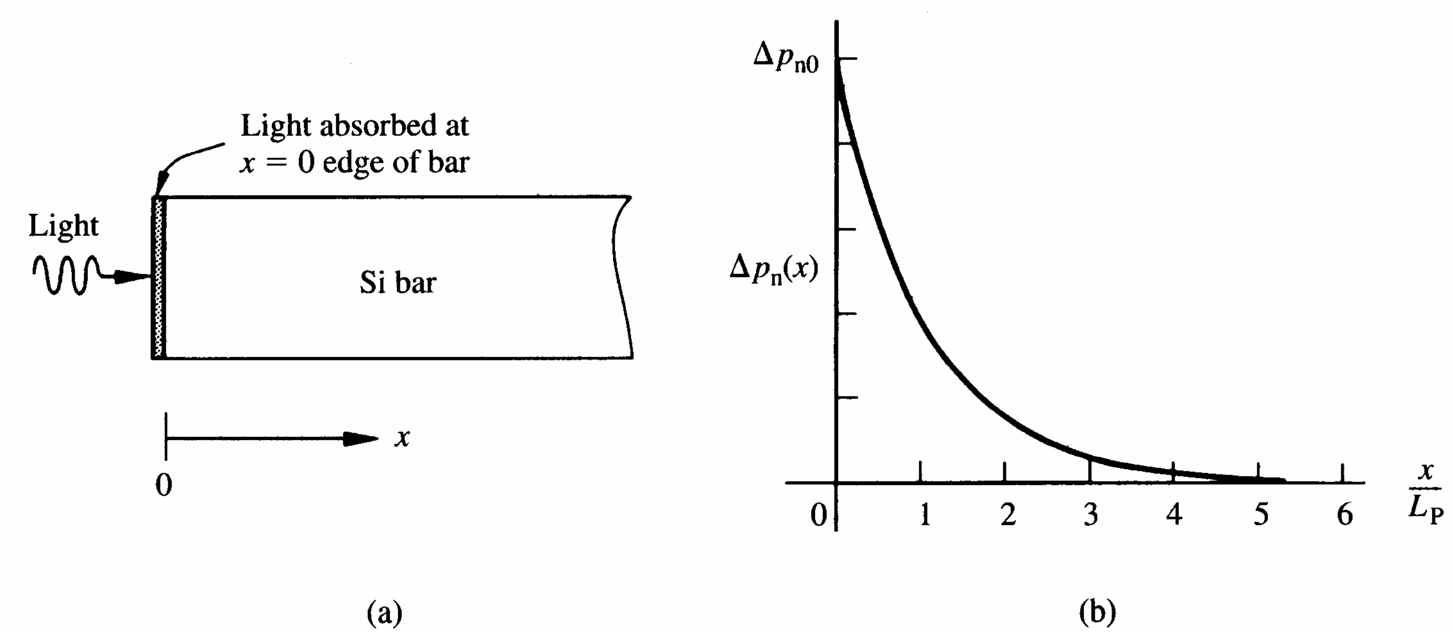
$t > 0$: $G_L = 0$; uniform ➔ d/dx = 0;

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{\Delta p_n}{\tau_p} \quad \text{so,} \quad \Delta p_n = \Delta p_n(0) \exp\left(-\frac{t}{\tau}\right)$$

$$\Delta p(t > 0) = G_L \tau_P \exp\left(-\frac{t}{\tau_p}\right) \quad \text{since} \quad \Delta p(0) = G_L \tau_p$$

Example 2

Consider a uniformly doped Si with $N_D = 10^{15}\ \text{cm}^{-3}$ is illuminated such that $\Delta p_{n0} = 10^{10}\ \text{cm}^{-3}$ at $x = 0$. No light penetrates inside Si. Determine $\Delta p_n(x)$. (see page 129 in text)



Solution is:

$$\Delta p_n(x) = \Delta p_{n0} \exp\left(-\frac{x}{L_p}\right) \quad \text{where} \quad L_p = \sqrt{D_p \tau_p}$$

Minority carrier diffusion length

In the previous example, the exponential falloff in the excess carrier concentration is characterized by a decay length, $\boldsymbol{L_p}$, which appears often in semiconductor analysis.

$L_p = (D_p \tau_p)^{1/2}$ associated with minority carrier holes in n-type materials

$L_n = (D_n \tau_n)^{1/2}$ associated with minority carrier electrons in p-type materials

Physically L_n and L_p represent the average distance minority carriers can diffuse into a sea of majority carriers before being annihilated. What are typical values for L_p and L_n?